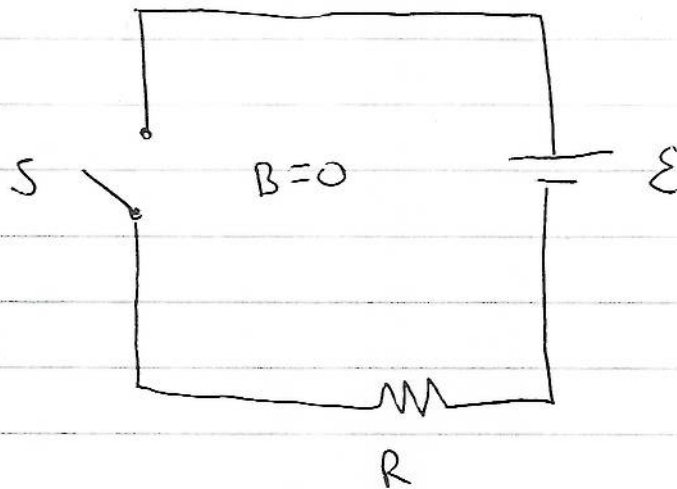
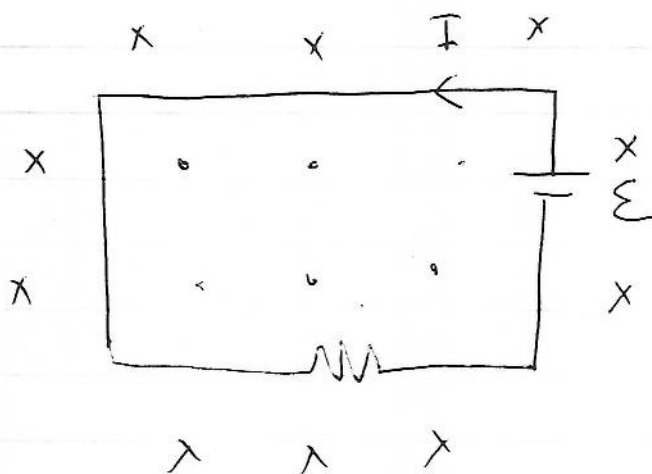


Inductance
Consider



close switch



Increasing flux in circuit

$\frac{d\Phi}{dt} \rightarrow$ induced emf opposing battery
self induced emf

Recall Faraday's law

$$\mathcal{E} = - \frac{d\Phi}{dt} \quad \text{where} \quad \Phi = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ inside circuit loop area of circuit loop

Ampere, Biot-Savart

$$B \propto I \quad \text{so} \quad \Phi \propto I$$

$$\mathcal{E}_{\text{induced}} \propto - \frac{dI}{dt}$$

$$\mathcal{E} = - L \frac{dI}{dt}$$



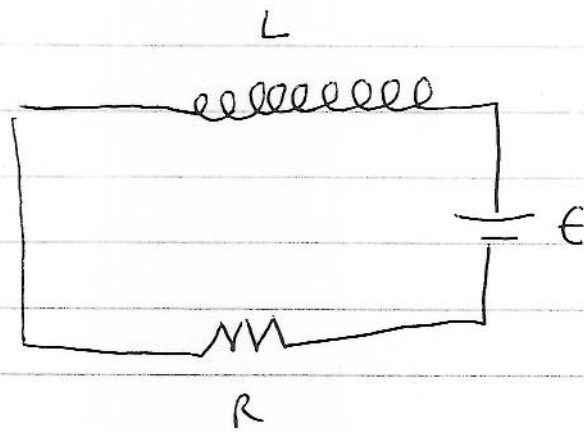
proportionality
constant

L : inductance

Define

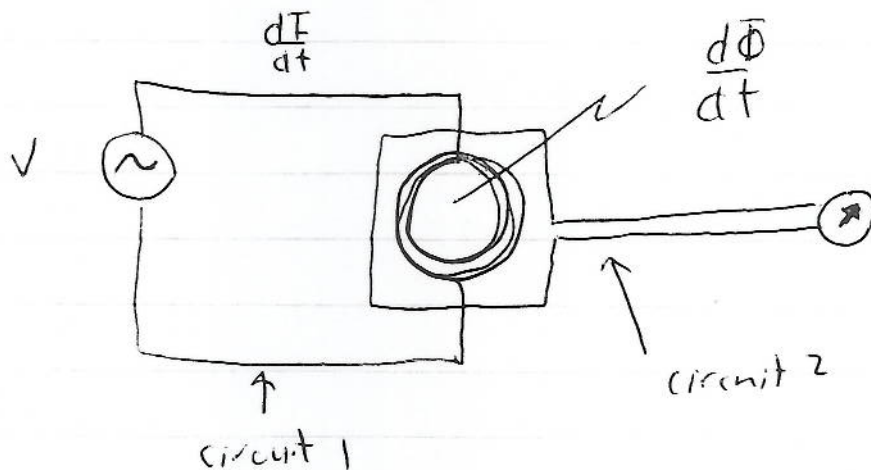
$$L = - \frac{\mathcal{E}}{\frac{dI}{dt}} \quad \text{henry}$$

To beef up inductance

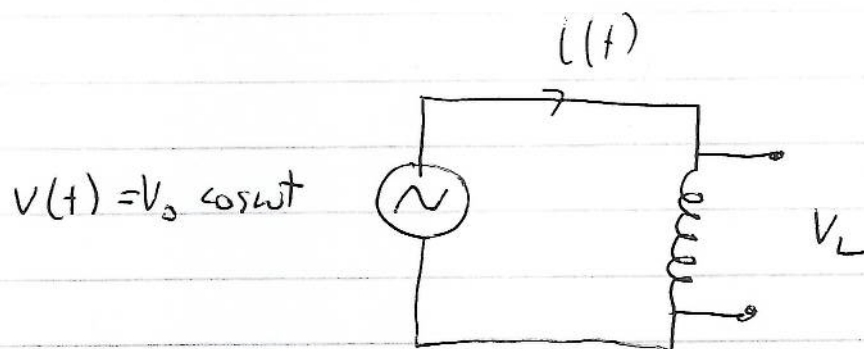


* Self induction - time varying current in some conductor produces emf in that same conductor

mutual induction - time varying flux in one circuit produces emf in another



Now consider



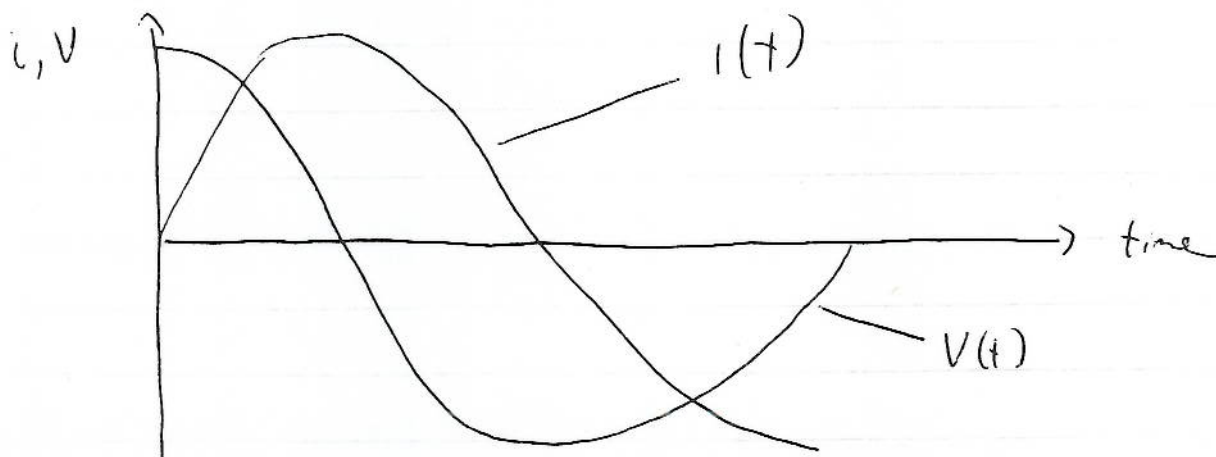
$$V = L \frac{di}{dt}$$

$$\int_0^t di = i(t) - i(0)$$

$$i(t) - i(0) = \frac{1}{L} \int_0^t V_0 \cos \omega t \, dt$$

$$i(0) = 0, \quad i(t) = \frac{V_0}{L\omega} \sin \omega t = \frac{V_0}{L\omega} \cos\left(\omega t - \frac{\pi}{2}\right)$$

V, i out of phase



ELI

$$i = \operatorname{Re} \left(\frac{V_0}{j\omega L} e^{j\omega t} \right) = \frac{V(t)}{j\omega L}$$

$$X_L = j\omega L \quad \text{inductive reactance}$$

Mechanical analogy

Inductance $\longleftrightarrow$ mass

Large L

$\rightarrow$ Hard to change current rapidly

Small L

$\rightarrow$ Easy to change current rapidly.

Large mass

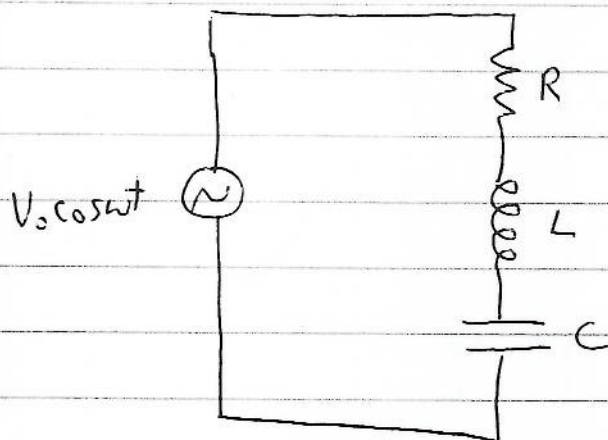
$\rightarrow$ Hard to move object

Small mass

$\rightarrow$ Easy to move it

9/99

X RLC Circuits
Consider



The point : $I = \frac{V}{Z}$ where $Z = R + j\omega L + \frac{1}{j\omega C}$

$$Z = R + X_L + X_C$$

Sketch of the hard way:

Kirchhoff says

$$V_0 \cos \omega t = iR + L \frac{di}{dt} + \frac{Q}{C}$$

$$\frac{d}{dt} : \quad L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = -\omega V_0 \sin \omega t$$

Assume soln of the form

$$i = I_0 \cos(\omega t - \theta)$$

Plug in ...

Get

$$I_0 \left[\left(\omega L - \frac{1}{\omega C} \right) \cos \theta - R \sin \theta \right] \cos \omega t + \left[I_0 \left(\omega L - \frac{1}{\omega C} \right) \sin \theta + I_0 R \cos \theta - V_0 \right] \sin \omega t = 0$$

$\cos \omega t$, $\sin \omega t$ orthogonal.

Each coefficient must be zero

$$\left(\omega L - \frac{1}{\omega C} \right) \cos \theta - R \sin \theta = 0$$

$$I_0 \left(\omega L - \frac{1}{\omega C} \right) \sin \theta + I_0 R \cos \theta - V_0 = 0$$

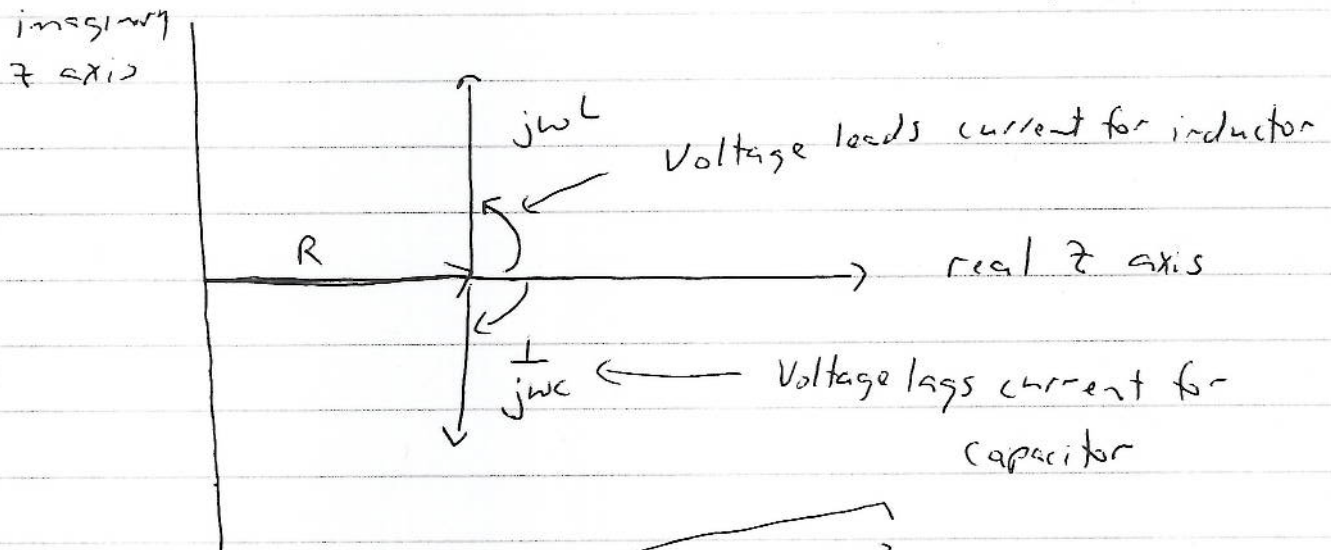
$$\tan \theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$I_0 = \frac{V_0}{R \cos \theta + \left(\omega L - \frac{1}{\omega C} \right) \sin \theta}$$

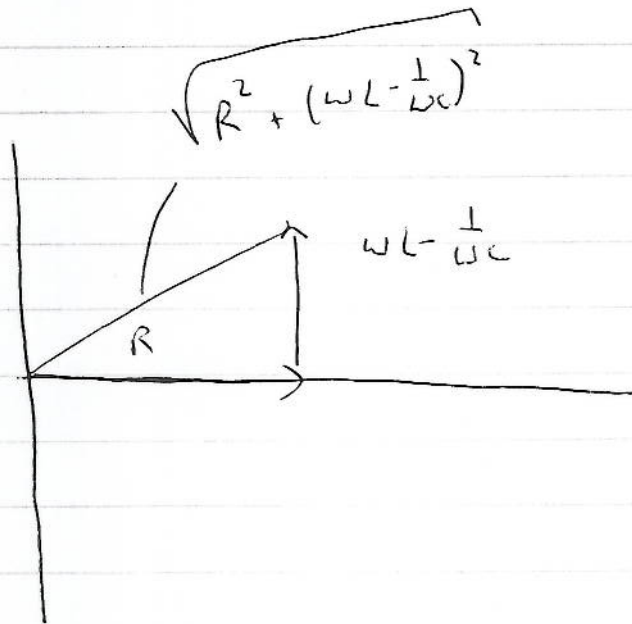
Easier way

Complex impedance plane

ELI ICE



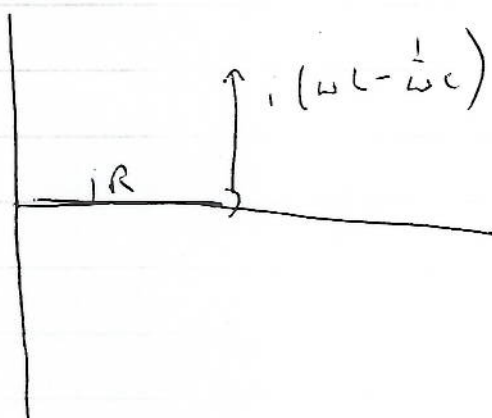
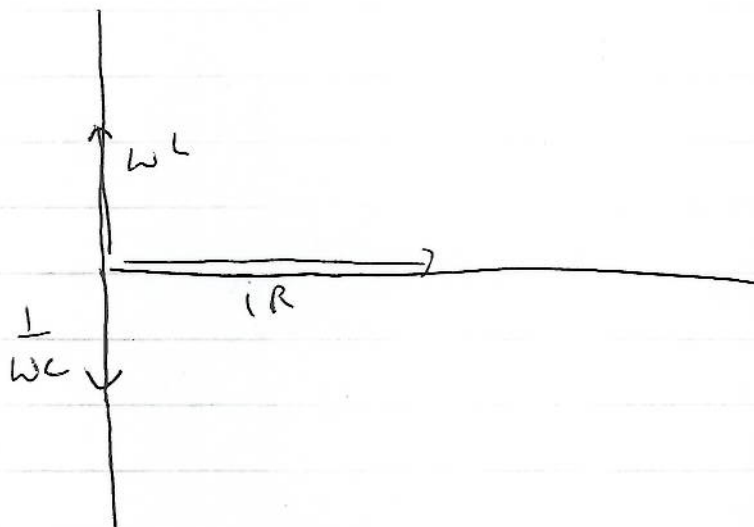
$\Rightarrow$



$$i(t) = \frac{V(t)}{Z} = \frac{V_0 e^{j\omega t}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$I_0 = \frac{V_0}{\left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}}$$

Complex voltage plane

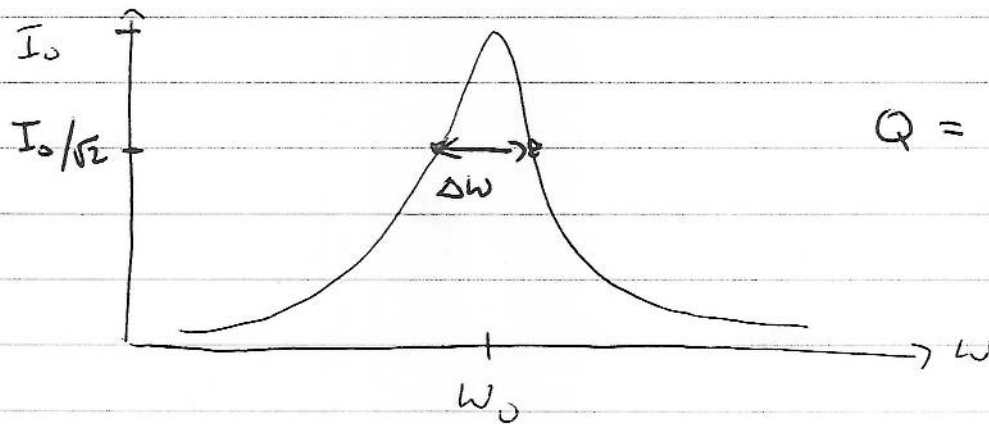


Resonance - series RLC circuit

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

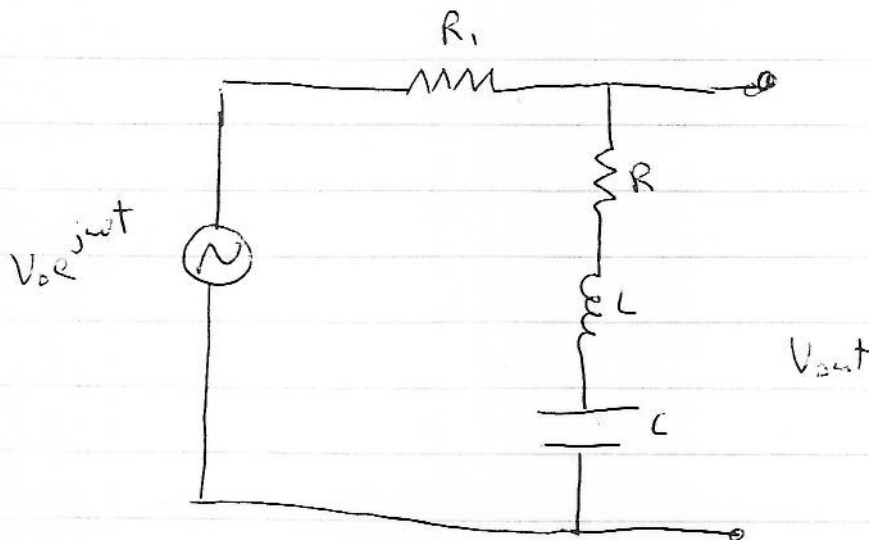
I_0 maximum when $\omega L - \frac{1}{\omega C} = 0$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Z = R$$



$$Q = \frac{\omega_0}{\Delta\omega}, \text{ Quality factor}$$

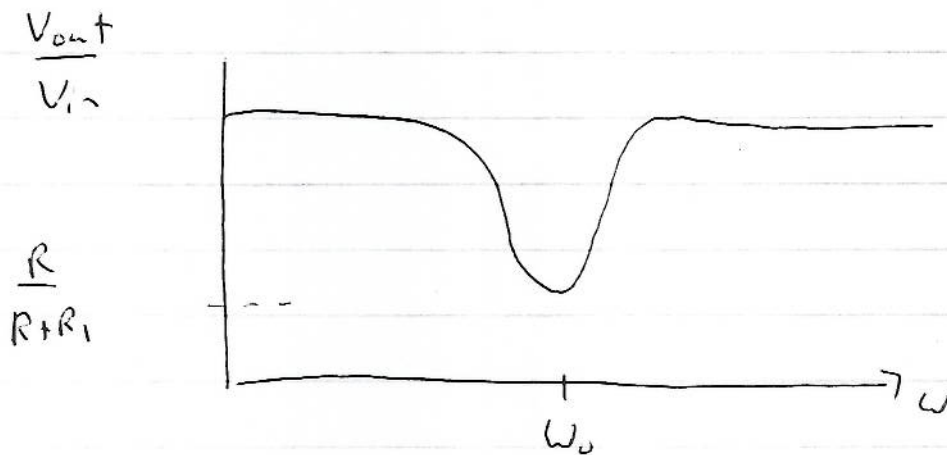
Now consider



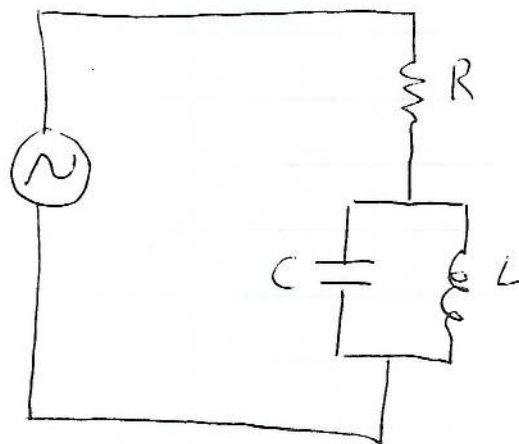
$$Z \text{ across } V_{out} = R + j\omega L + \frac{1}{j\omega C}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\text{When } \omega = \frac{1}{\sqrt{LC}}, \quad Z_{out} \text{ minimum}$$



Parallel RLC circuit - Tank circuit

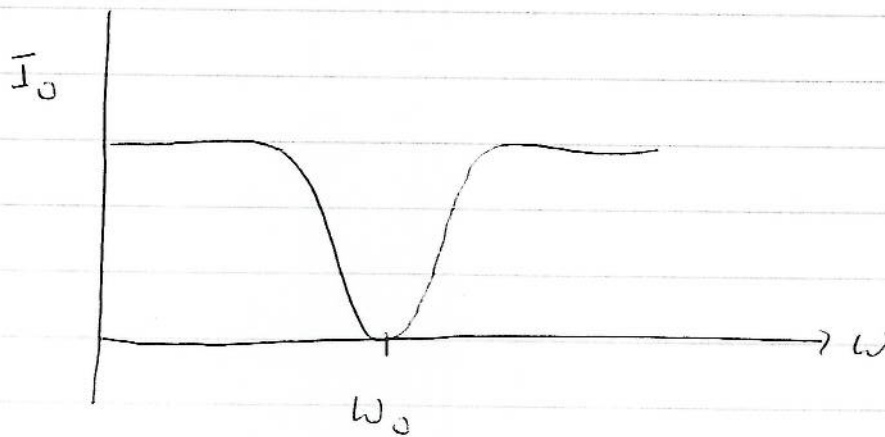


$$\frac{1}{Z_{LC}} = \frac{1}{j\omega L} + \frac{1}{1/j\omega C}$$

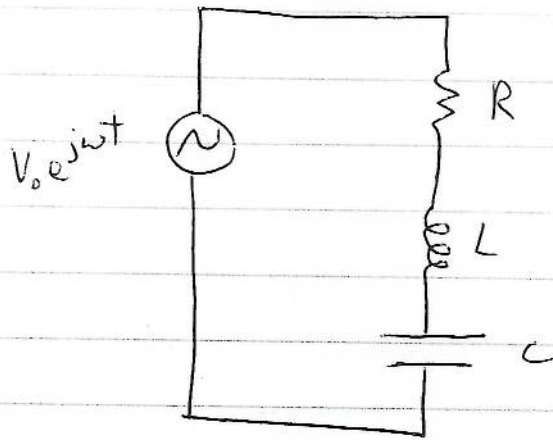
$$Z_{LC} = \frac{j\omega L \left(\frac{1}{j\omega C} \right)}{j\omega L + \frac{1}{j\omega C}} = \frac{-j \frac{L}{C}}{\omega L - \frac{1}{\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$= \frac{j\omega L}{1 - \frac{\omega^2}{\omega_0^2}}$$

$$\omega \rightarrow \omega_0 \quad Z_{LC} \rightarrow \infty$$



Numerical Example - with a point



$$V_0 = 10 \text{ V}$$

$$R = 5 \Omega$$

$$L = 4 \text{ mH}$$

$$C = 0.1 \mu\text{F}$$

At resonance, give I , V_L , V_C

$$I = \frac{V}{Z} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 10^{-3} \text{ H} \times 10^{-7} \text{ F}}} = 5 \times 10^4 \text{ rad/s}$$

$$V_L = I X_L = I \omega L = 2 \text{ A} \times 5 \times 10^4 \frac{\text{rad}}{\text{sec}} \times 4 \times 10^{-3} = 400 \text{ V}$$

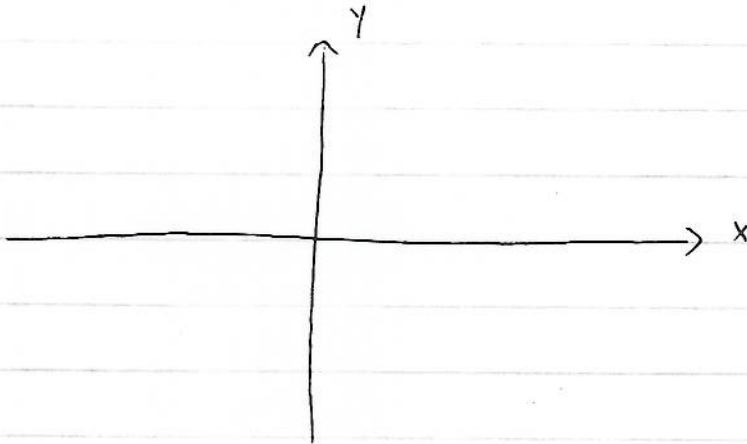
$$V_C = I X_C = \frac{I}{\omega C} = \frac{2 \text{ A}}{5 \times 10^4 \times 10^{-7}} = 400 \text{ V}$$

V_L , V_C both greater than V_0 !

Fourier Analysis

Vector Analogy (important for quantum mechanics & electronics)

Consider 2-D Cartesian plane.



Define $\hat{x}$: unit vector in x direction
 $\hat{y}$: y direction

Claim: Any vector $\vec{v}$ in 2 D plane can be described by

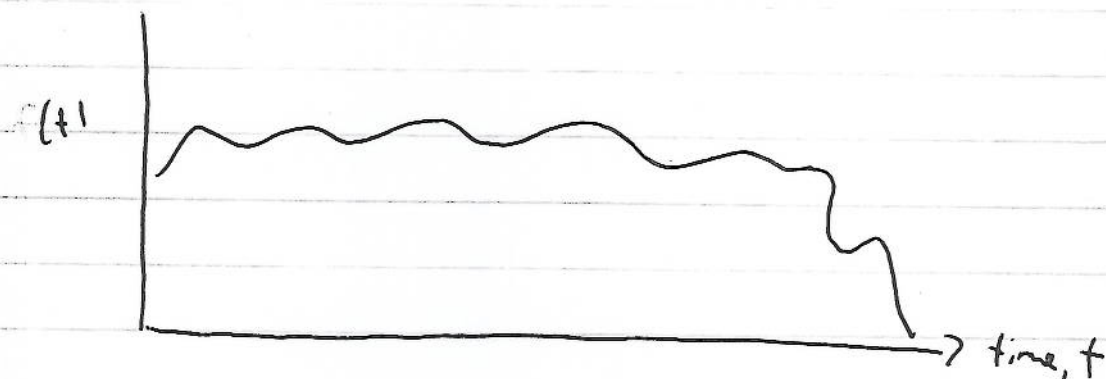
$$\vec{v} = a\hat{x} + b\hat{y}$$

$\hat{x} \ \& \ \hat{y}$ are complete $\rightarrow$ they span the entire space & can express any vector.

$\hat{x} \ \& \ \hat{y}$ are orthogonal $\rightarrow \hat{x} \cdot \hat{y} = 0$, perpendicular

Functions:

Just as any vector is a combination of $\hat{x}$'s, $\hat{y}$'s & $\hat{z}$'s
 (almost) any function is a combination of sines and cosines.

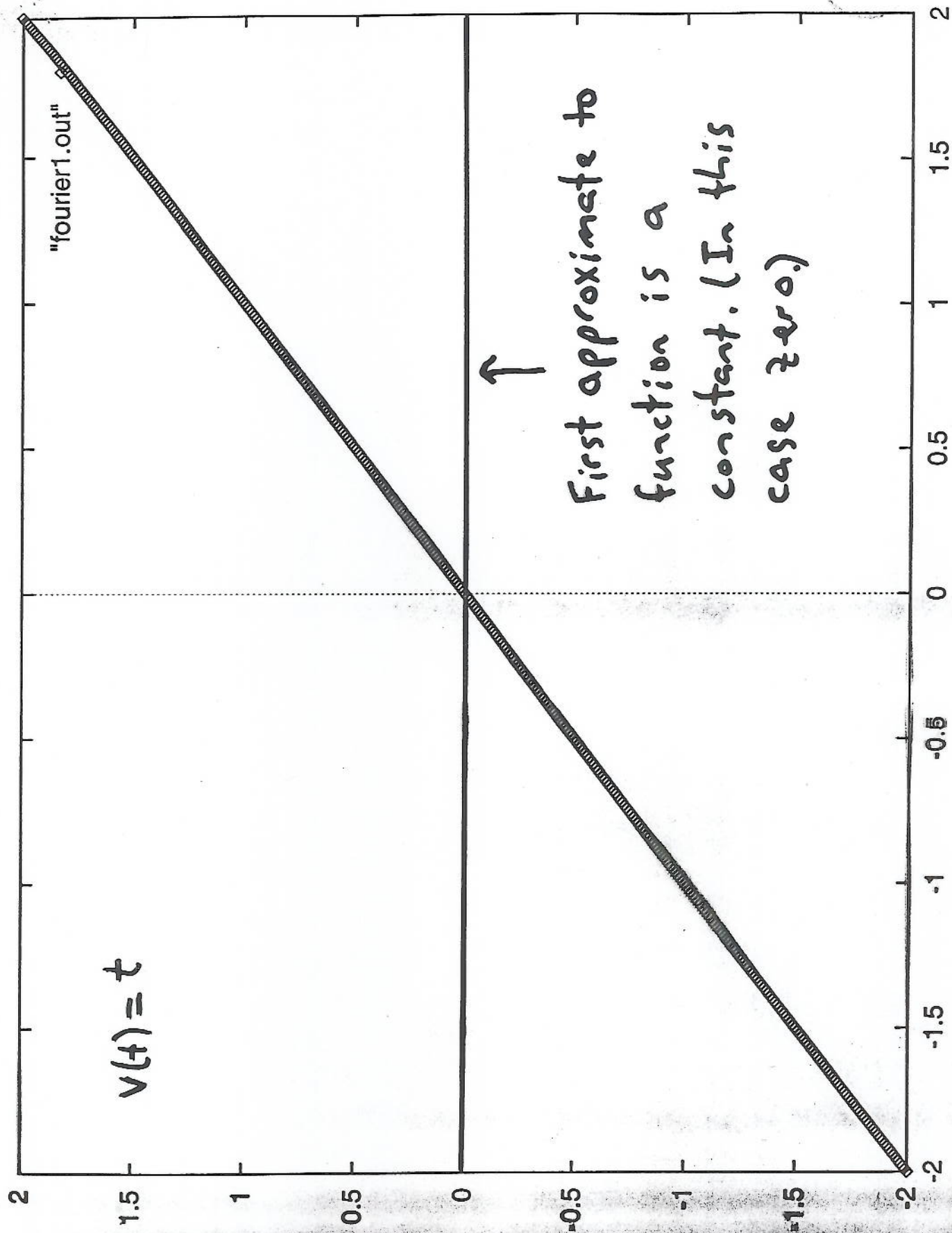


$$V(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

constant (D.C. offset) $\nearrow$ $\frac{a_0}{2}$
 $\nwarrow$ $\sum_{n=1}^{\infty} a_n \cos n\omega t$ cosines of different amplitudes & frequencies
 $\nwarrow$ $\sum_{n=1}^{\infty} b_n \sin n\omega t$ $\nwarrow$ amplitude $\nwarrow$ frequency

Example

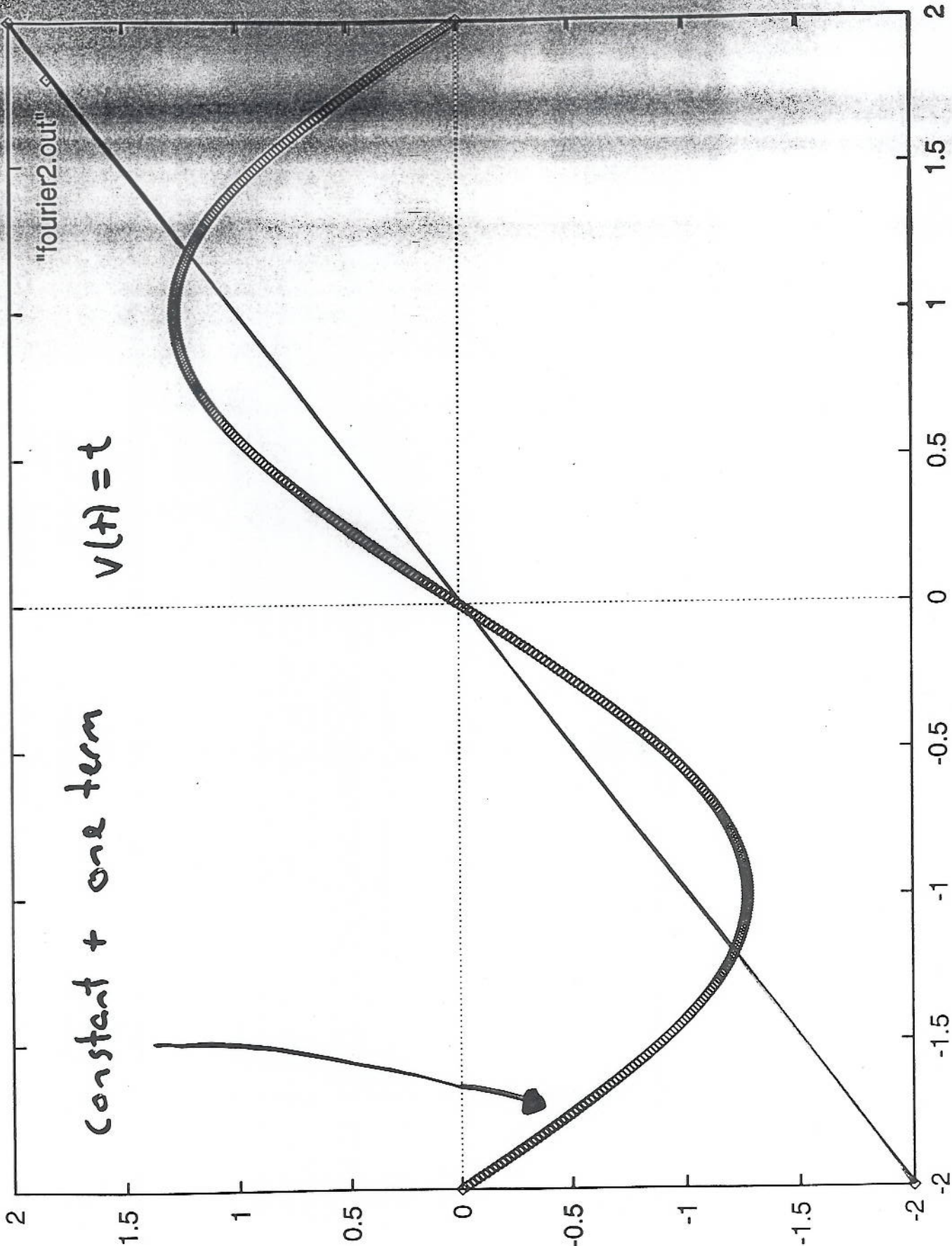
$$V(t) = t \quad -2 < x < 2$$

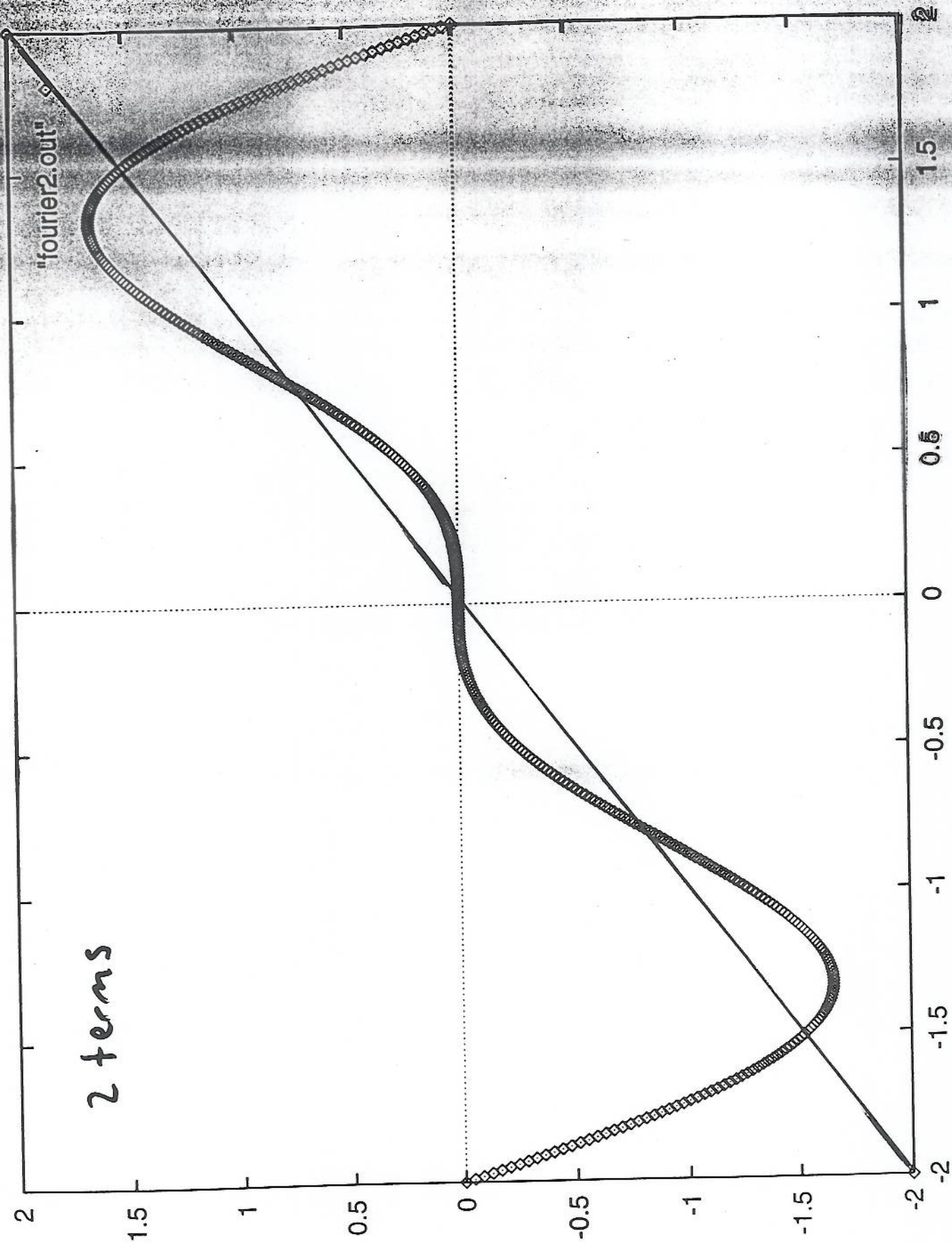


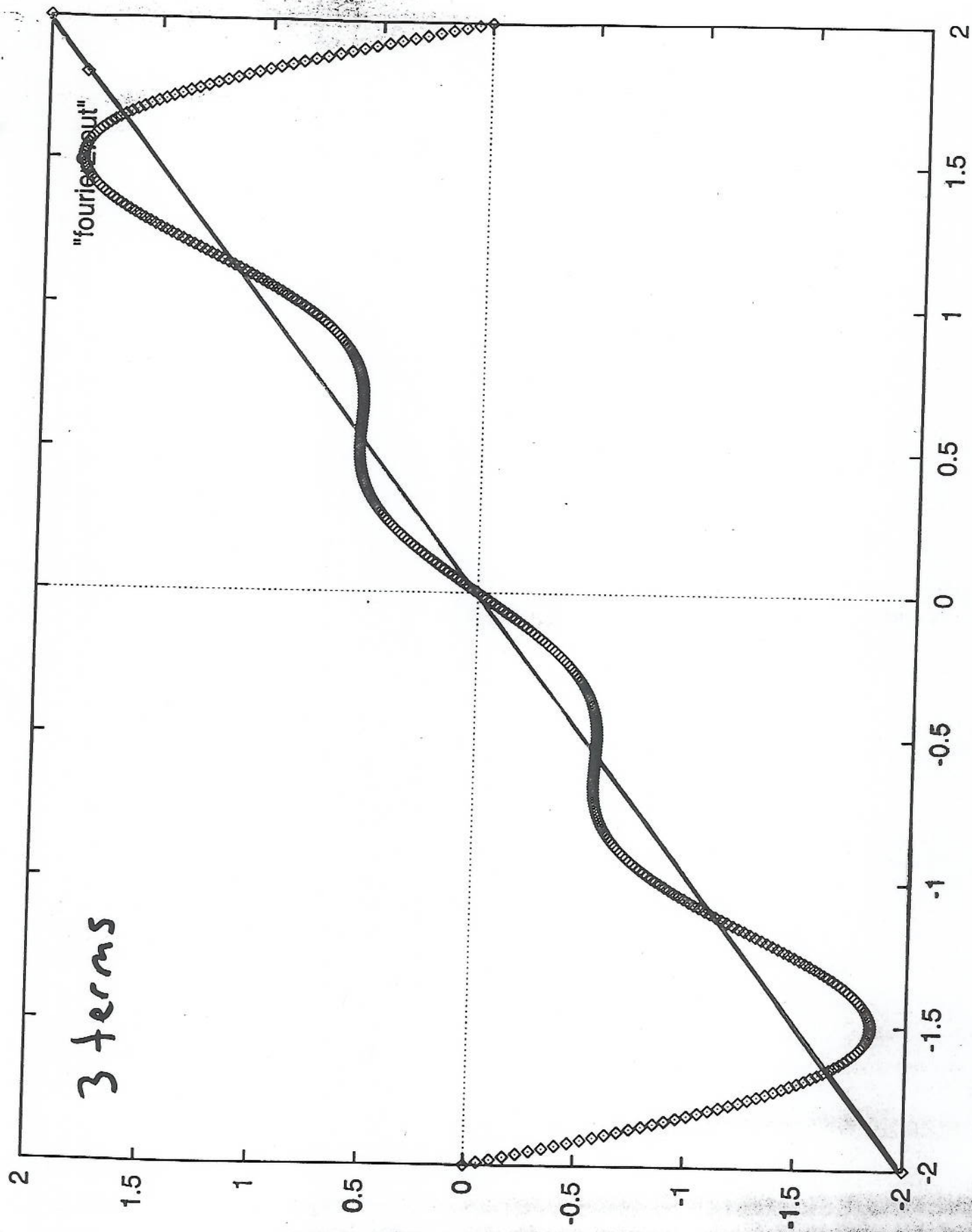
constant + one term

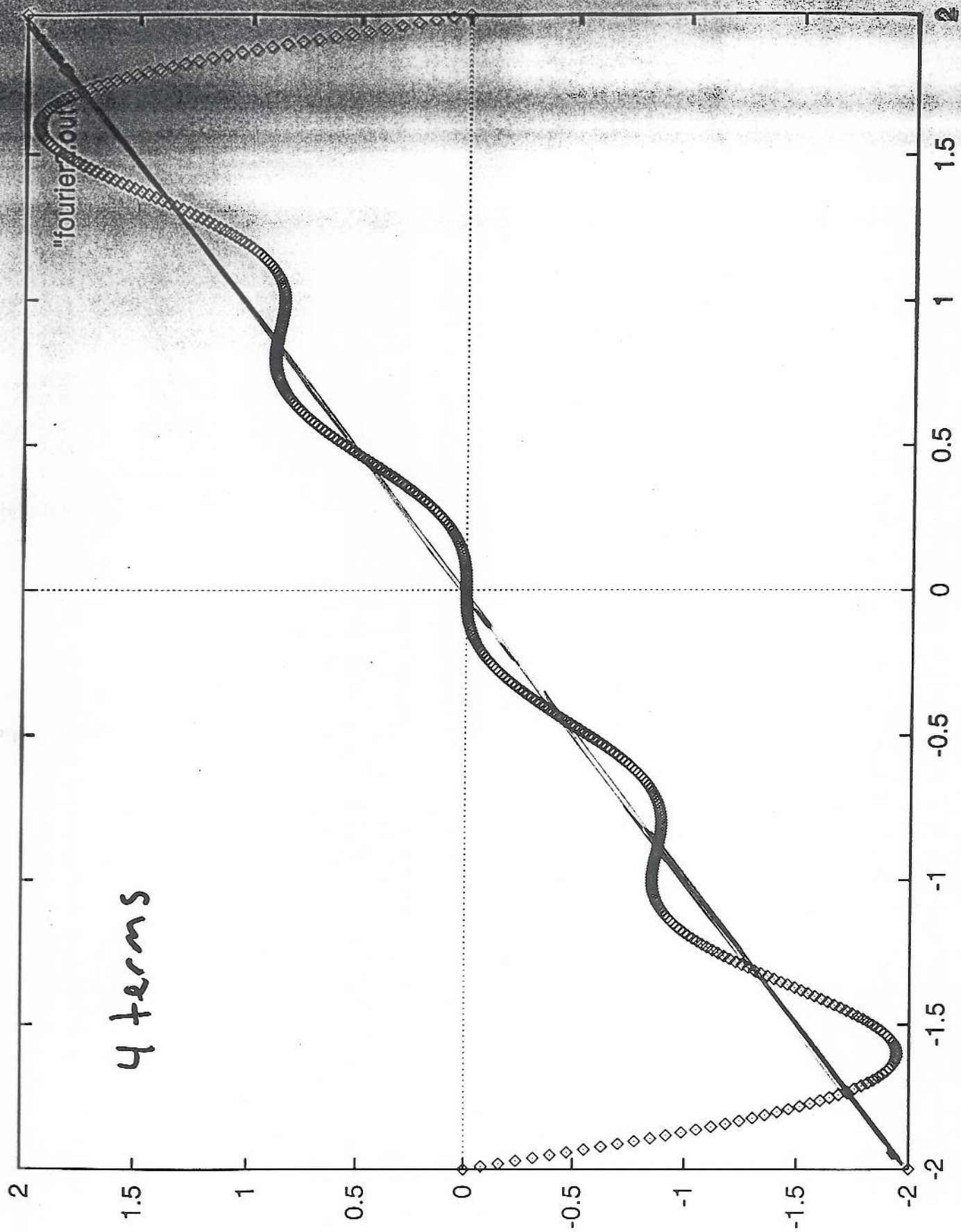
$v(t) = t$

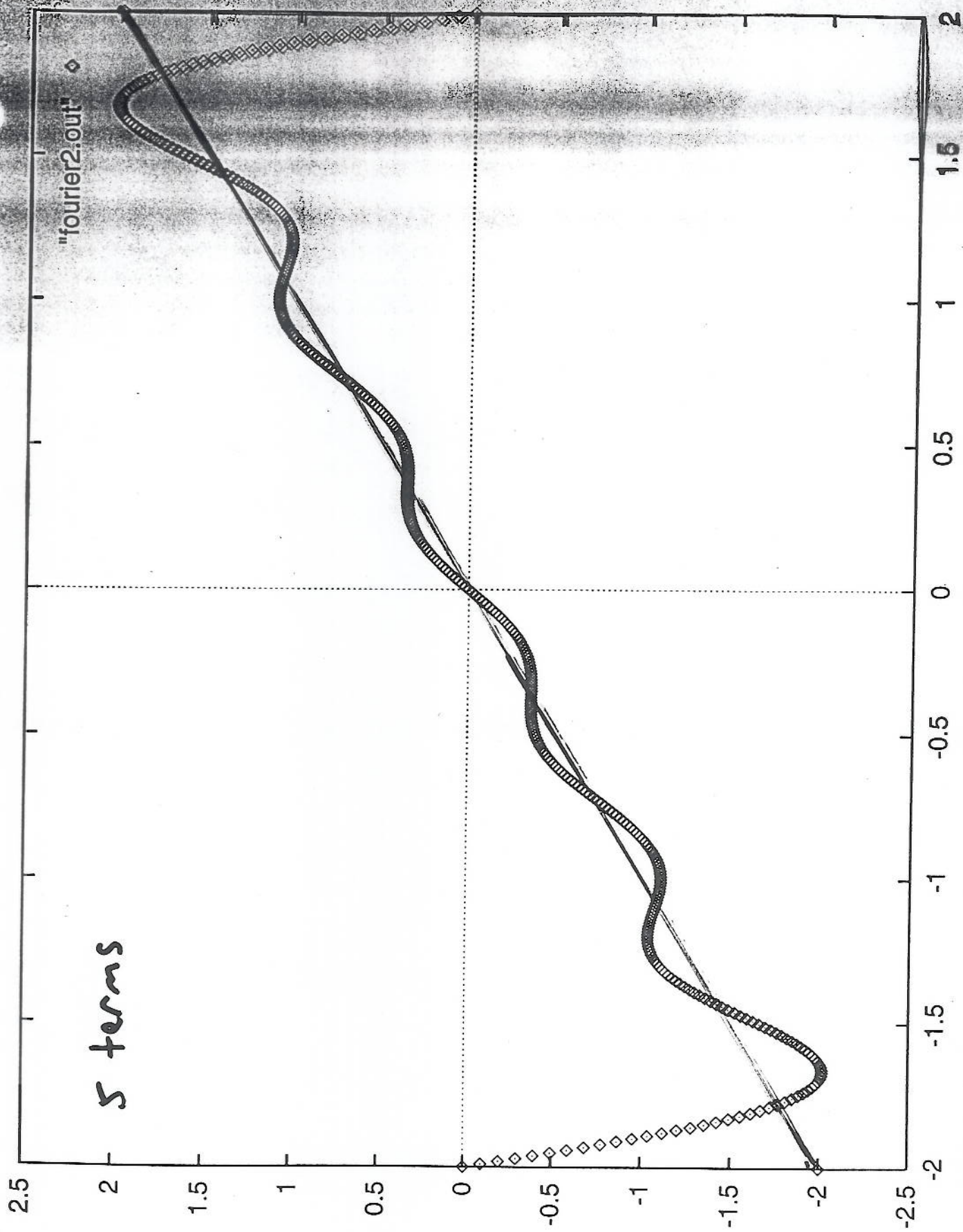
"fourier2.out"

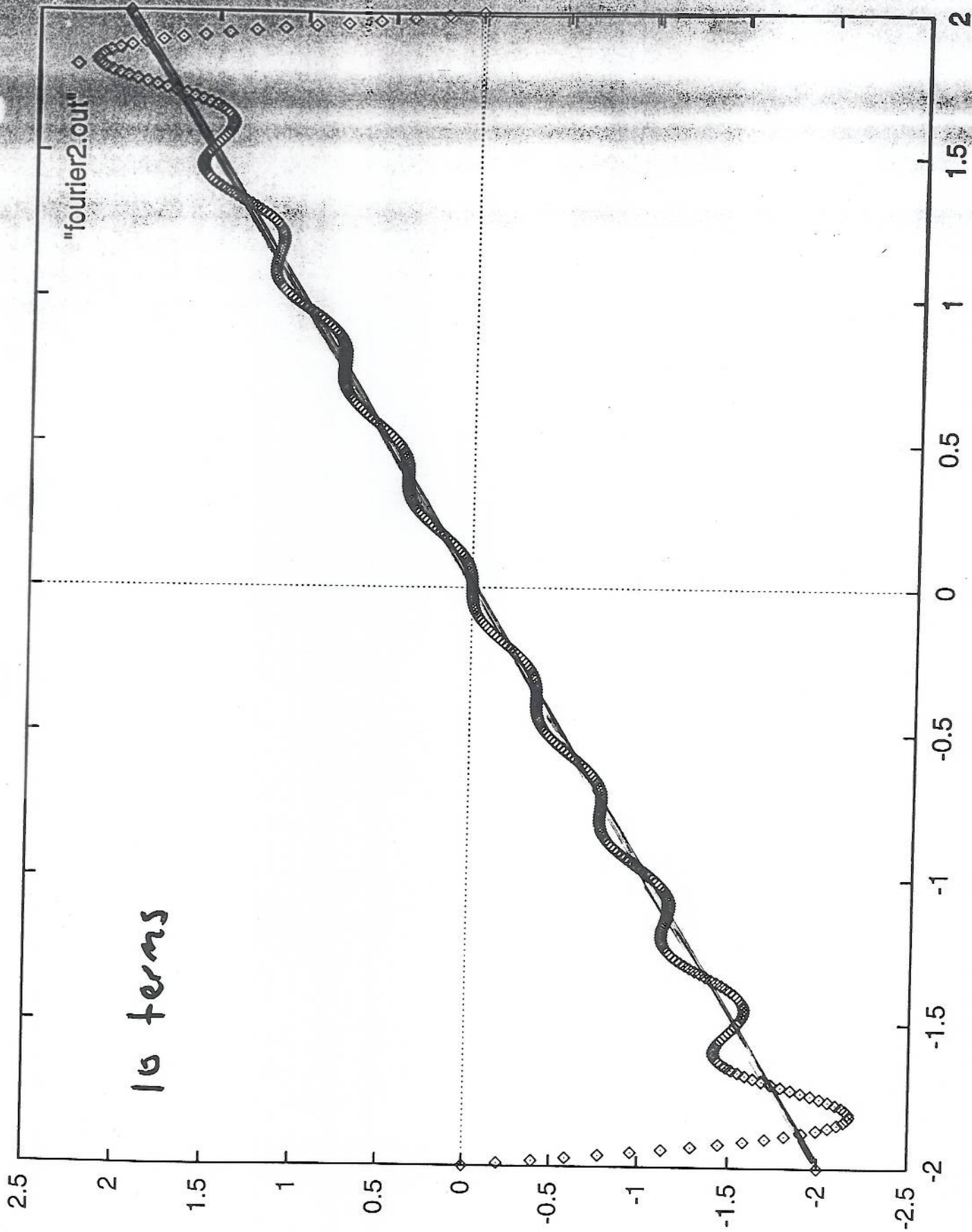


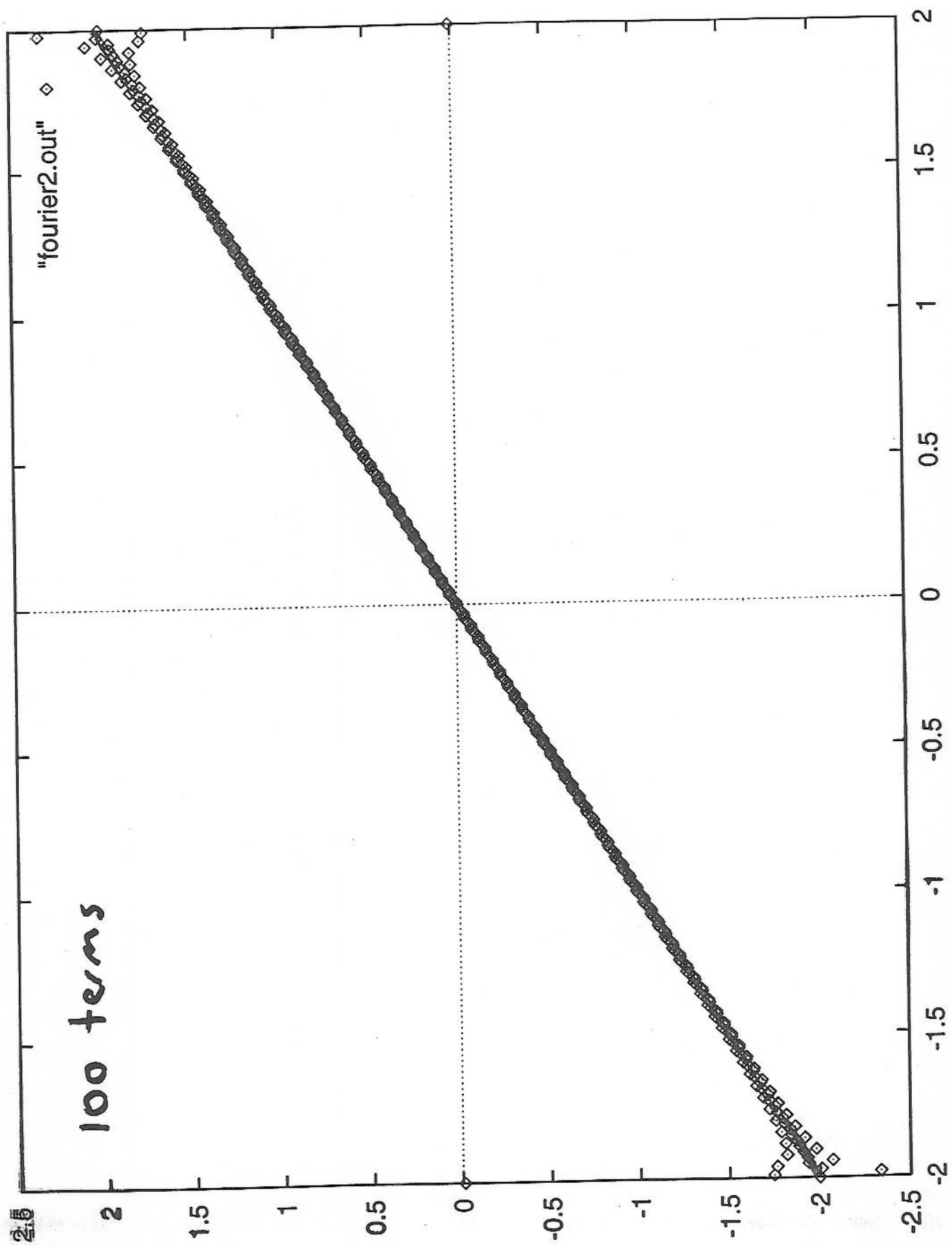










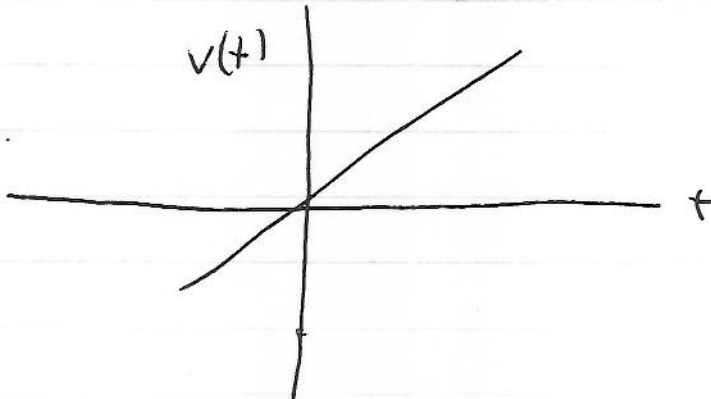


$$S_0 \quad a_k = \frac{2}{T} \int_{-T/2}^{T/2} v(t) \cos \frac{2\pi kt}{T} dt$$

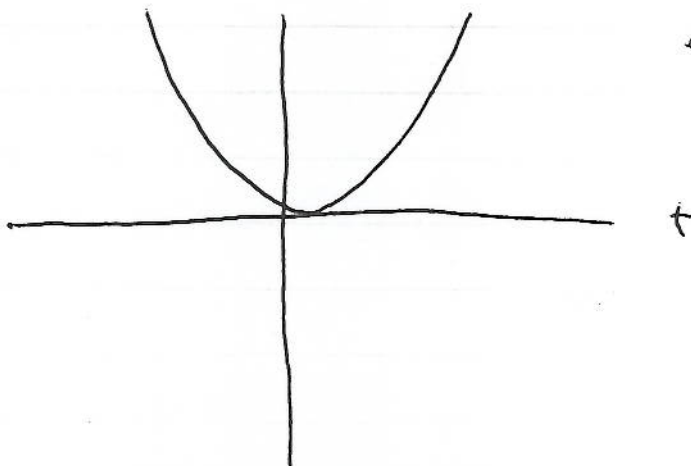
multiply $v(t)$ by $\sin \frac{2\pi kt}{T}$, same argument

$$b_k = \frac{2}{T} \int_{-T/2}^{T/2} v(t) \sin \frac{2\pi kt}{T} dt$$

Often, can use symmetry

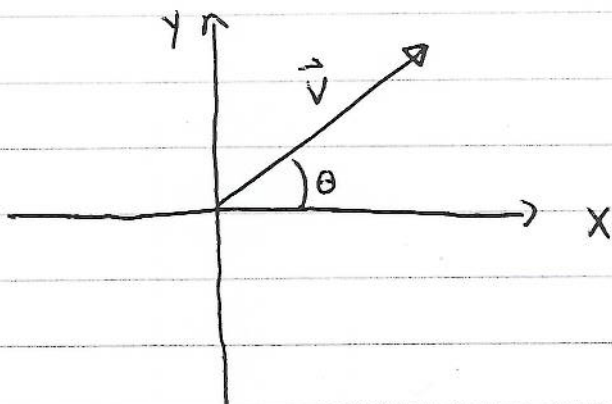


odd, only sine series



even, only cosine series

For some vector, $\vec{v}$,



$$V_x = \vec{v} \cdot \hat{x} = |\vec{v}| \cos \theta$$

$$V_y = \vec{v} \cdot \hat{y} = |\vec{v}| \sin \theta$$

For 3D

$$\vec{v} = \vec{v} \cdot \hat{x} \hat{x} + \vec{v} \cdot \hat{y} \hat{y} + \vec{v} \cdot \hat{z} \hat{z}$$

↑
picks off
x component

↑
y component

↑
z component

How do we determine a_n 's & b_n 's?

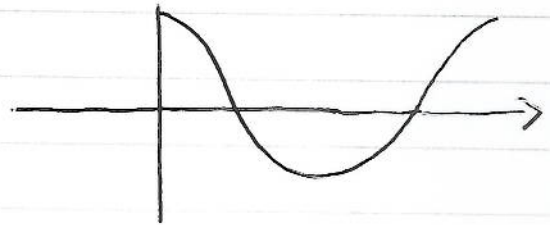
Use orthogonality.

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n t}{T} + \sum_{n=1}^{\infty} b_n \frac{2\pi n t}{T}$$

Multiply both sides by $\cos \frac{2\pi k t}{T}$ k is same integer
integrate from $-\frac{T}{2}$ to $\frac{T}{2}$

$$\int_{-T/2}^{T/2} v(t) \cos \frac{2\pi k t}{T} dt = \int_{-T/2}^{T/2} \left(\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n t}{T} + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n t}{T} \right) \cos \frac{2\pi k t}{T} dt$$

$$\int_{-T/2}^{T/2} \frac{a_0}{2} \cos \frac{2\pi k t}{T} dt = 0$$

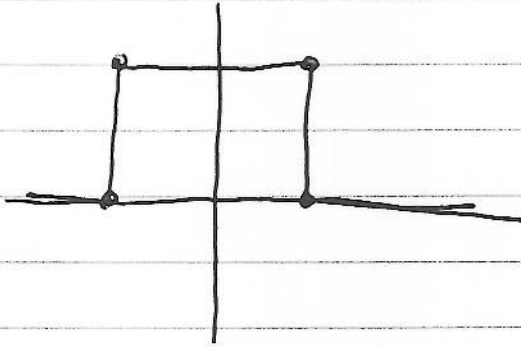


$$\int_{-T/2}^{T/2} \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n t}{T} \cos \frac{2\pi k t}{T} dt = 0$$

$$a_n \int_{-T/2}^{T/2} \cos \frac{2\pi k t}{T} \cos \frac{2\pi k t}{T} dt = \frac{T}{2} a_n \quad \text{Picks off } a_n$$

THIS IS LIKE DOT PRODUCT

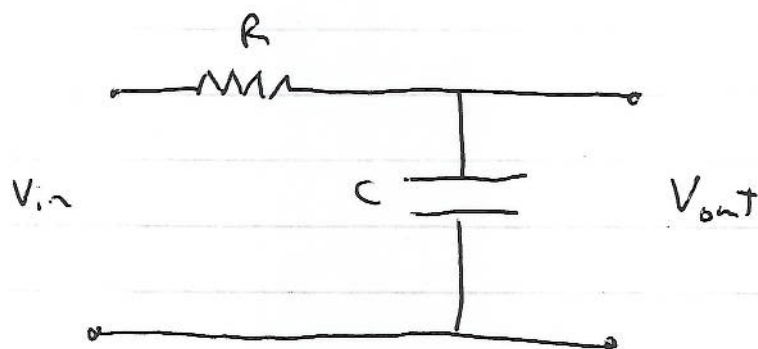
sharp edges



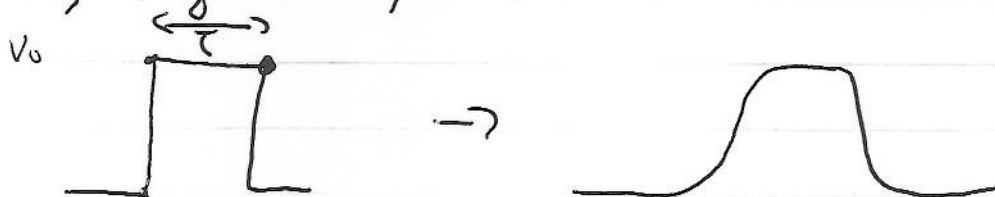
Need high frequency
components

Integrating circuit

Consider low pass filter



High frequencies get attenuated.



Kirchhoff says:

$$V_{in} = V_0 = V_R + V_C = iR + \frac{Q}{C}$$

$$\frac{dQ}{dt} R + \frac{Q}{C} = V_0$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{V_0}{R}$$

$$Q = CV_0 (1 - e^{-t/RC})$$

$$Q = VC$$

$$V_{out} = V_0(1 - e^{-t/RC})$$

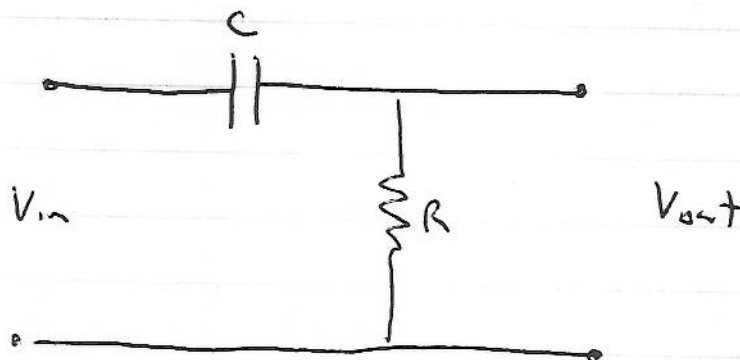
If RC large $RC \gg \tau$

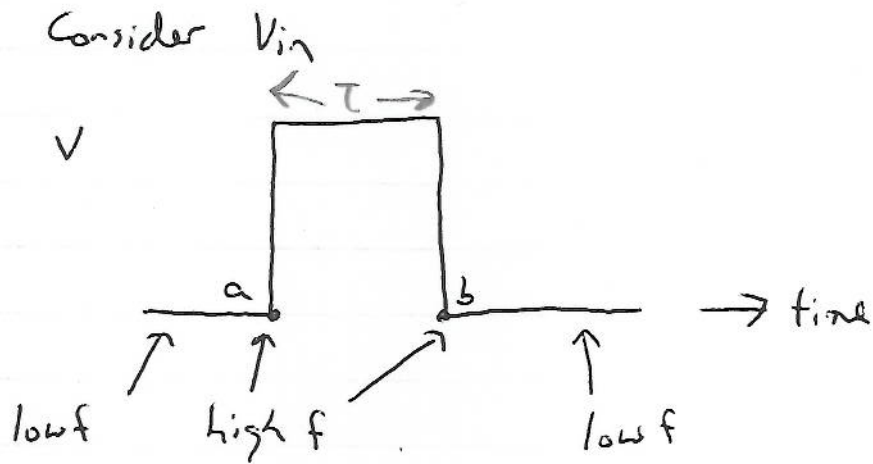
$$e^{-t/RC} = 1 - \frac{t}{RC} + \dots$$

$$V_{out} = \frac{V_0 t}{RC}$$

$$V_{out} \propto \int_0^t V_{in} dt$$

Differentiating circuit (High Pass)





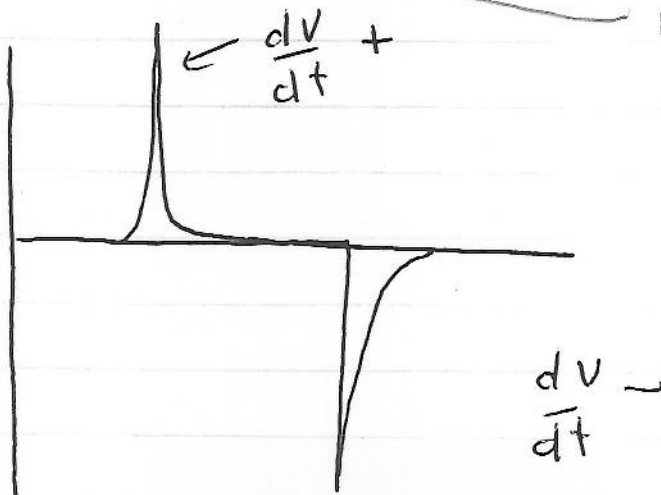
High Pass filter: expect $|V_{out}|$ large at points a & b
where $\frac{dV}{dt}$ is large

$$Q = CV_c \quad i = C \frac{dV_c}{dt}$$

$$V_{out} = iR = RC \frac{dV_c}{dt}$$

$$\propto \frac{dV_c}{dt} \approx \frac{dV_{in}}{dt}$$

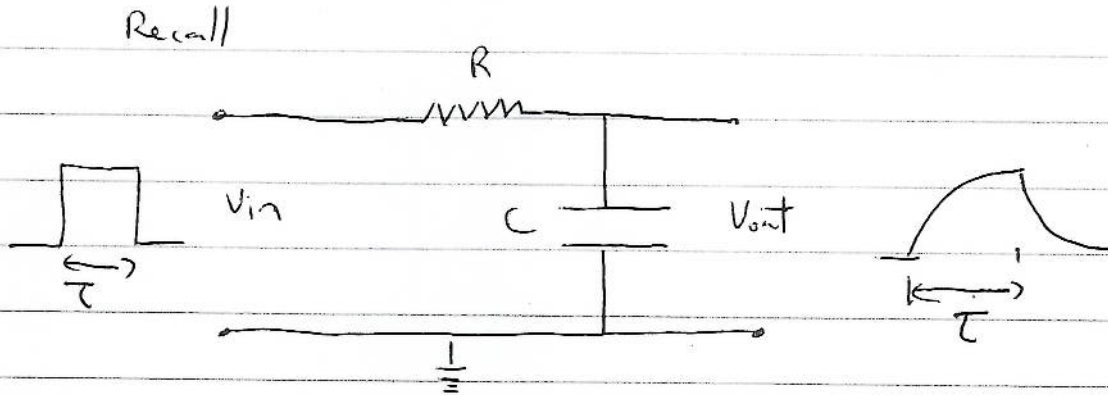
= charges up quickly



if $\frac{RC}{\tau} \ll 1$ then $V_r \ll V_c$
then $V_c \approx V_{in}$

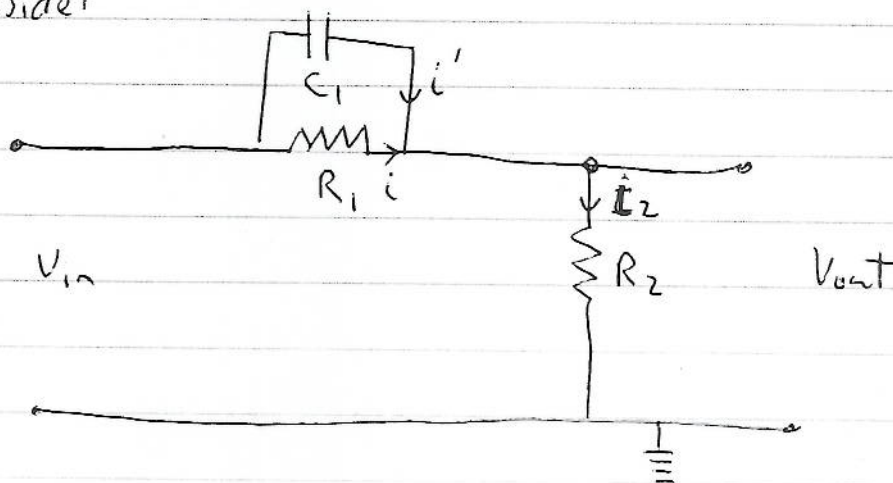
$$\frac{|V_R|}{|V_c|} = \frac{R}{\frac{1}{\omega C}} = \omega RC$$

Pulse shaping circuit.



Often, we want sharper pulses for timing purposes

Consider



$$i + i' = i_2$$

$$\frac{V_{in} - V_{out}}{R_1} + \frac{d}{dt} C_1 (V_{in} - V_{out}) = \frac{V_{out}}{R_2}$$

$$C_1 \frac{dV_{out}}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_{out} = C_1 \frac{dV_{in}}{dt} + \frac{V_{in}}{R_1}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{12}}$$

$$\frac{dV_{out}}{dt} + \frac{1}{C_1 R_{12}} V_{out} = \frac{dV_{in}}{dt} + \frac{1}{R_1 C_1} V_{in}$$

$$V_{in} \sim$$

$$V_{in} = V_0 (1 - e^{-t/\tau_{in}})$$

$$\frac{dV_{in}}{dt} = \frac{V_0}{\tau_{in}} e^{-t/\tau_{in}}$$

$$\frac{dV_{out}}{dt} + \frac{1}{C_1 R_{12}} V_{out} = V_0 \left(\frac{1}{\tau_{in}} - \frac{1}{R_1 C_1} \right) e^{-t/\tau_{in}} + \frac{V_0}{R_1 C_1}$$

Choose $\tau_{in} = R_1 C_1$

$$\frac{dV_{out}}{dt} + \frac{1}{R_{12} C_1} V_{out} = \frac{V_0}{R_1 C_1}$$

$$V_{out} = \frac{R_2 V_0}{R_1 + R_2} (1 - e^{-t/\tau'})$$

$$\tau' = R_{12} C_1$$

R_{12} is always less than R_1 . Why?

$$\tau' = \frac{R_2}{R_1 + R_2} \tau_{in}$$

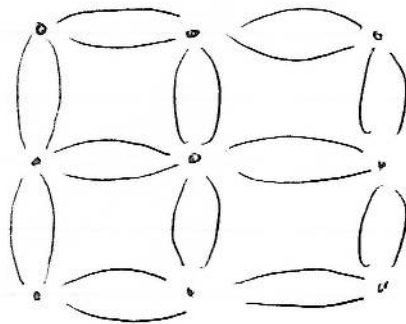
$$\tau' < \tau.$$

Semiconductor Physics

X

Types of matter

- gas - atoms interact only via hard collisions
- liquid - weak interaction
- plasma - ionized gas
- solid - rigid lattice
- crystal



amorphous - not regular pattern