Electroproduction of Nucleon Resonances with Polarized Leptons*

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(Received 19 June 1972)

The electroproduction of nucleon resonances with polarized leptons and a polarized nucleon target is considered and compared with what is expected to be the behavior of deep-inelastic polarized scattering. We find a set of nucleon resonances in the symmetric quark model such that their excitation gives the same results for the ratio of neutron to proton inelastic scattering and for the polarization asymmetry on protons and neutrons as does the naive quark-parton model. However, the symmetric quark model with harmonic forces predicts dramatic variations in the helicity structure of the excitation of the prominent nucleon resonances, which are not supported by the existing data. Additional tests of the symmetric quark model and a discussion of what is known of resonance excitation with what is expected in polarized deep-inelastic scattering are presented.

I. INTRODUCTION

In the near future there will exist polarized lepton beams (electrons at SLAC, muons at BNL and NAL) which, together with polarized targets, will permit the study of polarized lepton-nucleon collisions. Such experiments will provide a new avenue for exploring the properties of deep-inelastic scattering and therefore for testing additonal aspects of the various theories advanced to explain such processes. These experiments will also permit a detailed examination of the helicity structure of nucleon-resonance electroproduction.

In this paper we shall be primarily concerned with this latter aspect – the helicity structure of nucleon-resonance electroproduction. There are dynamical models, in particular the quark model with harmonic forces, which can be critically tested in this regard. However, we shall also be comparing nucleon resonance electroproduction by polarized leptons and nucleons with what is expected to be the behavior of the deep-inelastic polarized scattering, in light of the connections between their respective behaviors which seem to hold in the unpolarized case.

In Sec. II we review the kinematics of polarized scattering and define the relevant structure functions and cross sections. Then we present the naive quark-parton model predictions for the various observables to get an idea of the sign and magnitude of the effects to be expected in the deep-inelastic scattering. With the background thus set, we consider in Sec. III the possibility of obtaining the naive quark-parton model results through a sum of nucleon resonance contributions to the relevant structure functions. Naturally we turn to a quark model of nucleon resonance states in order to try and realize this possibility. We find that it is indeed possible to construct a set of nucleon resonances in the symmetric quark model such that their excitation gives the same results as the predictions of the naive quark-parton model for the scattering of polarized as well as unpolarized leptons on both neutrons and protons. In Sec. IV we pass to the problem of the behavior of specific resonances undergoing polarized electroproduction within the symmetric quark model with harmonic forces. Dramatic variations of the helicity structure of the prominent resonances with the momentum transfer to the leptons are predicted by the quark model, but are not supported by existing data, as noted in an earlier paper.[1] Some additional tests of the basic structure of the symmetric quark model are presented and a discussion of the comparison of resonance excitation with what is expected in deep-inelastic scattering is found in Sec. V.

II. INELASTIC SCATTERING WITH POLARIZED LEPTONS

We consider inelastic scattering of polarized leptons (incident four-momentum k and helicity $\pm\frac{1}{2}$, final four-momentum k') on polarized nucleons (four-momentum p and covariant spin vector s_μ such that $s\cdot p=0$, $s\cdot s=+1$).[2] Assuming one-photon exchange, the double differential cross section for detecting the final lepton only in the laboratory can then be written as

$$\frac{d^2\sigma}{d\Omega' dE'}=\frac{1}{(2\pi)^2}\frac{E'}{E}\left(\frac{e^2}{q^2}\right)^2 L_{\mu\nu}^{(\pm)}W_{\mu\nu}\,, \tag{1}$$

where $L_{\mu\nu}^{(\pm)}$ arises from the square of the matrix element of the lepton current and $W_{\mu\nu}$ arises similarly from the hadronic current. Neglecting lepton masses, initial helicity $\pm\frac{1}{2}$ leptons give

$$L_{\mu\nu}^{(\pm)} = \tfrac{1}{2}(k_\mu k'_\nu + k'_\mu k_\nu + \tfrac{1}{2}q^2\delta_{\mu\nu} \pm \epsilon_{\mu\nu\lambda\sigma}k_\lambda k'_\sigma)\,, \tag{2}$$

where $q^2 = (k-k')^2$ is the invariant four-momentum squared carried by the virtual photon. The quantity $W_{\mu\nu}$ involves the two familiar form factors W_1 and W_2, which occur for spin-averaged scattering, as well as two spin-dependent form factors, which are chosen differently by each new paper on spin-dependent inelastic scattering. We chose here to use the two functions d and g defined by[3]

$$W_{\mu\nu} = W_1(\nu, q^2)(\delta_{\mu\nu} - q_\mu q_\nu/q^2) + \frac{W_2(\nu, q^2)(p_\mu - p\cdot q q_\mu/q^2)(p_\nu - p\cdot q q_\nu/q^2)}{M_N{}^2}$$
$$+ \frac{1}{4\pi M_N}[-\epsilon_{\mu\nu\lambda\sigma}q_\lambda s_\sigma d(\nu, q^2) + s\cdot q\epsilon_{\mu\nu\lambda\sigma}q_\lambda p_\sigma g(\nu, q^2)]\,, \tag{3}$$

where M_N is the nucleon mass and $\nu = -p\cdot q/M_N$ is the virtual photon's energy in the laboratory.

Clearly one needs *both* a polarized lepton beam and a polarized target to determine experimentally the structure functions $d(\nu, q^2)$ and $g(\nu, q^2)$. Denoting by

$$\frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega' dE'}\quad\left(\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega' dE'}\right)$$

the cross section with the beam and target spins polarized parallel (antiparallel) to each other along the beam direction, one has

$$\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega' dE'} - \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega' dE'} = \frac{4\alpha^2 E'}{q^2 E}\left(\frac{1}{4\pi M_N}\right)[(E + E'\cos\theta)d(\nu, q^2) + (E - E'\cos\theta)(E+E')M_N g(\nu, q^2)] \tag{4}$$

for leptons scattered by an angle θ. Polarizing the nucleon in the scattering plane but perpendicular to the incident lepton direction leads to a cross section with a different dependence on d and g, which may be useful in separating out their individual contributions.[4]

Just as in the case of unpolarized scattering, one can work in terms of total cross sections for virtual photons (mass squared $= -q^2$) on nucleons. For the transverse scattering one defines total cross sections for "γ" $+ N \to$ hadrons where the spin of the photon and nucleon are parallel and the net spin component along the photon's momentum direction is $\pm\frac{3}{2}$, $\sigma_{3/2}(\nu, q^2)$, and correspondingly where the spin of the photon and nucleon are antiparallel and the net spin component is $\pm\frac{1}{2}$, $\sigma_{1/2}(\nu, q^2)$. It is then simplest to choose a normalization such that the spin-averaged total cross section is just the transverse total cross section of Hand[5]:

$$\sigma_T(\nu, q^2) = \tfrac{1}{2}[\sigma_{1/2}(\nu, q^2) + \sigma_{3/2}(\nu, q^2)]\,. \tag{5}$$

One defines the transverse asymmetry A as

$$A(\nu, q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}\,, \tag{6}$$

which then must lie in the region $-1 \leq A \leq +1$. There is then a relation between the spin-dependent structure functions, and the alternate description of the transverse scattering in terms of A and the spin-averaged structure function W_1:

$$\frac{\nu d(\nu, q^2) + M_N(\nu^2 + q^2)g(\nu, q^2)}{4\pi M_N} = A(\nu, q^2)W_1(\nu, q^2)\,. \tag{7}$$

One of the most interesting aspects of spin-averaged deep-inelastic scattering is of course the scaling behavior of W_1 and νW_2, i.e., that for $q^2 \gtrsim 1$ GeV^2 they appear to be functions of the dimensionless variable $\omega = 2M_N\nu/q^2$ rather than ν and q^2 independently as would be the case in general. The analogous behavior for spin-dependent scattering is that νd and $\nu^2 g$ should scale.[3,4,6]

To get an idea of the expected magnitude and sign of νd and $\nu^2 g$, let us look at the simplest quark-parton model[7] in which the nucleon is considered (in an infinite-momentum frame) as composed of three pointlike spin-$\frac{1}{2}$ quarks, $\mathcal{PPN}$ for the proton and $\mathcal{NNP}$ for the neutron, with an arbitrary longitudinal-momentum distribution. In such a model the deep-inelastic scattering is transverse (the longitudinal-to-transverse cross-section ratio vanishes in the Bjorken limit of $\nu, q^2 \to \infty$ at fixed ω) so that $\nu W_2 = (q^2/\nu)W_1$ and the ratio of neutron to proton scattering is $\frac{2}{3}$ (the ratio of the sum of the squares of the charges of their constituents).

For the spin-dependent structure functions one finds in the same model that in the scaling limit

$$\nu^2 g(\nu, q^2) = 0 \tag{8a}$$

for either the neutron or proton, while

$$\frac{\nu d(\nu, q^2)}{4\pi M_N} = \tfrac{5}{9}W_1(\nu, q^2) = \tfrac{5}{9}W_1(\omega) \tag{8b}$$

for the proton (i.e., $A_p = \frac{5}{9}$), and

$$\frac{\nu d(\nu, q^2)}{4\pi M_N} = 0 \tag{8c}$$

for the neutron (i.e., $A_n = 0$). That $\nu^2 g = 0$ is a simple consequence of the point-fermion (with no anomalous magnetic moment) assumption of the quark-parton model.

Thus we expect inelastic scattering on the proton to have large positive asymmetries on the basis of the quark model.[8] In fact, in almost any simple parton model of deep-inelastic scattering one would expect positive (or possibly zero) asymmetries, for the asymmetry has the simple interpretation as the net spin of the partons, weighted by their charges squared, aligned along the nucleon's spin. Thus, since one does not expect the constituents to align themselves dominantly opposite to the nucleon's spin, one expects that $A \geq 0$, or $\sigma_{1/2} > \sigma_{3/2}$ in the deep-inelastic region. The expected sign of A in the deep-inelastic and resonance regions will be a central theme to be returned to again in succeeding sections.

III. INELASTIC ELECTRON-NUCLEON SCATTERING AND NUCLEON RESONANCES IN THE QUARK MODEL

The direct experimental observation of sizable differences between electron-proton and electron-neutron spin-averaged inelastic scattering indicates the presence of a nondiffractive component in virtual photon-nucleon interactions.[9] Within the framework of two-component duality[10] this nondiffractive component should be interpretable in terms of a sum of direct-channel (nucleon) resonances. Indeed, a very close correlation between the behavior of nucleon resonance electroproduction and deep-inelastic scattering is observed to hold.[11] It is also possible to construct theoretical models where inelastic electron-nucleon scattering is expressible in terms of a sum of (infinitely) many direct-channel resonances with scaling and other desirable properties built into the model.[12]

A nonzero asymmetry in polarized inelastic lepton-nucleon scattering should also have its origin in a nondiffractive component of virtual photon-nucleon interactions. This follows directly from performing an analysis of possible t-channel exchanges in forward virtual photon-nucleon scattering in which one finds that the spin-dependent amplitudes (whose imaginary parts are the structure functions d and g) do not receive a contribution in leading order in the energy from Pomeranchukon exchange.[13] This also follows from one's naive expectation that forward diffraction scattering should not depend on the spin orientation of the particles involved, so that $\sigma_{1/2} = \sigma_{3/2}$ or $A = 0$.

One possible program would try to reproduce both the (observed) unpolarized and polarized lepton-nucleon inelastic scattering in terms of an infinite sum of nucleon resonances.[14] As a start in this direction we attempt here to reproduce the naive quark-parton model results (ratio of neutron to proton inelastic scattering, $\sigma_n/\sigma_p = \frac{2}{3}$, $A_p = \frac{5}{9}$, $A_n = 0$) in terms of a sum of direct-channel nucleon resonance contributions. In other words, we attempt to construct a set of nucleon resonance states, the sum of whose contributions to inelastic lepton-nucleon scattering duplicates the naive (three-) quark-parton model results for σ_n/σ_p, A_p, and A_n.

To achieve such a model we turn to the symmetric quark model with orbital angular momentum excitation for the nucleon resonances. The ground state is assumed to be a totally symmetric state of three quarks with the nucleon [and its SU(3) partners] corresponding to a total quark spin of $\frac{1}{2}$, and the 3-3 resonance [and its SU(3) partners] to a total quark spin of $\frac{3}{2}$. We make the standard assumption that the excitation of the nucleon by virtual photons is such that the photon acts on only one quark at a time. Since the nucleon wave function is totally symmetric, only final-state resonance wave functions which are totally symmetric or of mixed symmetry are excitable, corresponding to the 56- and 70-dimensional representations of SU(6), respectively, but not the 20-dimensional representation, which is totally antisymmetric. Furthermore, we will take only the interaction of the photon with the magnetic moments of the quarks, and neglect terms arising from their orbital motion. This immediately forces the photon-nucleon interaction to be purely transverse, in agreement with theories of deep-inelastic scattering containing spin-$\frac{1}{2}$ partons and as suggested experimentally.[15] As we will see in the next section, in explicit models with harmonic forces between the quarks the interaction arising from the spin term dominates that from the orbital term at large

TABLE I. Contributions to $\sigma_{1/2}$ and $\sigma_{3/2}$ in the quark model for proton and neutron targets coming from the various SU(3) octets and decuplets which make up the $\underline{56}$ and $\underline{70}$ representations of SU(6). A and B are dynamical factors related to the O(3) structure of the supermultiplet wave function, and S is the total quark spin. $A = B$ reproduces the quark-parton model results for σ_n/σ_p, A_p, and A_n.

	$\underline{56}$		$\underline{70}$		
	$\underline{8}$ $S=\frac{1}{2}$	$\underline{10}$ $S=\frac{3}{2}$	$\underline{8}$ $S=\frac{1}{2}$	$\underline{8}$ $S=\frac{3}{2}$	$\underline{10}$ $S=\frac{1}{2}$
$\sigma^p_{1/2}$	$2A$	$\frac{4}{9}A$	$2B$	0	$\frac{2}{9}B$
$\sigma^p_{3/2}$	0	$\frac{4}{3}A$	0	0	0
$\sigma^n_{1/2}$	$\frac{8}{9}A$	$\frac{4}{9}A$	$\frac{2}{9}B$	$\frac{2}{9}B$	$\frac{2}{9}B$
$\sigma^n_{3/2}$	0	$\frac{4}{3}A$	0	$\frac{6}{9}B$	0

q^2. This is the situation we are interested in here, so that our neglect of the terms arising from orbital motion is not really an additional assumption.

With such a model we can now proceed to calculate the neutron-to-proton ratio, proton asymmetry, and neutron asymmetry obtained from excitation of all the states in a 56- or 70-dimensional representation of SU(6) (all states assumed degenerate in mass). For the $\underline{56}$ we find (see Table I)

$$\sigma_n/\sigma_p = \tfrac{12}{17}, \tag{9a}$$

$$A_p = \tfrac{5}{17}, \tag{9b}$$

$$A_n = 0, \tag{9c}$$

while for the $\underline{70}$

$$\sigma_n/\sigma_p = \tfrac{3}{5}, \tag{10a}$$

$$A_p = 1, \tag{10b}$$

$$A_n = 0. \tag{10c}$$

Thus, for any mixture of the two we have

$$0.60 = \tfrac{3}{5} \leq \sigma_n/\sigma_p \leq \tfrac{12}{17} \simeq 0.71, \tag{11a}$$

$$0.29 \simeq \tfrac{5}{17} \leq A_p \leq 1.0, \tag{11b}$$

$$A_n = 0. \tag{11c}$$

We immediately note that the quark-parton model results correspond to neither of Eqs. (9) or (10), but they do lie in the range of Eqs. (11). It is easy to show that there does in fact exist a linear combination of $\underline{56}$ and $\underline{70}$ states which gives

$$\sigma_n/\sigma_p = \tfrac{2}{3}, \tag{12a}$$

$$A_p = \tfrac{5}{9}, \tag{12b}$$

$$A_n = 0, \tag{12c}$$

which are exactly the naive quark-parton model results. Thus, while the actual experimental σ_n/σ_p ratio lies outside the range given by Eq. (11a), it is possible to construct a set of direct-channel resonances which reproduce the naive quark-parton model results for σ_n/σ_p, A_p, and A_n. Such a representation is very useful in that it permits one to see what corresponds in an s-channel picture to various parton-model results. Conversely one can see what effect will result from realistic modifications of the idealized situation in the symmetric quark model for nucleon resonances.

In particular, we refer to Table I, which lists the contributions to $\sigma_{1/2}^{p,n}$ and $\sigma_{3/2}^{p,n}$ from the various octets and decuplets in the $\underline{56}$ and $\underline{70}$ representations of SU(6). From these one may form σ_n/σ_p, A_p, and A_n by summing over the states in the $\underline{56}$ or $\underline{70}$, and reproduce the results in Eqs. (9) and (10). Using this table one also may easily deduce the effect upon σ_n/σ_p, A_p, and A_n of making *ad hoc* assumptions as to the importance of the various s-channel resonances. For example, it is possible that the contribution[16] of decuplets might fall faster with q^2 than that of octets, and hence will be unimportant at large q^2. If one suppresses the decuplet contributions in Table I, one finds that for the $\underline{56}$

$$\sigma_n/\sigma_p = \tfrac{4}{9}, \tag{13a}$$

$$A_p = 1, \tag{13b}$$

$$A_n = 1, \tag{13c}$$

and for the $\underline{70}$

$$\sigma_n/\sigma_p = \tfrac{5}{9}, \tag{14a}$$

$$A_p = 1, \tag{14b}$$

$$A_n = -\tfrac{1}{5}. \tag{14c}$$

The resulting magnitudes of σ_n/σ_p are in better agreement with experimental observations[9] near $\omega = 1$ than those in Eqs. (9) and (10). If the suppression of decuplets in Table I corresponded to reality near $\omega = 1$, then the similarity of σ_n/σ_p and A_p for the $\underline{56}$ and $\underline{70}$ [Eqs. (13) and (14)] would make it necessary to measure A_n in order to determine their relative contributions to inelastic scattering.

IV. HELICITY STRUCTURE OF THE PROMINENT NUCLEON RESONANCES

The symmetric quark model, with harmonic forces acting between pairs of quarks, has been rather successfully employed, both in classifying hadron states[17] and in calculating the electromagnetic transitions between different hadron states due to the emission or absorption of real photons.[18] Recently, a relativistic quark model with harmonic forces has been developed by Feynman, Kislinger, and Ravndal[19] and used to calculate the matrix elements of both the vector and axial-vector currents, again with considerable quantitative success. With such a quark model it becomes possible to treat the very relativistic processes involved in the electroproduction of nucleon resonances[20,21] and to examine in detail the s-channel model for inelastic scattering discussed in the last section and its comparison with the real world.

Let us first examine the behavior of the whole set of nucleon resonances in the $\underline{56}$ or $\underline{70}$ representations of SU(6) using the model of Feynman, Kislinger, and Ravndal[19] as applied by Ravndal[20] to electroproduction. In Figs. 1 and 2 we show the behavior of σ_n/σ_p, A_p, and A_n for the sum of resonances in the $\underline{56}$ and $\underline{70}$ representations of SU(6) as a function of q^2 using Ravndal's formulas. As can be seen in the figures, σ_n/σ_p, A_p, and A_n either are constant with q^2 or rapidly approach their $q^2 \to \infty$ values as q^2 departs from zero. This is due to the fact that with increasing q^2 the terms in the ampli-

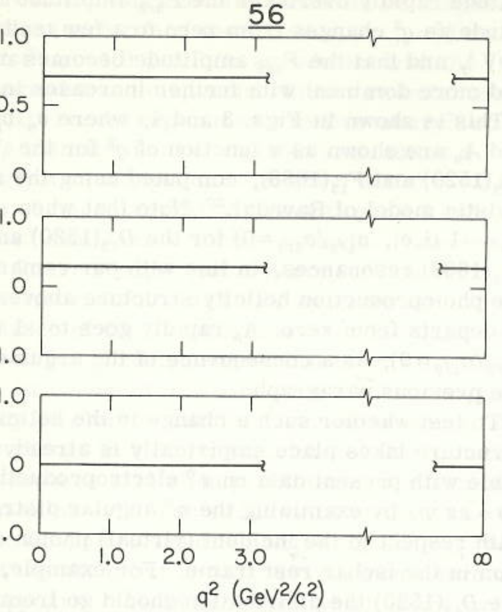

FIG. 1. The q^2 dependence of σ_n/σ_p, A_p, and A_n for electroproduction of the sum of resonances in the $\underline{56}$ representation of SU(6) with $L=0$. All resonances are taken to have an arbitrary but common mass.

tudes arising from the spin of the quarks dominate over those arising from their orbital motion. Is this helicity structure and its q^2 dependence manifest in the behavior of the individual nucleon resonances which make up the $\underline{56}$ and $\underline{70}$ in the usual baryon classification scheme?

In the case of photoproduction ($q^2=0$), one of the major successes of either the relativistic or nonrelativistic versions of the symmetric quark model was in fact the prediction of the remarkable helicity structure of the photoproduction amplitudes for the first three prominent nucleon resonances: the $P_{33}(1236)$, $D_{13}(1520)$, and $F_{15}(1690)$. These are experimentally the best-identified nucleon resonances in the ($\underline{56}$, $L=0$), ($\underline{70}$, $L=1$), and ($\underline{56}$, $L=2$) representations of SU(6), respectively.

Let us work in the center-of-mass system of the photon and nucleon (isobar rest frame) and consider the two independent amplitudes for formation of a given resonance to be $F_{1/2}$ and $F_{3/2}$ corresponding to net spin component λ equal to $\frac{1}{2}$ and $\frac{3}{2}$ along the photon's direction of motion. Then experimentally it is known[22] that the excitation of the $P_{33}(1236)$ is dominantly of a magnetic dipole character ($F_{3/2}/F_{1/2}=\sqrt{3}$), while the excitations of the D_{13} and F_{15} from protons proceed almost entirely through the $\lambda=\frac{3}{2}$ state ($F^p_{1/2}\approx 0$ for D_{13} and F_{15}). This can be seen directly in the forward and backward differential cross sections for $\gamma p\to\pi^+ n$ or $\pi^0 p$, where only the $\lambda=\frac{1}{2}$ amplitude contributes by

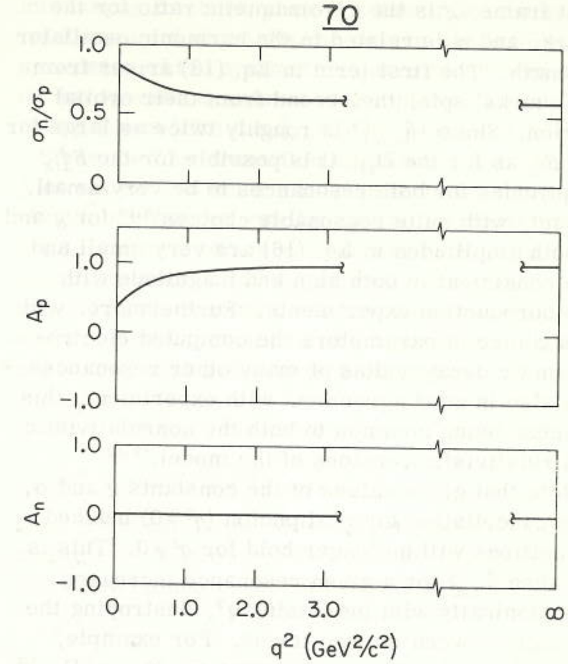

FIG. 2. The q^2 dependence according to Ref. 20 of σ_n/σ_p, A_p, and A_n for electroproduction of the sum of resonances in the $\underline{70}$ representation of SU(6). All resonances are taken with a common mass of 1.625 GeV/c^2.

angular momentum conservation, and where there is no appropriate structure on passing through the energy region of the $D_{13}(1520)$ and $F_{15}(1690)$. We note also that the Drell-Hearn-Gerasimov sum rule,[23]

$$(\mu_A)^2=\frac{M_N{}^2}{2\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}[\sigma_{3/2}(\nu)-\sigma_{1/2}(\nu)], \tag{15}$$

which equates the square of the nucleon's anomalous moment (a manifestly positive quantity) to an integral over cross sections proportional to $|F_{3/2}|^2-|F_{1/2}|^2$, has for protons the possibility of being saturated by low-lying resonances precisely because $|F_{3/2}|^2>|F_{1/2}|^2$ for all the prominent nucleon resonances.[24]

In the symmetric quark model with harmonic forces the dominance of the $\lambda=\frac{3}{2}$ excitation for the D_{13} and F_{15} comes about because of a cancellation between terms arising from the quarks' orbital motion and terms arising from the magnetic moments of the quarks. Explicitly, in the nonrelativistic model of Copley, Karl, and Obryk,[18] one has for the $\lambda=\frac{1}{2}$ amplitude with a proton target[25]

$$F^p_{1/2}\propto|\vec{q}_{\rm c.m.}|^2-\alpha^2/g,\quad\text{for the } D_{13}$$

and (16)

$$F^p_{1/2}\propto|\vec{q}_{\rm c.m.}|^2-2\alpha^2/g,\quad\text{for the } F_{15}$$

where $\vec{q}_{\rm c.m.}$ is the three-momentum in the isobar rest frame, g is the gyromagnetic ratio for the quark, and α is related to the harmonic oscillator strength. The first term in Eq. (16) arises from the quarks' spin, the second from their orbital motion. Since $|\vec{q}_{\rm c.m.}|^2$ is roughly twice as large for the F_{15} as for the D_{13}, it is possible for the $F_{1/2}^p$ amplitudes for both resonances to be very small. In fact, with quite reasonable choices[18,19] for g and α both amplitudes in Eq. (16) are very small and are consistent in both sign and magnitude with photoproduction experiments. Furthermore, with this choice of parameters the computed electromagnetic decay widths of many other resonances are also in good agreement with experiment, this success being common to both the nonrelativistic and relativistic versions of the model.[18,19]

Note that given values of the constants g and α, this cancellation for real photon ($q^2=0$) induced transitions will no longer hold for $q^2\neq 0$. This is because $\vec{q}_{\rm c.m.}{}^2$ for a given resonance increases monotonically with increasing q^2, destroying the balance between the two terms. For example, while the ratio of cross sections $\sigma_{3/2}/\sigma_{1/2}=|F_{3/2}|^2/|F_{1/2}|^2$ is predicted by the relativistic quark model[19] to be more than 10 at $q^2=0$ for the $D_{13}(1520)$ resonance excited from protons, by $q^2=0.3$ GeV^2 (spacelike) this ratio is predicted to be less than unity. By $q^2\approx 1$ GeV^2 the ratio is predicted to be $\sim\frac{1}{10}$. We find that for both the D_{13} and F_{15} the $F_{1/2}$ amplitude rapidly overtakes the $F_{3/2}$ amplitude in magnitude as q^2 changes from zero to a few tenths of a GeV^2, and that the $F_{1/2}$ amplitude becomes more and more dominant with further increases in q^2.[25]

This is shown in Figs. 3 and 4, where σ_n/σ_p, A_p, and A_n are shown as a function of q^2 for the $D_{13}(1520)$ and $F_{15}(1688)$, computed using the relativistic model of Ravndal.[20] Note that when $q^2\to 0$, $A_p\approx -1$ (i.e., $\sigma_{1/2}/\sigma_{3/2}\approx 0$) for the $D_{13}(1520)$ and $F_{15}(1688)$ resonances, in line with our remarks on the photoproduction helicity structure above. As q^2 departs from zero, A_p rapidly goes to +1 (i.e., $\sigma_{3/2}/\sigma_{1/2}\simeq 0$), as a consequence of the arguments of the previous paragraph.

To test whether such a change in the helicity structure takes place empirically is already possible with present data on π^0 electroproduction, $ep\to e\pi^0 p$, by examining the π^0 angular distribution with respect to the incident (virtual) photon direction in the isobar rest frame. For example, for the $D_{13}(1520)$ the distribution should go from being nearly $\sin^2\theta$ at $q^2=0$, where $\sigma_{1/2}/\sigma_{3/2}\simeq 0$, to isotropic at $q^2\simeq 0.3$ GeV^2, where $\sigma_{3/2}\simeq\sigma_{3/2}$, to approximately $1+3\cos^2\theta$ at $q^2=1.0$ GeV^2, where $\sigma_{3/2}/\sigma_{1/2}\simeq 0$. Thus there should be a *dramatic* change in the angular distribution of the π^0 between $q^2=0$ and $q^2\simeq 1$ GeV^2 for the $D_{13}(1520)$.

Experiment, on the other hand, gives no indication for such a change. While the excitation of the

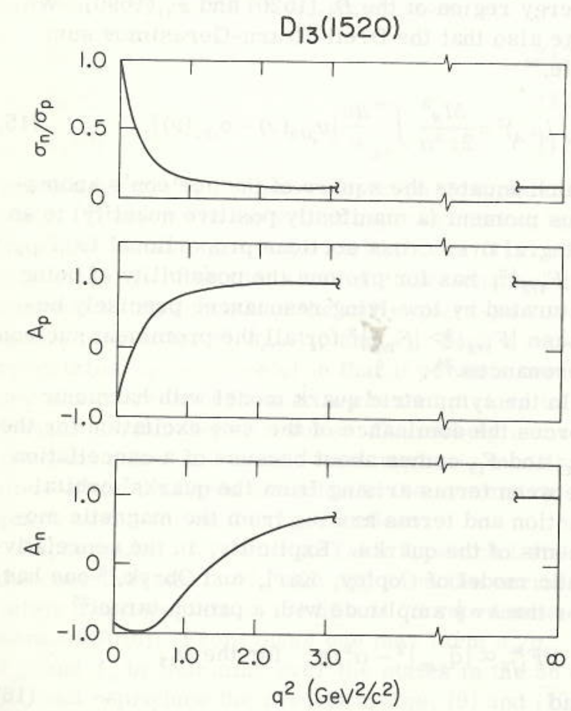

FIG. 3. The q^2 dependence of σ_n/σ_p, A_p, and A_n for the $D_{13}(1520)$ according to the relativistic model of Ref. 20.

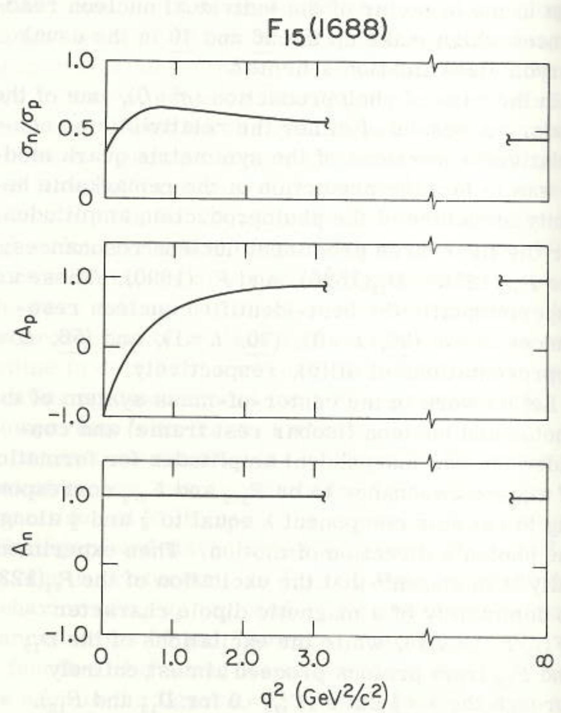

FIG. 4. The q^2 dependence of σ_n/σ_p, A_p, and A_n for the $F_{15}(1688)$ according to the relativistic model of Ref. 20.

first resonance is known to maintain its magnetic dipole character (and therefore $\sigma_{3/2}/\sigma_{1/2}=\frac{3}{1}$) out to at least[26] $q^2=1.0\ \mathrm{GeV}^2$ (in agreement with the quark-model and dispersion-theory calculations[27] as well), recent experiments at Daresbury[28] on $ep \to e\pi^0 p$ indicate that the $D_{13}(1520)$ maintains a strongly $\lambda=\frac{3}{2}$ excitation from $q^2=0$ out to $q^2=0.6\ \mathrm{GeV}^2$, the angular distributions at $q^2=0.4$ and $0.6\ \mathrm{GeV}^2/c^2$ exhibiting the same behavior ($\sim\sin^2\theta$) as at $q^2=0$ (see Fig. 5). An experiment[29] on backward π^0 electroproduction at DESY suggests the same dominance of the $\lambda=\frac{3}{2}$ amplitude for the third resonance (the F_{15}) region out to at least $q^2\approx 0.5\ \mathrm{GeV}^2$. Thus, at values of q^2 where such a change should already be clearly visible, there is no indication of the change in helicity structure of the D_{13} and F_{15} resonance excitation predicted by the symmetric quark model. The small value of the $\lambda=\frac{1}{2}$ amplitude for photoproduction of the D_{13} and F_{15} "predicted" by the quark model with harmonic forces thus appears to be an accident, which evaporates as q^2 changes even slightly.

The predicted helicity structure of the $P_{33}(1236)$, however, is in agreement with the data. This result (for the P_{33}) is a consequence of only the SU(6) $\times$O(3) structure of the model.[30] It would thus be

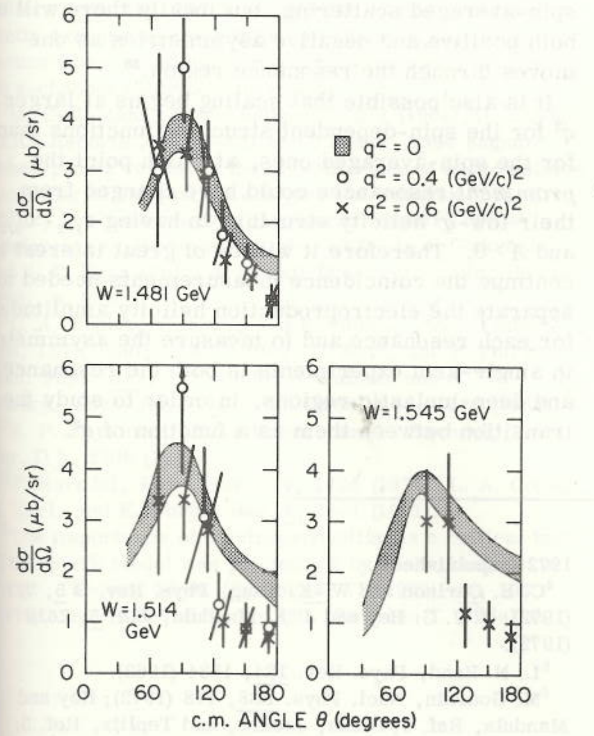

FIG. 5. Angular distributions of "γ" $+p \to \pi^0+p$ in the center-of-mass system, with θ the angle between the incoming photon and outgoing pion three momenta. Data are from Refs. 22 (for $q^2=0$) and 28 (for $q^2=0.4$ and $0.6\ \mathrm{GeV}^2/c^2$).

interesting to check whether other more general relations hold which depend only on the SU(6)$\times$O(3) symmetry of the symmetric quark model and not on the specific dynamics of the quark's interaction with photons [like the $\vec{q}_{\mathrm{c.m.}}{}^2$ term in Eq. (16)].

Since the four transition amplitudes ($\lambda=\frac{1}{2}$ and $\frac{3}{2}$ on proton and neutron) arise from only two terms, the quarks' orbital motion and their magnetic moments, there are two linear relations between amplitudes for the excitation of each resonance. In the case of the D_{13} these are (in an obvious notation for photoproduction and the transverse amplitudes in electroproduction)

$$\begin{aligned} F^n_{3/2} &= -F^p_{3/2}, \\ F^n_{1/2} &= -(2/3\sqrt{3})F^p_{3/2} - \tfrac{1}{3}F^p_{1/2}. \end{aligned} \tag{17}$$

Thus if the excitation of the D_{13} remains almost purely $\lambda=\frac{3}{2}$ on protons, there must be a finite $\lambda=\frac{1}{2}$ excitation on neutrons. Similarly, for the F_{15},

$$\begin{aligned} F^n_{3/2} &= 0, \\ F^n_{1/2} &= \tfrac{1}{3}\sqrt{2}\,F^p_{3/2} - \tfrac{2}{3}F^p_{1/2}. \end{aligned} \tag{18}$$

Decisive experimental information with neutron targets to test the relations for $F^n_{1/2}$ is lacking at present. However, recent phenomenological analysis,[31] while supporting the relations for the helicity-$\frac{3}{2}$ amplitudes, suggests that the helicity-$\frac{1}{2}$ amplitude relations might not be satisfied.

A complete set of such relations may be constructed from Table I of Ref. 18, where the explicit Clebsch-Gordan coefficients of the quark-model amplitudes are given. Relationships of the type (17) and (18) test a more fundamental aspect of the quark model for nucleon resonances than the magnitudes of individual amplitudes, which are interaction- and parameter-dependent.

V. DISCUSSION

As is evident from the discussion at the end of the last section, the near vanishing of the helicity-$\frac{1}{2}$ amplitude for the D_{13} and F_{15} resonances in photoproduction is not due to the SU(6)$\times$O(3) symmetry of the harmonic quark model only. In fact, if $F_{1/2}$ vanishes for proton targets it cannot do so for neutron targets without all the transition amplitudes to the D_{13} or F_{15} vanishing. The smallness of $F^p_{1/2}$ in photoproduction of the D_{13} and F_{15} thus depends on dynamics. The failure of the harmonic quark model to give the correct q^2 dependence of $F^p_{1/2}$ for the D_{13} and F_{15} transitions must then be blamed on the dynamics of that model, and in particular on the harmonic potential and resulting wave functions for the resonant states.

A possible way out of the difficulties of the previous section might then be to change the potential binding the quarks, and in particular to modify the short-distance behavior of the potential.[32] It is, after all, the smoothness of the short-distance behavior of the harmonic potential which gives Gaussian form factors, in strong disagreement with experiment at large q^2. Presumably a considerable modification of the potential near the origin is needed if one is to get rid of the very rapid variation of the helicity amplitudes with q^2 of the quark model with harmonic forces. At the same time one does not want to destroy the "good" predictions for the photoproduction ($q^2=0$) amplitudes (including their magnitudes) of the oscillator model, nor the level structure which has proved so successful in classifying the baryon states. The construction of a suitable model is thus nontrivial, and it is not clear that in doing so one will not be forced to introduce additional parameters and complications, losing the simplicity of the harmonic quark model in the process.

The prediction of helicity-$\frac{1}{2}$ dominance for very large q^2 which we found in the quark model (see Figs. 1–4) does appear to be more general, however. If we write the ratio of the helicity-$\frac{1}{2}$ and -$\frac{3}{2}$ amplitudes for the D_{13} in terms of electric dipole, E, and magnetic quadrupole, M, amplitudes, we have

$$\frac{F_{1/2}(q^2)}{F_{3/2}(q^2)}=\tfrac{1}{3}\sqrt{3}\left(\frac{-E(q^2)+3M(q^2)}{E(q^2)+M(q^2)}\right). \tag{19}$$

Near threshold (which occurs for q^2 timelike) angular momentum arguments tell us that $M/E \propto |\vec{q}|^2$. If we take this behavior to be true even when q^2 is spacelike, then M/E increases without limit as $q^2 \to \infty$ and $F_{1/2}/F_{3/2} \to \sqrt{3}$ as $q^2 \to \infty$. The ratio of $F_{1/2}/F_{3/2}$ increases rather slowly with q^2 in this "threshold-behavior model," and appears to be in agreement[33] with present data at low q^2 on $ep \to ep\pi^0$. At sufficiently large q^2 the $F_{1/2}$ amplitude should still dominate, however, and an extension of the q^2 range of the present experiments would seem worthwhile.

Finally, what might be the relation between the resonance region and the deep-inelastic scattering where, as we saw, the naive quark-parton model predicts[34] large positive asymmetries ($\sigma_{1/2} > \sigma_{3/2}$) on protons? The elastic peak must have $\sigma_{3/2}=0$ or $A=+1$. Aside from that we have seen that the excitation of the *prominent* nucleon resonances from protons has just the opposite behavior ($\sigma_{3/2} > \sigma_{1/2}$ or $A<0$) out to at least $q^2=0.5\ \mathrm{GeV}^2$. The behavior of the other nucleon resonances, many of which are broad and in low partial waves, is unknown at present, except of course those with $J=\frac{1}{2}$, which can only contribute to $\sigma_{1/2}$ and hence have $A=+1$. Thus the behavior of the asymmetry for the sum of *all* nucleon states for $q^2 \simeq 1$, where scaling begins for spin-averaged scattering, is uncertain at present.

However, it is possible to speculate on the behavior of polarized deep-inelastic scattering by considering the saturation of certain sum rules,[6,8] duality near $\omega=1$,[35] and the observed values of σ_n/σ_p together with the results of Sec. III. We would guess that A_n will be positive near $\omega=1$, falling rapidly to zero for $\omega>1$, but that A_p will be $\simeq +1$ near $\omega=1$ and decreasing slowly with ω, so that it is still large and positive at $\omega \simeq 3$. We then expect the resonance region will globally average the deep-inelastic scaling behavior, as in spin-averaged scattering, but locally there will be both positive and negative asymmetries as one moves through the resonance region.[36]

It is also possible that scaling begins at larger q^2 for the spin-dependent structure functions than for the spin-averaged ones, at which point the *prominent* resonances could have changed from their low-q^2 helicity structure to having $\sigma_{1/2} > \sigma_{3/2}$ and $A>0$. Therefore it will be of great interest to continue the coincidence measurements needed to separate the electroproduction helicity amplitudes for each resonance and to measure the asymmetry in single-arm experiments in both the resonance and deep-inelastic regions, in order to study the transition between them as a function of q^2.

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*Work supported in part by the U. S. Atomic Energy Commission.

†NATO Fellow, 1970–1972.

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