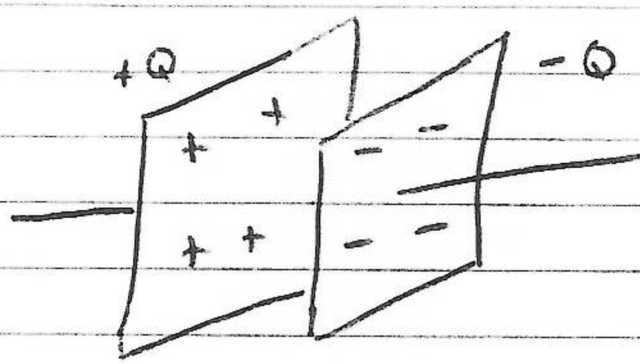
Capacitance

Suppose I have two conductors



- V is constant over conductor
  why?

- Can speak of *the* potential difference between them

$$V_{ab} = -\int_a^b \vec{E} \cdot d\vec{l}$$

If I double Q
$\rightarrow$ V doubles

$$V = \frac{Q}{C}$$

$\frac{1}{C}$: proportionality constant btwn V & Q

C: capacitance

- >0, always a + quantity
- measure of ability to store charge
- units $\frac{\text{Coulombs}}{\text{Volts}} \equiv$ farad
- symbol —||—

C is a <u>geometrical</u> quantity

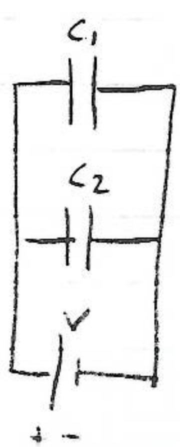
Determined by

- sizes of conductors
- shapes of conductors
- relative orientation
- separation of conductors

→ A <u>constant</u> which describes the two conductor system

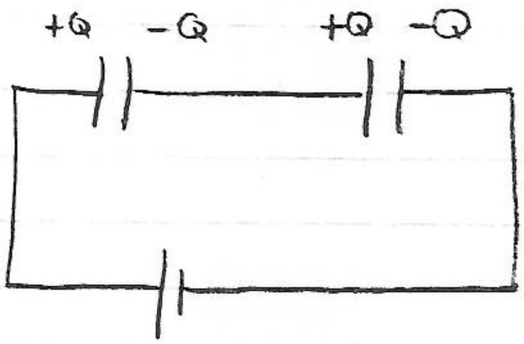
<u>Circuit configurations</u>

Connected in <u>parallel</u>



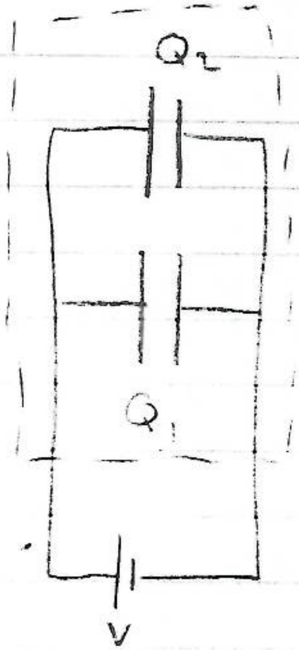
Same <u>potential</u> across each

Connected in <u>series</u>



Same <u>charge</u> on each

Consider C's in parallel



Important trick:

Replace  w/ one capacitor

C_{eq} has same effect on circuit as C_1 and C_2

$$Q_1 = C_1 V \qquad Q_2 = C_2 V$$

$$Q = (C_1 + C_2) V$$

$$= C_{eq} V$$

$$C_{eq} = C_1 + C_2$$

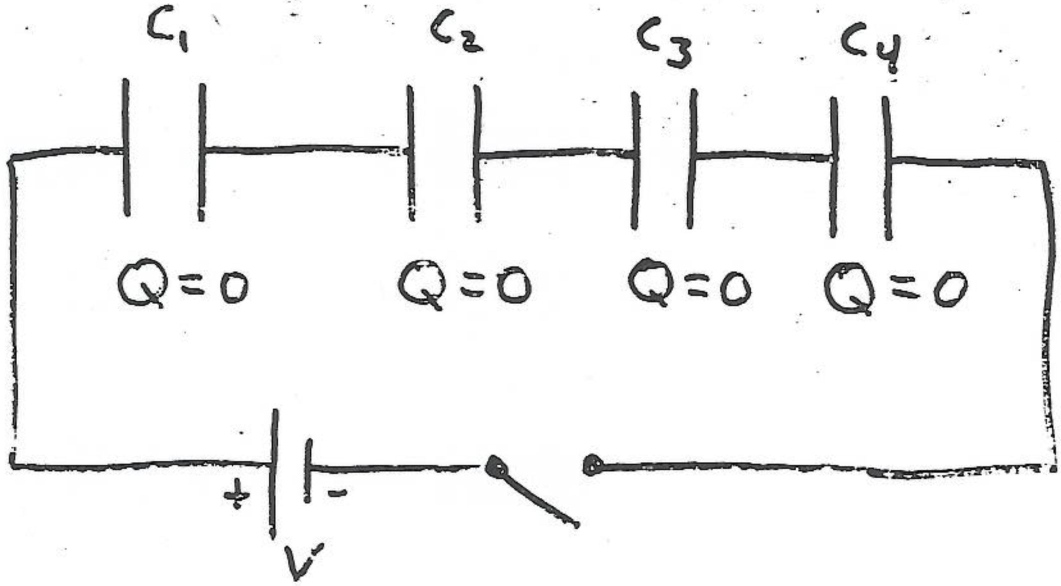
For more than two capacitors

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

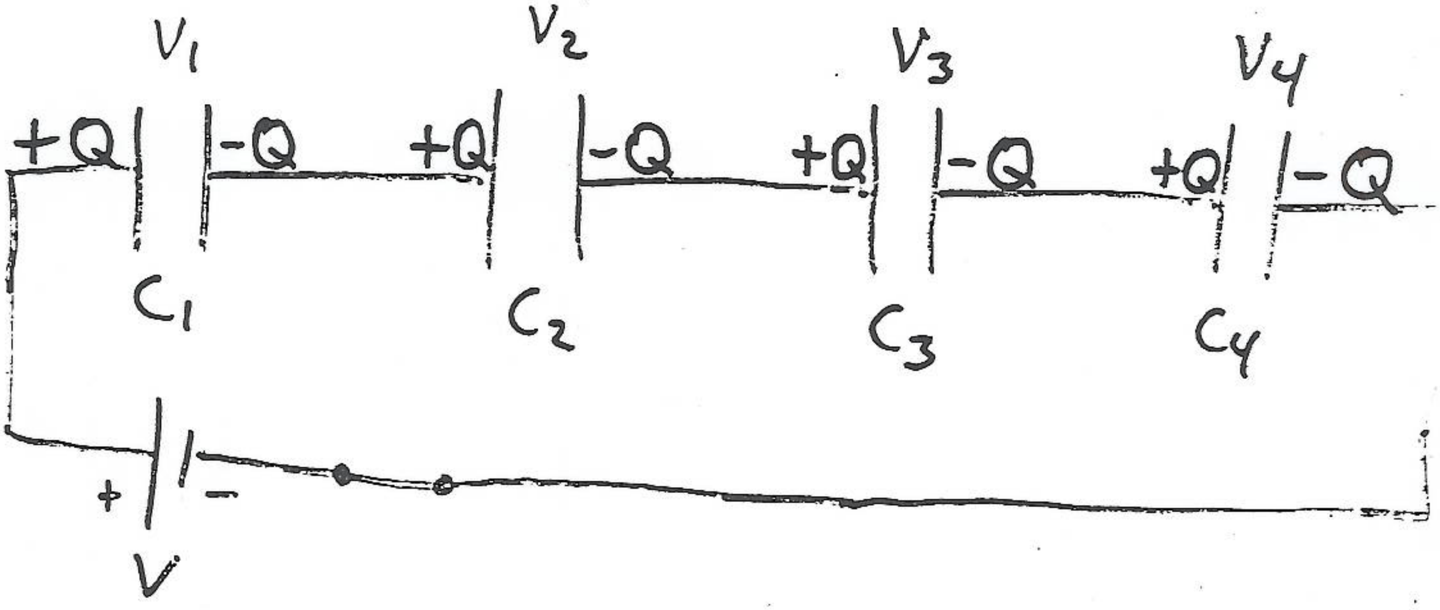
Equivalent capacitance of capacitors in parallel is the sum of the individual capacitances.

→ Capacitors in parallel, C_{eq} is greater than individual capacitances

Capacitors in series - Open switch



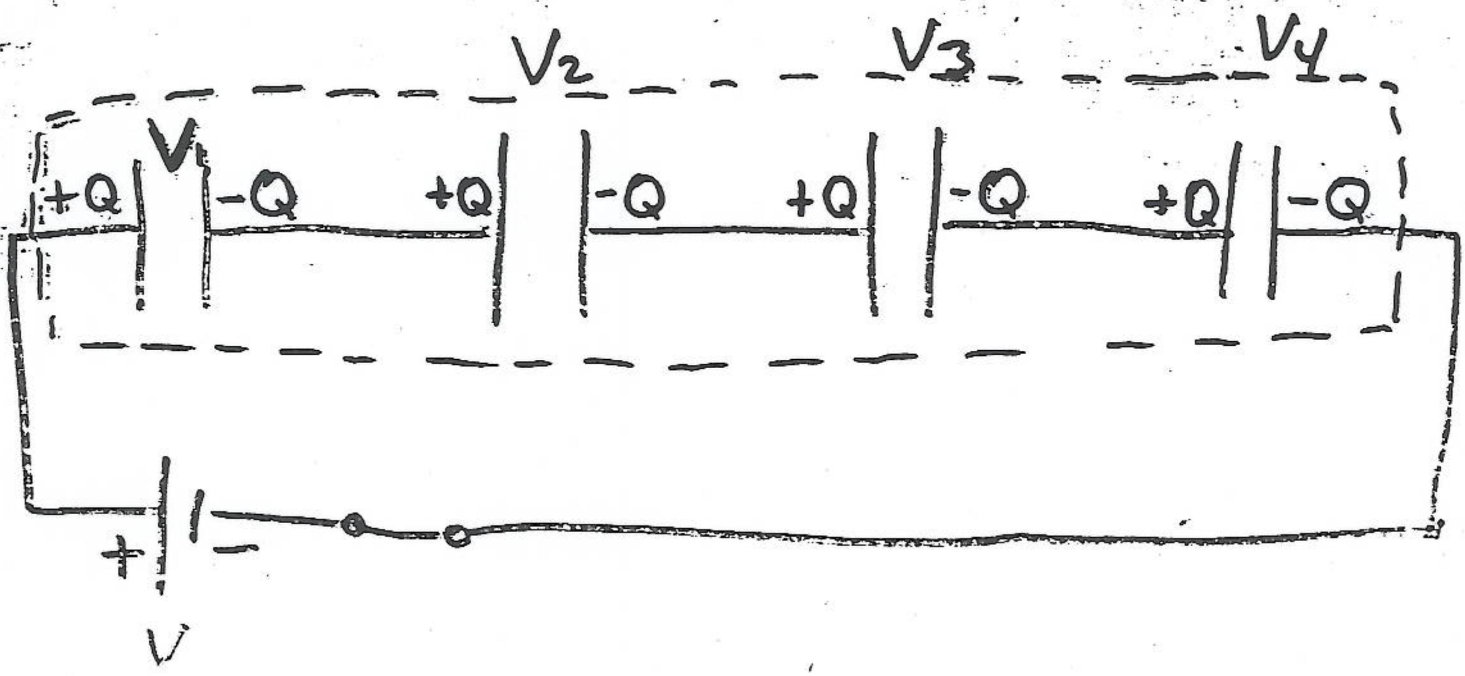
Closed switch



Charge does not jump gap, yet it flow

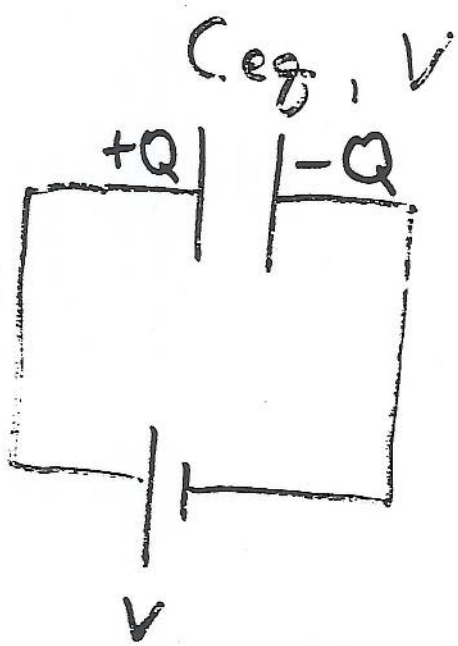
→ induction

What's our equivalent capacitor?



Replace 

$\Longrightarrow$



So, we have

$$V = \frac{Q}{C_{eq}}$$

$$V = V_1 + V_2 + V_3 + V_4$$

$$V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad V_3 = \frac{Q}{C_3} \quad V_4 = \frac{Q}{C_4}$$

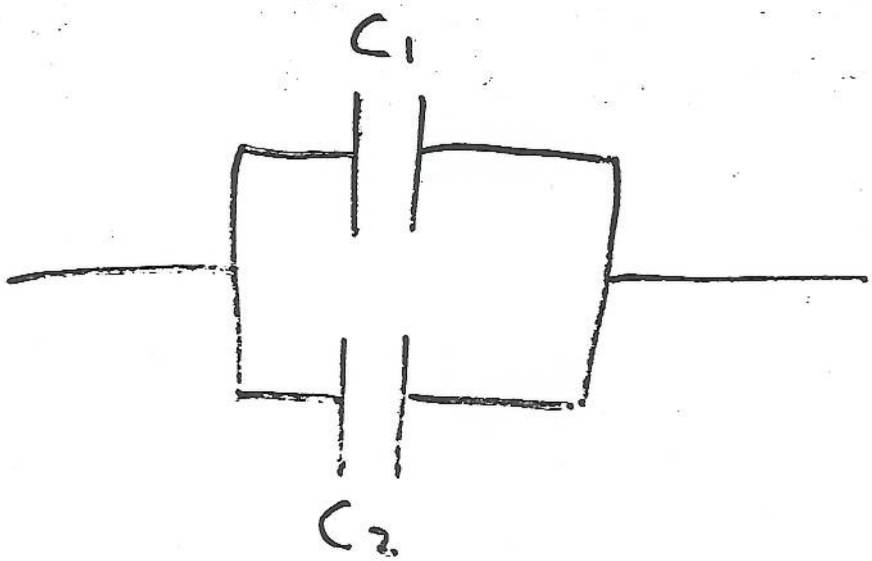
$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}\right)$$

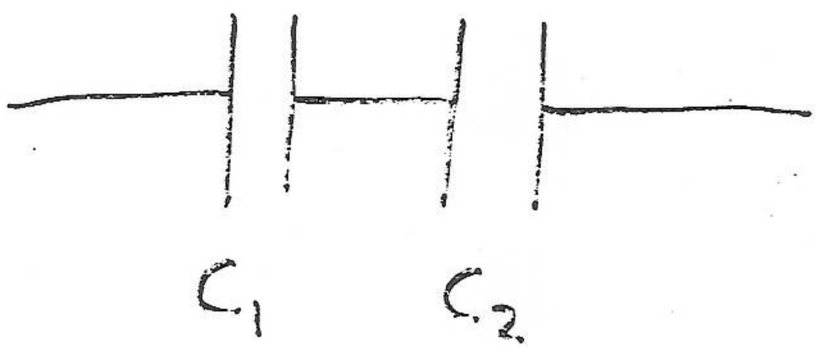
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots}$$

Capacitors in series

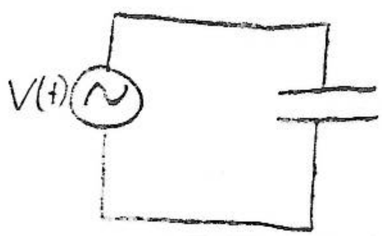
C_{eq} is always <u>less</u> than individual C's when in series

Which has greater C_{eq}?





Capacitive AC circuit



$Q = VC$

$V(t) = V_0 \cos\omega t$

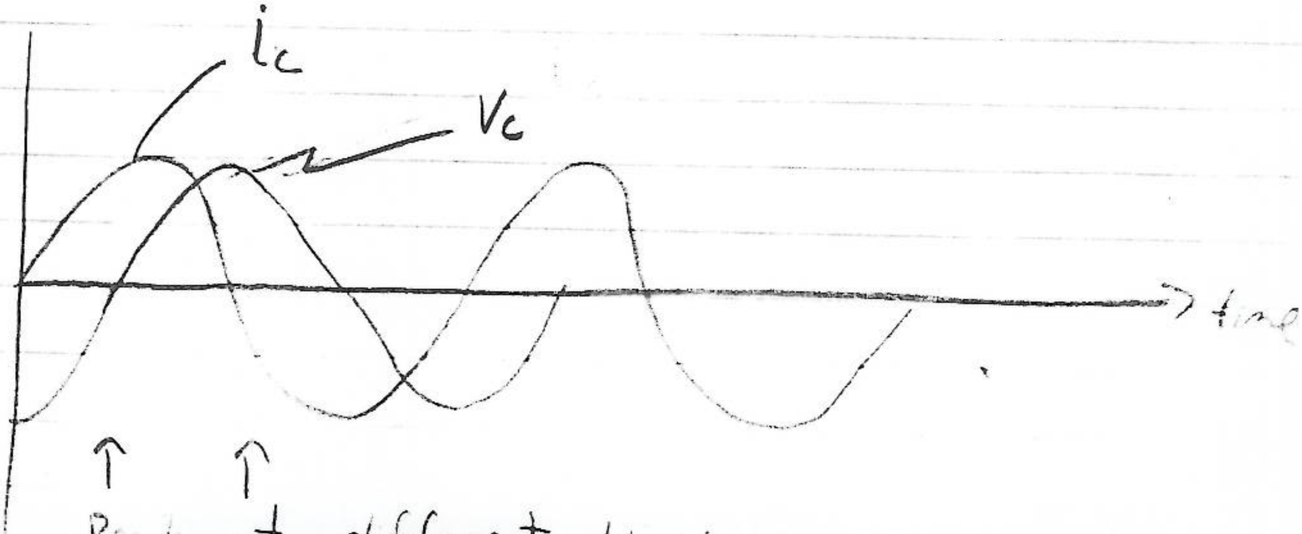
$Q(t) = V(t)C = V_0 C \cos\omega t$

$i(t) = \dfrac{dQ}{dt} = -\omega V_0 C \sin\omega t$

$= \omega V_0 C \cos(\omega t + \frac{\pi}{2})$ *phase

Large ω, large I → capacitor shorts at high ω
Small ω, small I → open circuit for DC

* What's this $\pi/2$?



Peak at different times

I leads V

$$V(t) = V_0 \cos\omega t = V_0 \mathrm{Re}\, e^{j\omega t}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$i = \omega C V_0 \mathrm{Re}\, e^{j(\omega t + \pi/2)}$$

Want eqn like $V = IR$, $I = V/R$

$$i = \omega C V_0 \mathrm{Re}\left[e^{j\omega t} e^{j\pi/2}\right]$$

$$= \mathrm{Re}\left[j\omega C V_0 e^{j\omega t}\right]$$

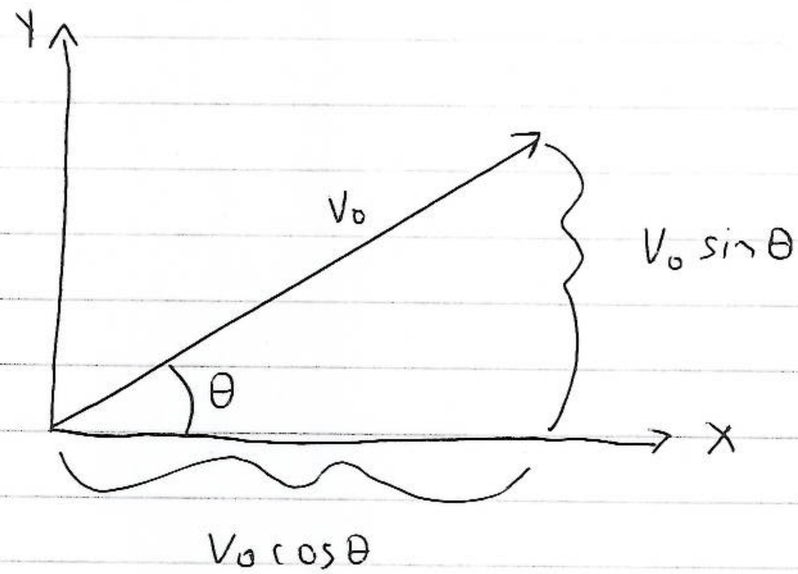
$$i(t) = \frac{V_0 e^{j\omega t}}{\boxed{1/j\omega C}} \longleftarrow \text{like "R"}$$

$$X_c = \frac{1}{j\omega C}$$

Big $\omega \rightarrow$ small X_c
Small $\omega \rightarrow$ big X_c

Phasor diagrams

Recall $e^{j\theta} = \cos\theta + j\sin\theta$



Rotating vector $\theta = \omega t$

Adding two constants

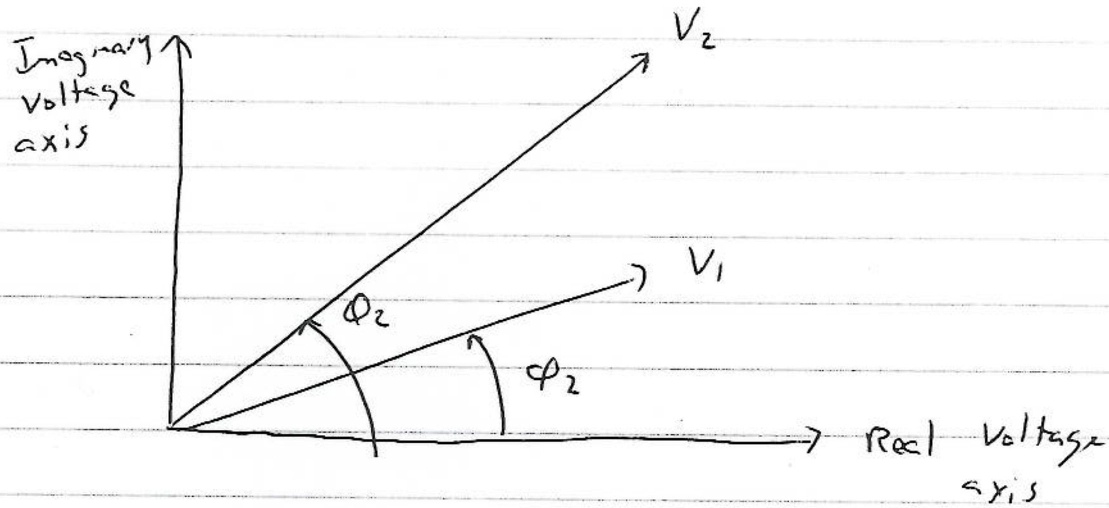
$3+4=7$ easy

Adding two waves - add x comps of vectors

$$V(t) = V_1 \cos(\omega t + \varphi_1) + V_2 \cos(\omega t + \varphi_2)$$

sum depends on time

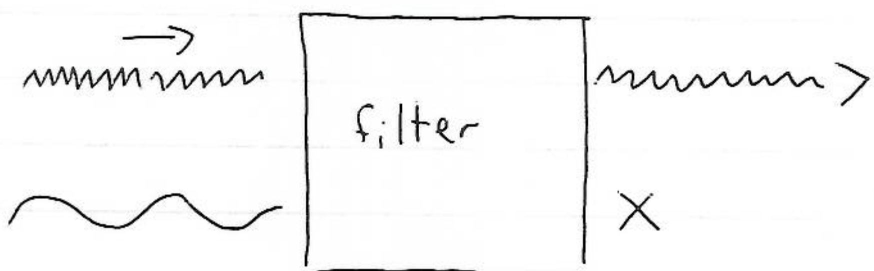
Complex voltage plane



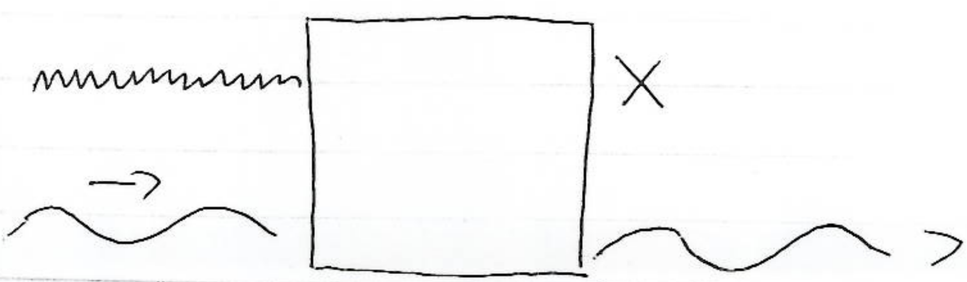
$v(t) \rightarrow$ sum of vectors in 2-d complex plane

Filters (Labs 3 & 4)

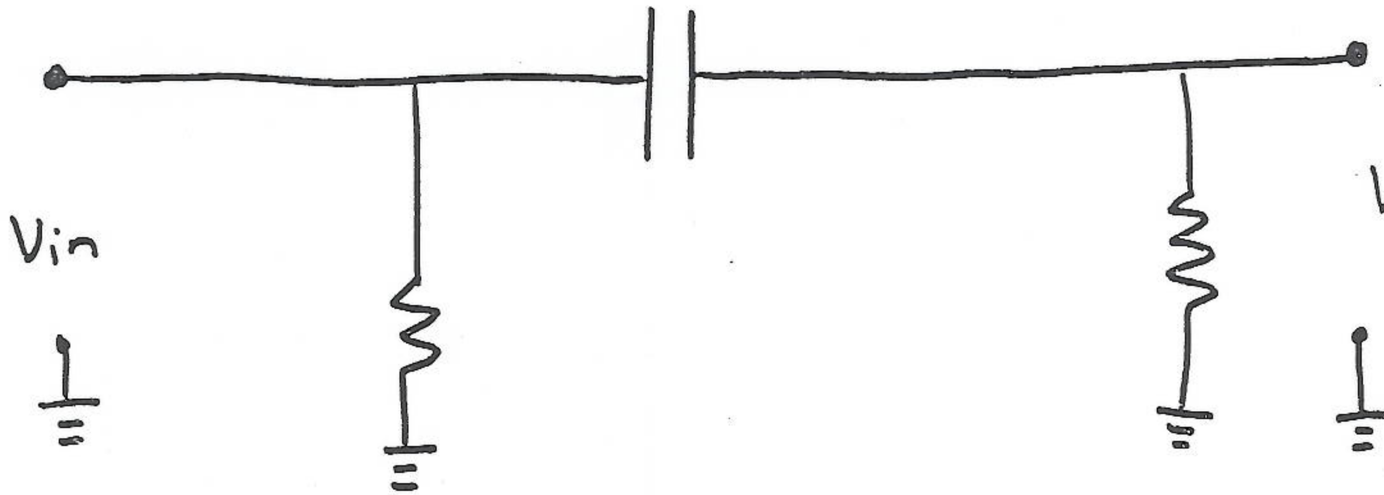
High Pass Filter

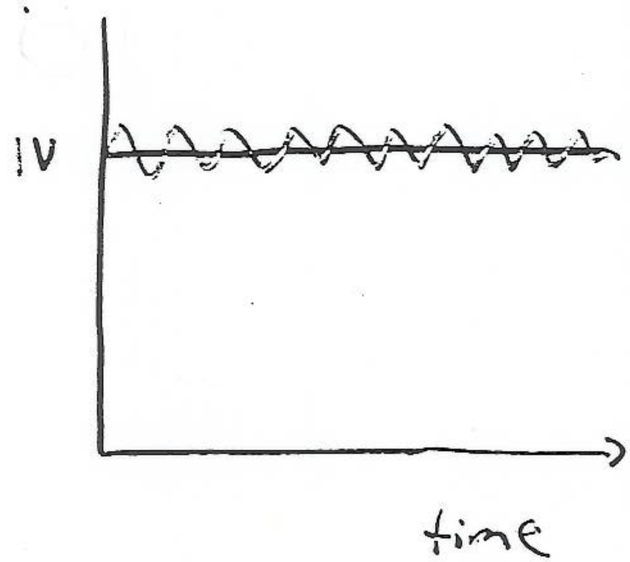


Low Pass Filter

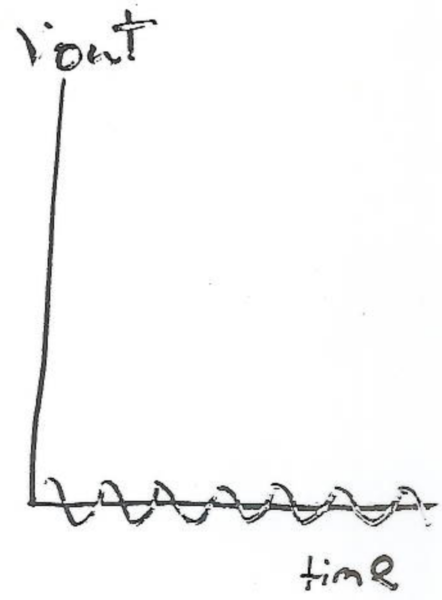


Ex:





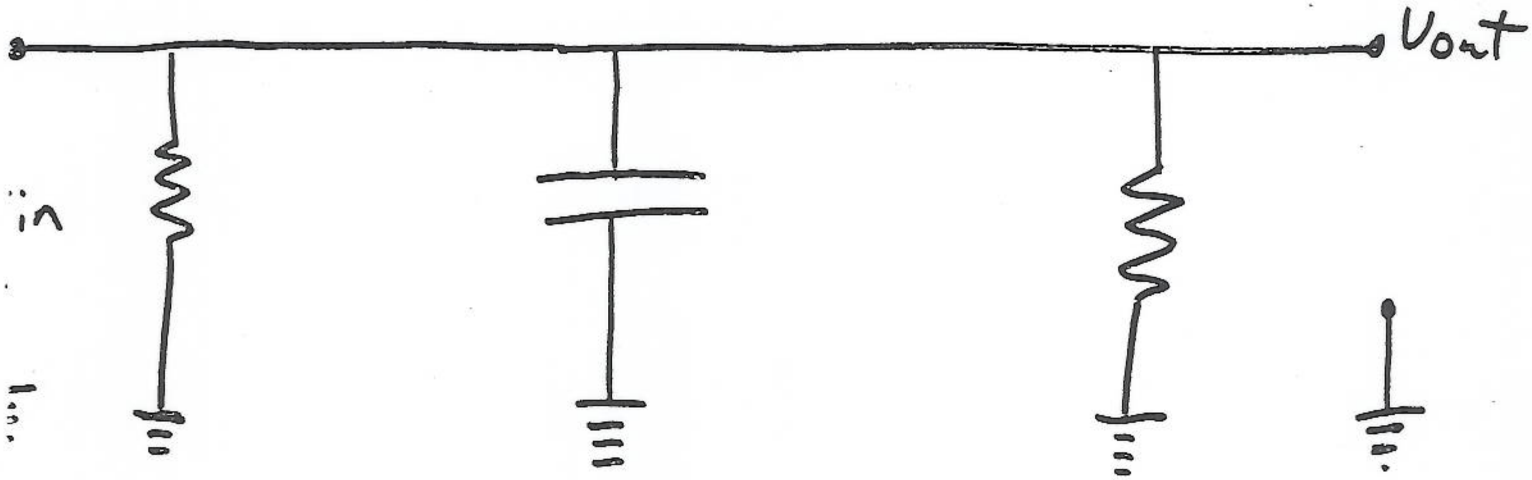
$\boxed{\text{C blocks D.C.}}$

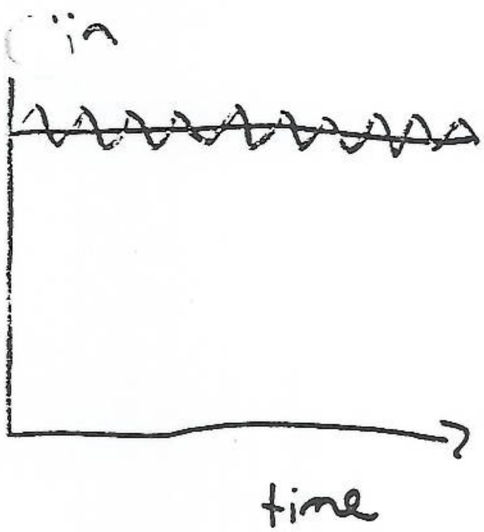


High voltage D.C. level
ith signal riding on it.

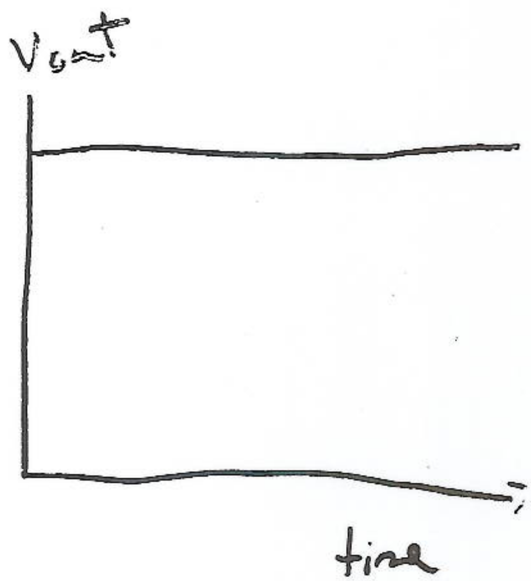
Output voltag
with just
signal

Ex:





C shorts AC to ground

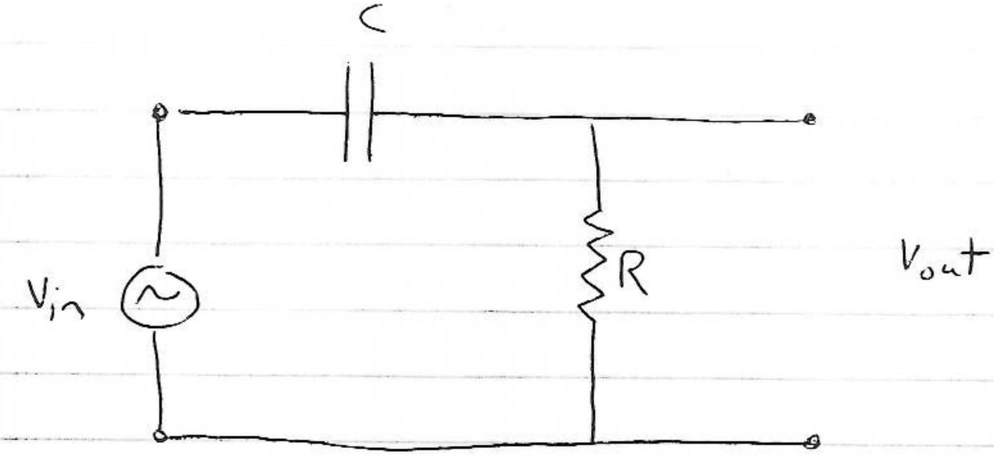


D.C. level with unwanted noise on it.

Cleaned up signal

High Pass filter

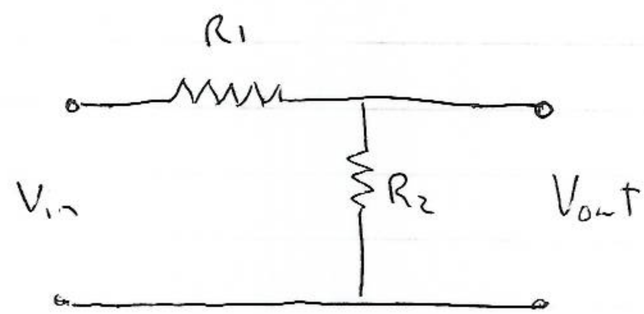
Consider



$$\text{gain} \equiv \frac{V_{out}}{V_{in}} \qquad \text{attenuation} \equiv \frac{V_{in}}{V_{out}}$$

Aside:

If I had



$$V_{out} = \frac{R_2}{R_1+R_2} V_{in}$$

Now, same deal

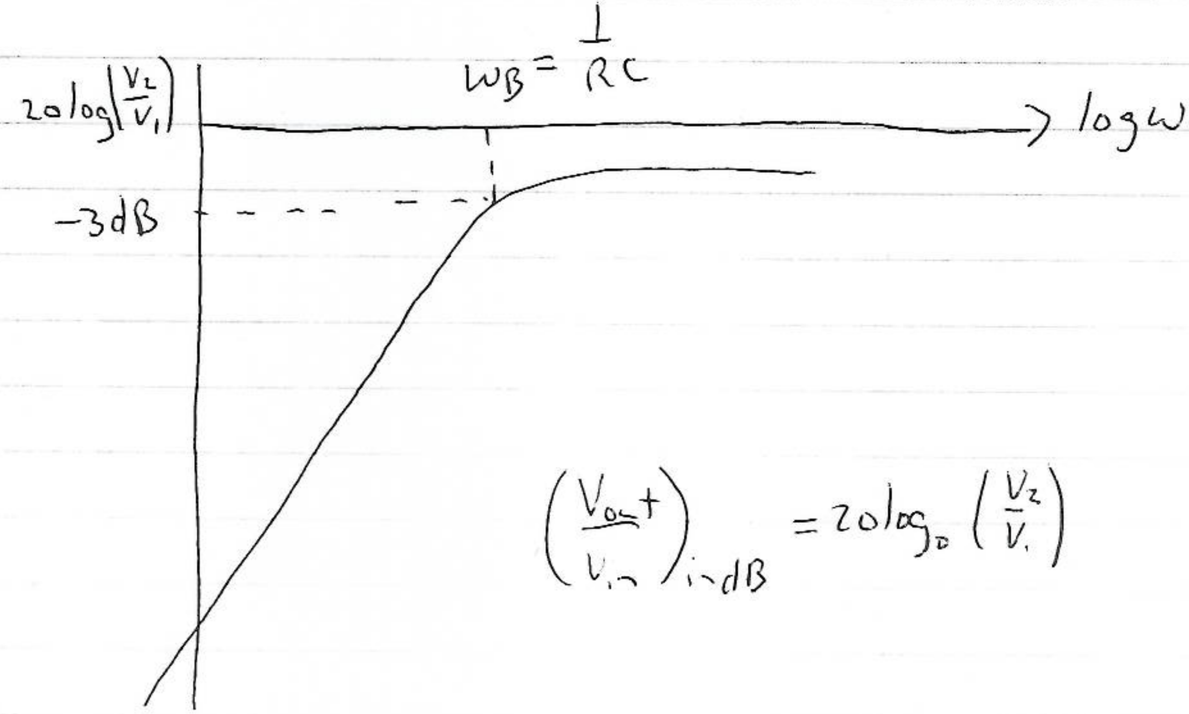
90° out of phase

$$V_{out} = \frac{R}{R+X_C} V_{in} = \frac{R}{R+\frac{1}{j\omega C}} V_{in} = \frac{j\omega CR}{1+j\omega CR} V_{in}$$

$$\left|\frac{V_{out}}{V_{in}}\right|^2 = \left(\frac{V_{out}}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}}\right)^* = \left(\frac{j\omega CR}{1+j\omega CR}\right)\left(\frac{-j\omega CR}{1-j\omega CR}\right)$$

$$= \frac{\omega^2 C^2 R^2}{1+\omega^2 C^2 R^2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$

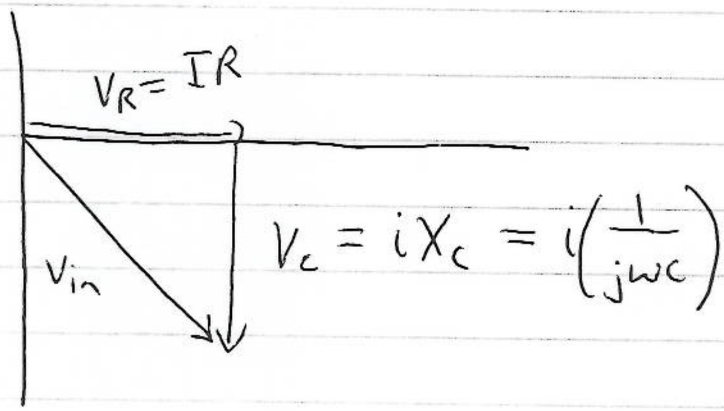


$$\left(\frac{V_{out}}{V_{in}}\right)_{in\,dB} = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

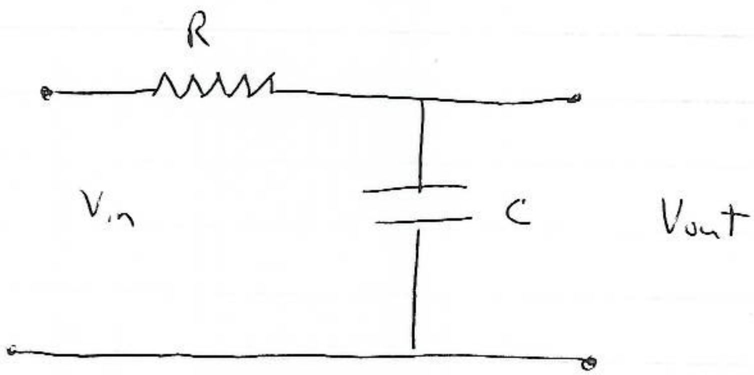
Phasor diagram

Kirchhoff says

$$V_{in} = V_c + V_R \qquad \text{ELI ICE}$$

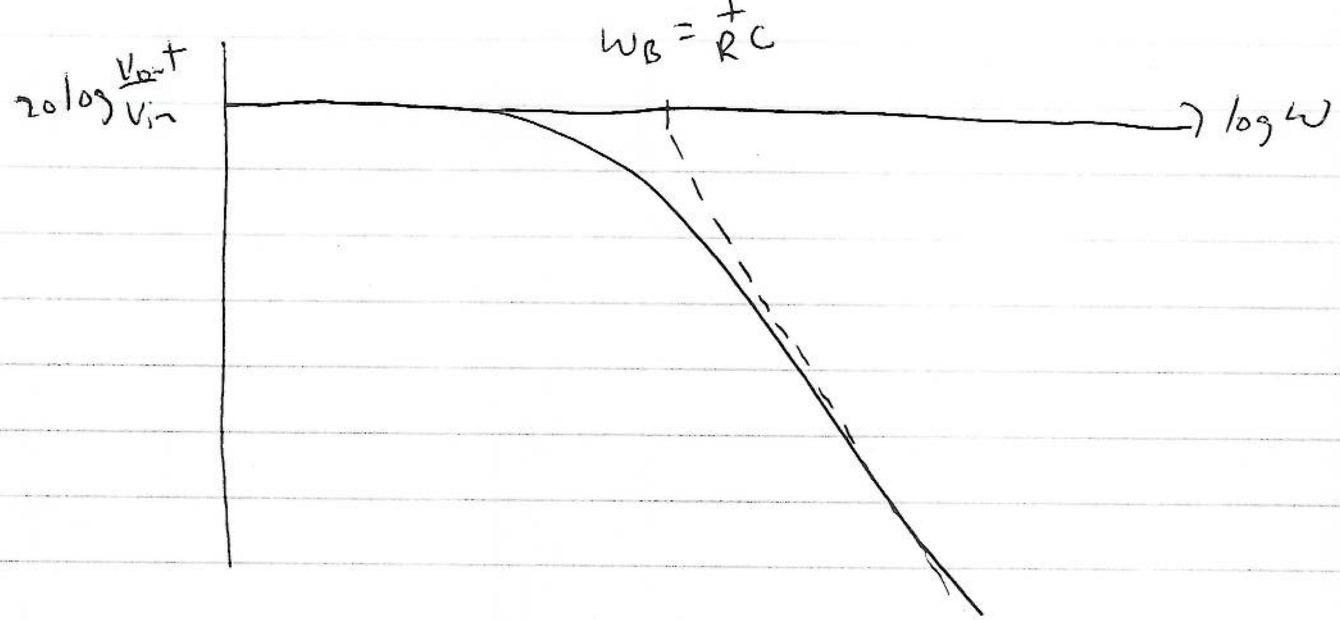


RC Low Pass Filter



$$V_{out} = \frac{X_c}{X_R + X_c} V_{in} = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} V_{in} = \frac{1}{1 + jwCR} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1 + w^2 R^2 C^2}}$$



Phasor diagram