# Global analysis of hadron-production data in $e^+e^-$ annihilation for determining fragmentation functions

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### Abstract

Fragmentation functions of pion, kaon, and nucleon are determined by global analyses of hadron-production data in  $e^+e^-$  annihilation. It is particularly important that uncertainties of the fragmentation functions are estimated for the first time. We found that light-quark and gluon fragmentation functions have large uncertainties, so that one should be careful in using these functions for hadron-production processes in heavy-ion collisions and lepton scattering. The analysis is extended to possible exotic hadron search by fragmentation functions. We found that internal structure of  $f_0(980)$ , such as  $s\bar{s}$  or tetraquark configuration, can be determined by noting differences between favored and disfavored fragmentation functions.

## 1. Introduction

Fragmentation functions are becoming important recently because they are used in describing hadron-production cross sections in lepton scattering and heavy-ion collisions. There are a number of parametrization studies; however, obtained functions are much different depending on analysis groups. It suggests that the functions should not be well determined at this stage although accurate determination could be crucial for discussing nucleon spin and quark-hadron matters. Considering this situation, we determined uncertainties of the fragmentation functions in Ref. [1]. The uncertainties are obtained in both leading order (LO) and next-to-leading order (NLO). This study is then extended to propose that the fragmentation functions can be used for searching exotic hadrons by noting differences between favored and disfavored functions [2]. In the following, these studies are explained.

# 2. Fragmentation functions and their uncertainties for $\pi$ , K, and p

The fragmentation function for a hadron h is defined by  $F^h(z,Q^2) = d\sigma(e^+e^- \to hX)/dz/\sigma_{tot}$ , where  $d\sigma(e^+e^- \to hX)/dz$  is the hadron production cross section and  $\sigma_{tot}$  is the total hadronic one. The variable  $Q^2$  is given by the center-of-mass energy squared  $(Q^2 = s)$ , and z is defined by  $z = E_h/(\sqrt{s}/2)$  with the hadron energy  $E_h$ . The fragmentation should be described by the sum of partonic contributions  $F^h(z,Q^2) = \sum_i \int_z^1 dy \, C_i(y,\alpha_s) D_i^h(z/y,Q^2)/y$ , where  $C_i(z,\alpha_s)$  is a coefficient function and  $D_i^h(z,Q^2)$  is the fragmentation function of the hadron h from a parton i. The fragmentation functions are usually parametrized in the form:  $D_i^h(z,Q_0^2) = N_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}$ , where  $N_i^h,\alpha_i^h$ , and  $\beta_i^h$  are the parameters which are determined by a  $\chi^2$  analysis of the  $e^+e^- \to hX$  data. Uncertainties of the determined functions are calculated by the Hessian method.

Determined LO and NLO fragmentation functions and their uncertainties are shown in Fig. 1 for  $\pi^+$  at  $Q^2=1~{\rm GeV}^2$ ,  $m_c^2$ , or  $m_b^2$  [1,3]. The uncertainties indicate that the functions are determined more accurately in the NLO in comparison with the LO. However, the uncertainties are large even in the NLO analysis particularly in the gluon and light-quark functions. Since these functions are used for investigating the origin of nucleon spin and properties of quark-hadron matters in hadron-production processes, the uncertainties should be taken into account in drawing any conclusion from hadron-production data. Our codes for calculating the determined functions and their uncertainties can be obtained from our web site [3].

Next, the obtained functions denoted as HKNS (Hirai, Kumano, Nagai, Sudoh) [1] are compared with other analysis results by KKP (Kniehl, Kramer, Pötter), Kretzer, AKK (Albino, Kniehl, Kramer), and DSS (de Florian, Sassot, Stratmann) for the pion  $((\pi^+ + \pi^-)/2)$  in Fig. 2 at  $Q^2=2$ , 10, or 100 GeV<sup>2</sup>. Although there are huge differences between the parametrizations in the gluon and strange-quark functions, all the curves are roughly within our uncertainty bands. It means that all the analyses are consistent with each other in spite of the large differences.

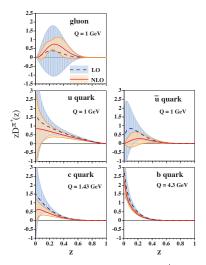


Fig. 1. Fragmentation functions for  $\pi^+$  and their uncertainties are shown in the LO and NLO.

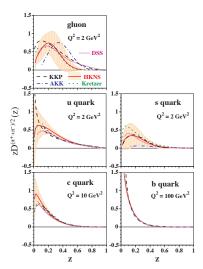


Fig. 2. Fragmentation functions for  $(\pi^+ + \pi^-)/2$  are compared with other NLO analysis results.

## 3. Exotic hadron search by fragmentation functions

We applied our analysis to exotic hadron search. There are two types in the fragmentation functions: favored and disfavored ones. The favored fragmentation means a fragmentation from a quark or an antiquark which exists in a hadron as a constituent in a quark model, and the disfavored means a fragmentation from a sea quark. Finding differences between these function, we should be able to find internal structure of hadrons [2]. As an example, we investigated  $f_0(980)$ , whose structure has been controversial. In a naive quark model, it may be interpreted as  $(u\bar{u} + d\bar{d})/\sqrt{2}$ ; however, it contradicts with experimental data of strong-decay and  $\gamma\gamma$  widths. According to lattice QCD analysis, its mass is too small to be interpreted as a glueball state. Therefore, remaining possibilities are  $s\bar{s}$  and tetraquark  $(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$  (or  $K\bar{K}$ ) states.

Type	Configuration	Status	Second moments	Peak positions
Nonstrange $q\bar{q}$	$(u\bar{u}+d\bar{d})/\sqrt{2}$	Unlikely	$M_s < M_u < M_g$	$z_u^{\rm max}>z_s^{\rm max}$
Strange $q\bar{q}$	$sar{s}$	Possible	$M_u < M_s \lesssim M_g$	$z_u^{\rm max} < z_s^{\rm max}$
Tetraquark (or $K\bar{K}$ )	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	Possible	$M_u \sim M_s \lesssim M_g$	$z_u^{\rm max} \sim z_s^{\rm max}$
Glueball	gg	Unlikely	$M_u \sim M_s < M_g$	$z_u^{\rm max} \sim z_s^{\rm max}$

Table 1. Possible  $f_0(980)$  configurations and their features in fragmentation functions at small  $Q^2$ .

If the  $f_0$  is an  $s\bar{s}$  state, the favored fragmentation from s is possible if a gluon is radiated from s, and then it splits into an  $s\bar{s}$  pair to form the  $f_0$  meson. This process is of the order of  $g^2$ , where g is the coupling constant. The disfavored fragmentation is proportional to  $g^3$  by considering a gluon radiation to have a color singlet  $f_0$  state, so that its probability is expected to be smaller than the favored one  $(M_u < M_s$  in second moments). In this way, we obtain characteristic features in second moments  $(M_i)$  of the fragmentation functions and in

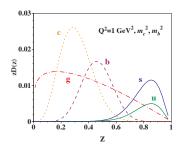


Fig. 3. Determined fragmentation functions of  $f_0(980)$ .

their functional peak positions  $(z_i^{\text{max}})$  at small  $Q^2$  ( $\sim 1 \text{ GeV}^2$ ) in Table 1 [2].

We show actual analysis results for the  $f_0$  fragmentation functions in Fig. 3. The s-and u-quark functions indicate valence-like structure which is peaked at large z. This fact suggests a tetraquark configuration. However, the relation  $M_u < M_s$  indicates an  $s\bar{s}$  type configuration. These conflicting results are obtained mainly because the functions are not accurately determined from the current  $e^+e^-$  data, although there is a possibility of an admixture state. If uncertainties of the functions are estimated, they are ten times larger than the functions themselves [2]. However, if accurate  $e^+e^- \to f_0 X$  data are obtained, internal structure should be determined.

#### References

- [1] M. Hirai, S. Kumano, T.-H. Nagai, and K. Sudoh, Phys. Rev. D75 (2007) 94009.
- [2] M. Hirai, S. Kumano, M. Oka, and K. Sudoh, arXiv:0708.1816 [hep-ph].

 $[3] \quad A \ code \ for \ calculating \ the \ fragmentation \ functions \ and \ their \ uncertainties \ can \ be \ obtained \ from \ http://research.kek.jp/people/kumanos/ffs.html.$