Detector Counting Efficiency

The detector counting efficiency (DE) relates the amount of radiation emitted by a radioactive source to the amount measured in the detector. The DE can be used to calculate the counting rate expected in a detector when the source strength is known or to calculate the source strength by measuring the counting rate in the detector. The DE is the ratio of the observed or measured counting rate (or total events in a known time interval) to the counting rate (or total events) emitted by the radiation source.

DE = D / N

where:

DE = the detector efficiency,
D = the number of photons counted in the detector and
N = number of photons emitted by the source.

Measuring D

For low energy photons, those that are absorbed as a photoelectric event in the detector, D is the net counts in the photopeak for that energy. For a NaI(Tl) detector, photoelectric events predominate for photons of 100 keV or less energy. Above 100 keV, Compton events become appreciable and, above 2 MeV, pair production events become appreciable. For these higher energy cases, it is necessary to know the fraction of events in the photopeak or the peak-to-total ratio R so that the net counts in the photopeak P can be related to the total counts in the detector.

R = P / D

where:

R = the peak-to-total ratio, and
P = the number of counts in the photopeak.

Thus, D is determined from the number of counts in the photopeak divided by the peak-to-total ratio. (Values of R are available in Figure 1.)

D = P / R

D can also be obtained by counting all pulses above the noise threshold with a sample in position and then counting all pulses with no sample in place. D is then obtained by subtracting the two sets of total counts. This latter technique is used only when no energy information or discrimination is necessary. Care must be taken to account for possible multiple counting due to events such as X-ray emission by the daughter atom or nuclear cascades from the daughter nucleus.

Calculating N

The activity of a radioactive source is usually given in Curies (abbreviated Ci). One Ci is defined to be 3.7 x 10E+10 disintegrations per second (dps):

1 Ci = 3.7 x 10E+10 dps

N can be calculated from the activity A by multiplying by the branching fraction BF for that mode of decay and the branching ratio BR for that photon energy and the counting time interval T. (Sometimes the total branching ratio TB, which is the product of BF and BR, are given. Some typical values are shown in Table 1.)

N = BF x BR x T x A
or
N = TB x T x A

where:

BF = the branching fraction for that mode of decay,
BR = the branching ratio for that photon energy,
TB = total branching ratio (TB = BF x BR),
T = the total counting time interval in seconds,
A = the activity in dps.

As an example, consider the 662 keV emission from a 10 µCi, ^{137}Cs source in 1 second.

10 µCi = 10 x 10E - 6 x 3.7 x 10E + 10 = 3.7 x 10E + 5 dps
BR = 0.944
BF = 0.9011
T = 1s

Substituting into N = BF x BR x T x A gives the following result for the total number of 662 keV photons emitted:

N = 0.944 x 0.9011 x 1s x 3.7 x 10E + 5 dps = 3.15 x 10E + 5 photons of 662 keV

Thus, a 10 µCi ^{137}Cs source emits 315,000 of the 662 keV photons in each second.

Source Decay

If the source calibration is not current, the source strength A must be corrected for the elapsed time by the equation:

$$A = A_0 \exp - (t / \tau)$$

where:

A = the activity when calibrated
t = time interval since the source strength was calibrated
τ = mean-life in the same units as the time interval (mean-life = half-life x 1.4427).

As an example, ^{137}Cs has a half-life of 30.07 years. If the source strength was calibrated 7.5 years ago, then t = 7.5 years, τ = 30.07 years x 1.4427 or 43.38 years and

$$A / A_0 = \exp - (7.5 / 43.38) = 0.84$$

This shows that a ^{137}Cs source loses 16% of its activity in 7.5 years.

Calculating DE

There are three factors, G, I and M, that affect the efficient absorption of the photons N emitted by the source. Their product is the detector efficiency DE.

More specifically:

G = The fraction of all space that the detector subtends. Unless the detector completely surrounds the source, the geometrical solid angle factor is less than 1.

I = The fraction of the photons transmitted by the intervening materials that reach the detector surface. There are losses due to absorption by material in the path of the photon. Air, detector housing materials and light reflectors around the detector are possible absorbers.

M = The fraction of the photons absorbed by the detector. The detector material is not always sufficiently thick to stop the radiation.

Example of right circular cylinder

Consider a 2-inch diameter NaI(Tl) detector 2 inches high and 4 inches from a ^{137}Cs source. In this case:

$$G = (\pi r^2) / (4 \pi R^2)$$

where:

πr^2 = area of detector face[1], and
$4\pi R^2$ = area of sphere with a radius equal to the source to detector distance

$$G = (\pi \times 1 \text{ inch} \times 1 \text{ inch}) / (4 \times \pi \times 4 \text{ inch} \times 4 \text{ inch})$$

$$G = 0.0156$$

This detector subtends or intercepts 1.56% of all space.

To calculate I, let us consider the effects of 4 inches of air and 0.020 inches of aluminum housing. For more materials, there are simply more factors in the following equation:

$$I = \exp - (\mu_1 \times d_1) \times \exp - (\mu_2 \times d_2)$$

where:

- μ_1 = 1.0 E-4 cm^{-1}, the attenuation coefficient of air for 662 keV photons,
- μ_2 = 0.20 cm^{-1}, the attenuation coefficient of aluminum for 662 keV photons,
- d_1 = 10 cm (4 inches), the distance traveled in air,
- d_2 = 0.05 cm (0.020 inch), the thickness of the aluminum container

$$I = (0.999) \times (0.990) = 0.99$$

In this case, the attenuation by intervening materials is only a 1% effect. (If the photon energy were lower, the losses would be greater, e.g., for 60 keV, I = (0.97) x (0.998) = 0.97.)

The fraction of the photons absorbed[(2)] by the detector M is calculated by subtracting the fraction that pass through the detector from 1:

$$M = 1 - \exp - (\mu \times d)$$

- μ = 0.30 cm^{-1}, the attenuation coefficient of NaI(Tl) for 662 keV photons,
- d = 5 cm (2 inches), the distance traveled in NaI(Tl),

$$M = 1 - 0.223 = 0.777$$

Note that M is directly available from Figure 4.

Then the detector efficiency becomes:

$$DE = G \times I \times M$$

$$= 0.0156 \times (0.99) \times (0.777) = 0.0120 \text{ or } 1.2\%$$

Thus a 2 x 2 NaI(Tl) detector can count only 1.2% of the radiation emitted by a point source of radiation 4 inches away. These are all events including the Compton events. If only the photopeak events are desired, then the number is reduced by the peak- to-total ratio (R ~ 0.26) to only 0.4%.

Example of a well detector

Consider a 2-inch diameter NaI(Tl) detector 2 inches high with a 0.75 inch diameter by 1.44 inch deep well. An ^{241}Am 60 keV source is in the bottom of the well. In this case, it is easier to calculate the fraction of space <u>not</u> subtended and then to subtract that value from 1 to get the fraction G subtended.

The fraction not subtended is the area of the hole of 0.75 diameter at the end of the well a distance of 1.44 inches:

$$1 - G = (\pi r^2) / (4\pi R^2)$$

where:

- πr^2 = area of hole in detector face[(3)], and
- $4\pi R^2$ = area of sphere with a radius equal to the distance from the source to the hole.
- 1 - G = (π x 0.375 inch x 0.375 inch) / (4 x π x 1.44 inch x 1.44 inch)

$$1 - G = 0.0170, \text{ and}$$

$$G = 0.983$$

This detector subtends or intercepts 98% of all space.

To calculate I, let us consider only the effects of 0.010 inches of aluminum well liner. For more materials, there are simply more factors in the following equation:

$$I = \exp - (\mu \times d)$$

where:

- μ = 0.7 cm^{-1}, the attenuation coefficient
- d = 0.025 cm (0.010 inch), the thickness of the aluminum container.

$$I = (0.983)$$

In this case, the attenuation by intervening materials is only a 1.7% effect.

The fraction of the photons absorbed[(4)] by the detector M is calculated by subtracting the fraction that pass through the detector from 1:

$$M = 1 - \exp - (\mu \times d)$$

- μ = 22 cm^{-1}, the attenuation coefficient of NaI(Tl) for 60 keV photons,
- d = 1.422 cm (.56 inch), the minimum distance traveled in NaI(Tl) at the bottom of the well,

$$M = 1 - 0.0 = 1.0$$

Then the detector efficiency becomes:

$$DE = G \times I \times M$$

$$= 0.983 \times (0.983) \times (1.0)$$

$$= 0.966 \text{ or } 97\% \text{ efficient for 60 keV}$$

In summary

The mathematical relationship between the amount of radiation emitted and that absorbed by a detector has been discussed. If the above equations are combined, the counting rate expected in a photopeak from a known source in a known geometry can be estimated.

$$P = DE \times R \times BF \times BR \times T \times A$$

where:

- **P** = the number of counts in the photopeak,
- **DE** = the detector efficiency,
- **R** = the peak-to-total ratio,
- **BF** = the branching fraction for that mode of decay,
- **BR** = the branching ratio for that photon energy,
- **T** = total counting time interval in seconds,
- **A** = the activity of the source in disintegrations per seconds (dps, corrected for decay).

Radioactive Source	E(keV)	Total Branching Ratio (TB)
^{241}Am	59.5	0.36
^{57}Co	122	0.86
^{57}Co	136	0.11
^{57}Co	122 & 136	0.97
^{60}Co	1173	1.00
^{60}Co	1332	1.00
^{60}Co	1173&1332	2.00
^{137}Cs	662	0.85
^{18}F	511	1.94
^{22}Na	511	1.8
^{22}Na	1274	1.0
^{228}Th	2615	0.36

Table 1. Branching Ratios

(1) Note that this is an approximation. The numerator should be the area of the spherical section obtained by integrating the area element instead of the area of the flat disc.
(2) It is assumed here that all photons are traversing the same amount of detector material. A correct procedure takes into account the different paths and their path lengths and sums or integrates to find the total fraction M. For more details, refer to advanced texts on the subject, such as Applied Gamma-Ray Spectroscopy, Edited by C. E. Crouthamel, Pergamon Press 1960.
(3) See footnote 1.
(4) See footnote 2.

Photopeak Efficiency of NaI(Tl) Detectors

In many applications, it is desirable to discriminate against background radiation or other spurious events. In these instances, it is necessary to count only the full energy or photopeak events generated by a detector.

There is no easy way to calculate the number of events that are expected in the photopeak only. The absorption efficiency of NaI(Tl), as shown in Figure 4 on page 7, gives the total counts (D) that can be expected in the channel or energy integrated spectrum. However, this includes the full energy peak, Compton edge, single and double escape peaks, backscattered and other Compton events. To get the number of events in the photopeak (P) only, the absorption efficiency should be multiplied by the photofraction (R), also called the peak-to-total ratio. These quantities are related by the following equation:

P = R x D

The photofraction or photopeak efficiency for various cylindrical geometries of NaI(Tl) detectors is shown in Figure 1. These were calculated using a Monte Carlo method.

Thus, to calculate the approximate number of 1274 keV events expected in the photopeak in a 1" x 1" NaI(Tl) detector with 10,000 counts (D) in the detector, D is multiplied by ~0.2 (R) to yield the total photopeak counts of ~2,000.

P = R x D

P = 0.2 x 10,000 counts

P = 2,000 counts

Materials that are less dense or have a lower atomic number have poorer peak-to-total ratios. In Figure 2, curves for a 3" x 3" NaI(Tl) scintillator and a 3" x 3" PVT plastic scintillator are plotted . The photopeak for the plastic scintillator is very weak and is not observed above ~60keV.

Peak-to-total charts for NaI and various other scintillators are in Appendix B of this document. These values are mainly for cylindrical geometries. A graph for NaI in certain special rectangular geometries is also included.

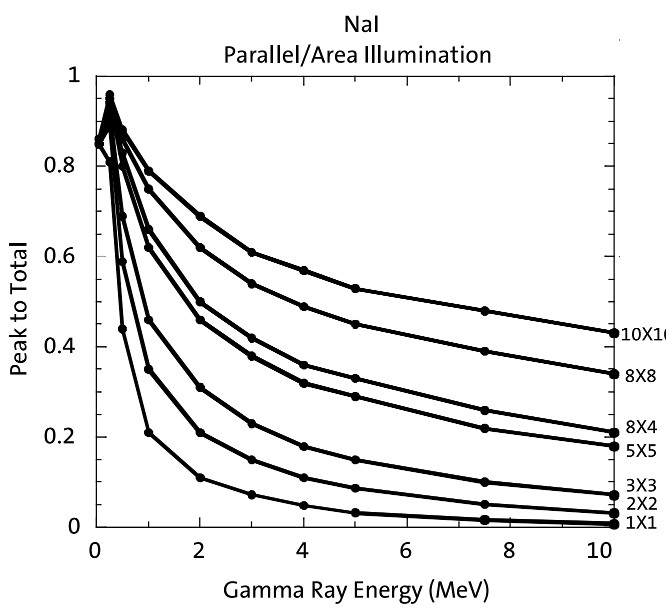


Figure 1.

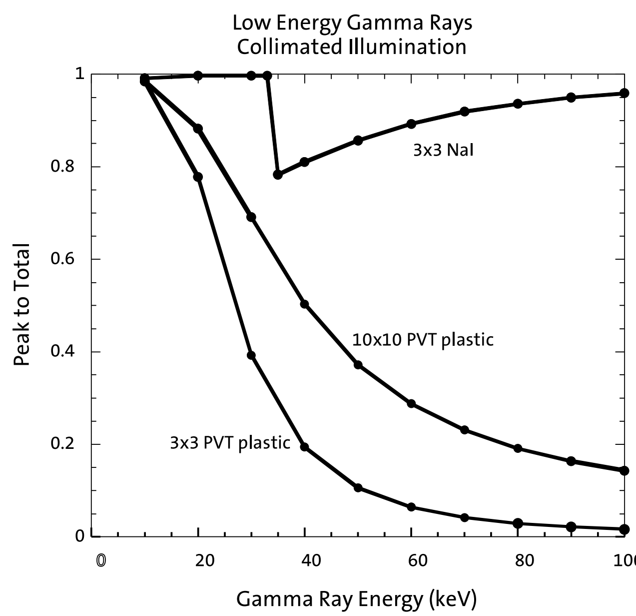


Figure 2.

Note: The numbers by each curve refer to the scintillator size; i.e., 10x10 refers to a scintillator 10 inches in diameter by 10 inches long

Transmission Efficiency of Window Materials

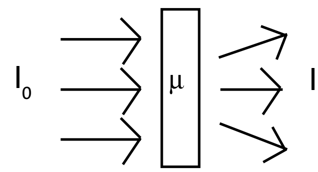
Gamma rays and X-rays are high energy photons. Gamma rays are produced by the nucleus of an atom and are typically in the 50 keV to 10 MeV energy range. X-rays are produced by the electrons around the nucleus and are typically in the 1 to 100 keV range.

Photons are absorbed in matter by statistical processes that lead to an exponential absorption that is a function of position. This function is normally written as:

$$I = I_0 e^{-\mu x}$$

where:

I_0 = the number of photons of a certain energy incident or entering the sheet of material,
x = the thickness of the sheet,
I = the number of photons that have passed through a layer of thickness x,
μ = the linear attenuation coefficient of the material for photons of this particular energy



The photons that do not get through have interacted within the sheet of material and are either absorbed or scattered. In this case, photons have interacted in the sheet.

The following chart (Figure 3) for various metal foils is based on calculations of this exponential function for certain thicknesses x. The number of photons transmitted by a certain thickness is the value of I. However, instead of calculating I for different I_0's, the ratio of I/I_0 is calculated and it is called the "Transmission."

As a sample calculation, a 5 mil foil (.005") of aluminum transmits 40% of the 10 keV photons incident on its face.

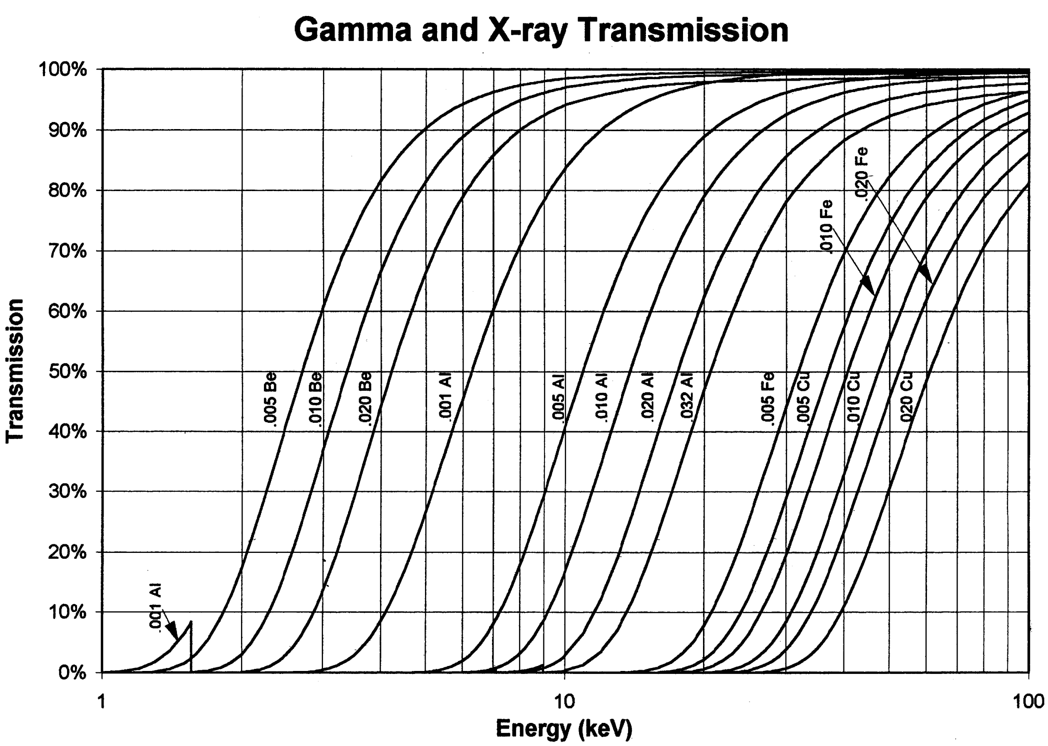


Figure 3.

Gamma and X-ray Absorption Efficiency

The calculation for the number of photons passing through a scintillator is similar to that used for Transmission Efficiency.

The following charts (Figures 4 through 15) are based on calculations of this exponential function for certain values of x for various scintillators. The number of photons absorbed by a certain thickness is the difference I_0 - I. However, instead of calculating for different I_0's, the ratio of $(I_0 - I)/I_0$ is calculated and it is called the "Percent Absorption."

For example, a NaI crystal of ¼" thickness would absorb 20% of the 500 keV photons incident on its face. (Refer to Figure 4.)

Materials represented in the Appendix A: Figures 4 to 15 – NaI, BaF_2, BGO, BrilLanCe®350, BrilLanCe®380, CaF_2, $CdWO_4$, CsI, PreLude™420, PVT Plastic Scintillator, and YAG.

Absorption Efficiency of NaI

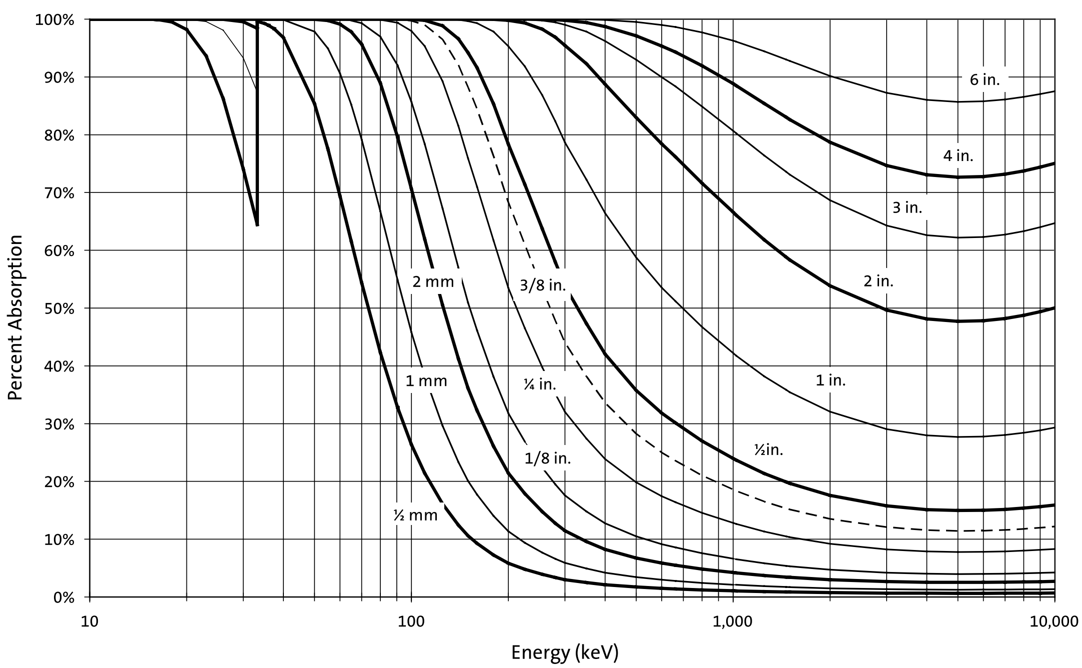


Figure 4.