Angular correction in reconstruction of electron spectra from depth dose distributions

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Techniques for reconstruction of electron spectra from the depth-dose curves used to date have ignored the angular distribution of incident electrons scattered at large angles. The approximation adopted is to employ a database of monoenergetic depth-dose curves generated for normal incidence of electrons at the surface. This approximation is acceptable for direct electrons with small angular spread. However, electrons scattered from the treatment head and collimating system may have large average angles of incidence which affects the depth-dose distribution significantly at shallow depths by increasing energy deposition close to the surface. We show that ignoring the electron incident angular distribution leads to systematic errors in the low energy region of reconstructed electron spectra. We propose a simple 1-D model to correct for these systematic errors using only electron angular distribution at the central beam axis. This model provides reconstructed spectra in excellent agreement with Monte Carlo simulation in the low energy region. [DOI: 10.1118/1.1592031]

Key words: electron, depth dose curve, energy spectrum, angular distribution, reconstruction

I. INTRODUCTION

Electron spectra may be reconstructed from the measured central-axis depth-dose distributions of broad beams produced by medical accelerators. These spectra are useful for the commissioning of advanced treatment planning systems (Ref. 1 and references therein) which require as input an initial phase space distribution of incident particles. However, there is a problem of degeneracy in the electron spectra obtained by presently used reconstruction procedures. This problem relates to the fact that dose is a scalar quantity of energy deposition within a volume of interest by a vector field of particles. In other words, dose is the energy deposition in a volume without regard to the direction of travel for the particles entering the volume. Thus the same scalar quantity of dose to a volume may be obtained for different joint energy-angular distributions of the particles entering the volume.

Reconstruction procedures used to date have not incorporated the angular distribution of incident electrons explicitly. These procedures are based on the depth-dose data that assumes the electrons were at normal incidence to the surface of the absorbing medium.[2–5] This assumption is not a good one for clinical electron beams and so the electron spectra produced by a reconstruction procedure invoking this assumption will inherently contain a systematic error. Depth-dose distributions from clinical electron beams are most affected in the shallow depth region by incident electrons having a large angular distribution. The reconstruction procedure interprets the dose deposited at shallow depths by these non-normally incident electrons as additional low energy electrons in the incident electron spectrum. Thus this introduces a systematic error in the reconstructed spectra for the low energy region. Use of a reconstructed spectrum containing this systematic error as an input to an advanced treatment planning system may lead to errors in prediction of the complete 3D dose distribution, or the dose perturbations caused by shallow-depth heterogeneities.

In our first article on electron spectrum reconstruction from depth-dose distribution,[5] we discussed the efficiency of an optimization method for solving the governing integral Fredholm equation of the first kind used in the reconstruction procedure. We showed improved accuracy and robustness of the reconstruction technique when the electron spectrum is separated into singular and regular components. Numerical artifacts originally caused by the ill-conditioned property of the governing equation were much reduced in the low energy part of the spectrum. This reduction allows us in this article to estimate the influence of the initial angular spread of incident electrons on the reconstructed electron spectra. We show that this influence is significant and should be taken into account.

II. ANGULAR WEIGHTING OF MONOENERGETIC DEPTH-DOSE CURVES

Techniques for reconstruction of electron spectra from broad beam central axis depth-dose distributions are based on numerical solutions to the equation

$$D(z)=\sum_{n=1}^{N} w_n d(z,E_n), \tag{1}$$

where w_n, $n=1,...,N$, are the spectral weights and $d(z,E_n)$, $n=1,...,N$, are the monoenergetic broad beam depth-dose distributions called response functions. Equation (1) is a discretized form of the integral Fredholm equation of the first kind. If the monoenergetic depth dose distributions $d(z,E_n)$, $n=1,...,N$, are known, the spectral weights w_n, $n=1,...,N$, may be found using one of the methods of statistical or deterministic optimization with different modifications to improve the quality of numerical solutions. Obviously, the accuracy of the reconstructed spectra will depend on (1) an optimization method used to solve the integral Fredholm equation of the first kind and (2) approximations in the monoenergetic depth-dose distributions $d(z,E_n)$, $n=1,...,N$. In this article, we discuss the approximations in the monoenergetic depth-dose distributions.

The monoenergetic depth-dose distributions $d(z,E_n)$, $n=1,...,N$, are usually simulated in RZ geometry for a point source located above the absorbing medium surface. A less accurate but more computationally efficient approach is to use parallel broad beam geometry and then apply an inverse square factor. These source model geometries correspond to a *normal* incidence of electrons at the central beam axis, an acceptable approximation for electron beams having a small angular spread. However, clinical electron beams typically have a component of low-energy electrons derived from primary electron interactions with the treatment head and collimating system, which are scattered at large angles to the beam central axis.

To improve the reconstructed spectra in the low energy region, the monoenergetic depth-dose curves must be corrected for the angular distribution of incident electrons. Angular distribution of incident electrons at the phantom surface is complicated and different at every point of incidence. Complete consideration of the angular distribution would require 3-D models. In this article, we propose a simplified 1-D model in the parallel plane geometry to correct the monoenergetic depth-dose distributions for the initial angular distribution of incident electrons. We assume that the angular distribution is the same at every incident position within the beam and equal to the angular distribution at the central axis. It is well known that the angular distribution at each lateral position may vary. However, typically, these assumptions are better for the electrons incident close to the beam central axis and in this study we use only the central axis depth-dose distribution data.

For the above assumptions on angular distribution, the corrected monoenergetic depth-dose distribution of a broad beam may be calculated in parallel plane geometry as

$$d(z,E_n)=\int_{-1}^{1}\chi_n(\mu)\Delta(z,E_n,\mu)d\mu, \tag{2}$$

where $\mu=\cos\theta$, θ is the angle between the normal to the surface and the direction of the incident electon, and $\chi_n(\mu)$ is the angular distribution of the incident electrons at the central axis. The energy-angular response functions $\Delta(z,E_n,\mu)$ represent the depth-dose distributions from a monoenergetic elementary azimuthally symmetric source

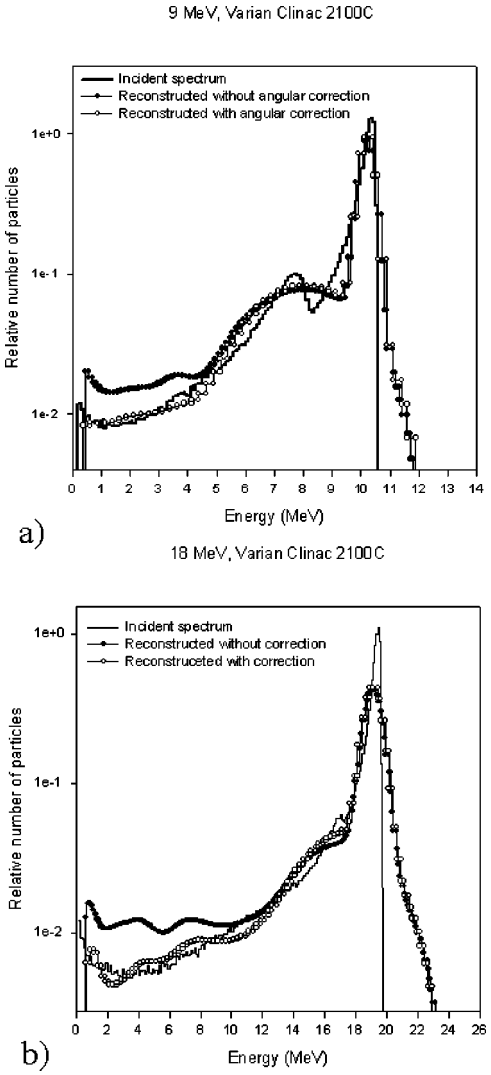

FIG. 1. Comparison of the incident and reconstructed electron spectra for the 9 (a) and 18 (b) MeV electron beams from a Varian Clinac 2100C accelerator. The incident electron spectra were obtained by Ding and Rogers using Monte Carlo treatment head simulation.[6]

with an energy $E=E_0$ and an angle of incidence $\mu=\mu_0$. Equation (2) can be used together with Eq. (1) only under the assumption that the angular distribution is not a function of energy within a given energy bin $[E_{n-1/2},E_{n+1/2}]$.[5] The angular distribution is, of course, made a function of energy from bin to bin and so over the entire spectrum. This step approximation does not worsen the reconstruction algorithm in practice because the energy bins are narrow (0.25 MeV) and the angular scattering cross-sections are smooth functions of energy.

Expressing Eq. (2) in a discrete form we obtain

$$d(z,E_n)=\sum_{m=1}^{M} w_m\chi_n(\mu_m)\Delta(z,E_n,\mu_m), \tag{3}$$

where w_m, $m=1,...,M$, are angular weights.

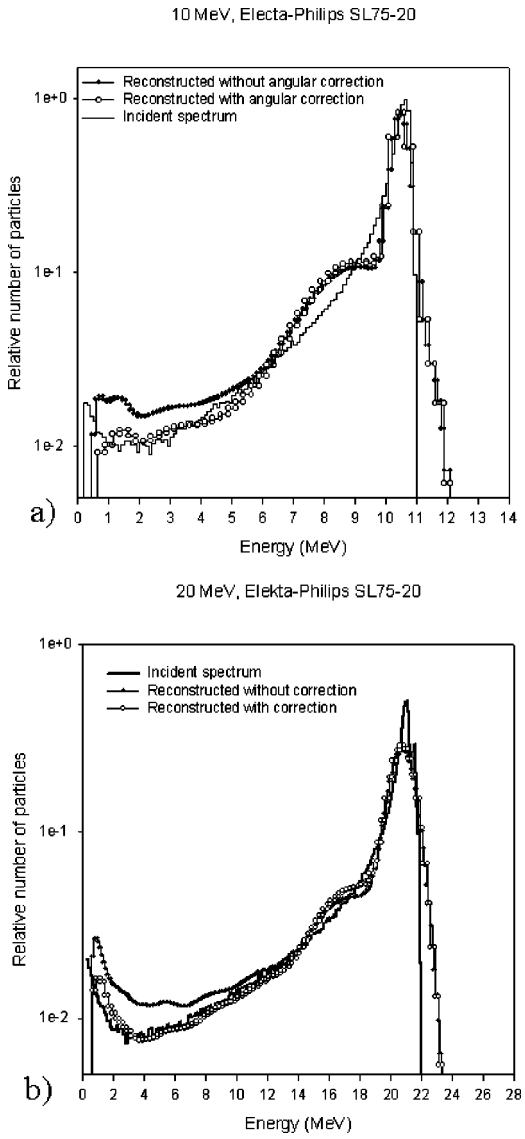

FIG. 2. Comparison of the incident and reconstructed electron spectra for the 10 (a) and 20 (b) MeV electron beams from an Elekta-Philips SL75-20 accelerator. The incident electron spectra were obtained by Ding and Rogers using Monte Carlo treatment head simulation (Ref. 6).

III. RESULTS

Electron energy spectra reconstructed without angular correction from depth-dose distributions produced by the 9 and 18 MeV electron beams of a Varian Clinac 2100 accelerator are shown in Figs. 1(a) and 1(b). Additionally, Figs. 2(a) and 2(b) provide reconstructed energy spectra for the 10 and 20 MeV electron beams generated by an Elekta-Philips SL75-20 accelerator. The depth-dose distributions used for the reconstruction were those calculated by Ding and Rogers using the EGS4-BEAM Monte Carlo code[6] without the depth dose component of the treatment head breamstrahlung. For electrons with energies $E \leqslant E_p/2$, where E_p is the most probable energy of the spectrum, the reconstructed spectra significantly overestimates, up to 100%, the electron number

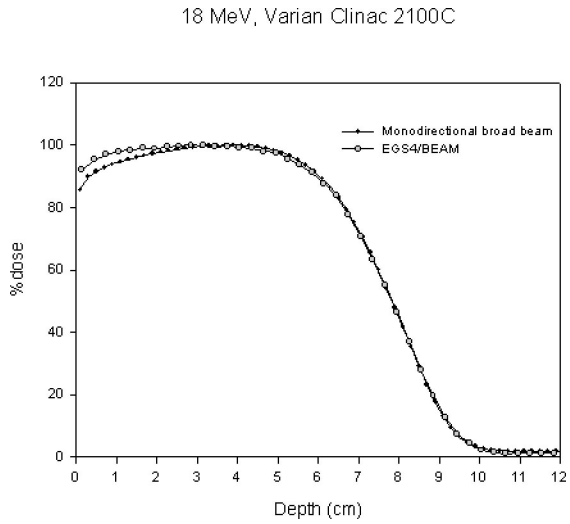

FIG. 3. Influence of the initial angular distribution of incident electrons on the depth-dose distribution in water for the 18 MeV electron beam of a Varian Clinac 2100C acelerator.

per energy interval in the known spectra. This trend is similar for the range of beam energies studied and for either accelerator.

In our earlier article,[5] we presented similar comparisons except that we avoided the influence of the electron incident angular distribution upon the verification of our optimization algorithms by using depth-dose distributions for reconstruction that were simulated in the same parallel broad geometry as the monoenergetic depth-dose distributions. Therefore the agreement of incident and reconstructed spectra in the low energy region was perfect. Obviously, the differences in the low energy region of the reconstructed and incident electron spectra in Figs. 1 and 2 are due to the neglect of initial angular distribution for incident electrons in the database of monoenergetic depth-dose distributions. The influence of the angular spread on depth-dose can be better understood by comparing the depth-dose distribution predicted using a complete Monte Carlo simulation[6] that includes angular distributions to the depth-dose distribution for a monodirectional beam (Fig. 3).

For the angular correction, the functions $\Delta(z,E_n,\mu_m)$ were calculated in the parallel plane geometry using the discrete ordinates method[7] and corrected for the divergence of the electron beam. We used 107 discrete energies between 0.5 and 27 MeV and within each discrete energy 16 angular intervals between 0° and 90° were chosen so the total number of depth-dose distributions to approximate the function $\Delta(z,E,\mu)$ was 1712 (i.e., 107×16). An example of energy deposition profiles for different incident angles is presented in Fig. 4 for 10 MeV electrons. These energy-angular response functions can be folded with any incident angular distribution $\chi_n(\mu)$ to obtain a database of corrected depth-dose distributions for reconstruction of electron spectra. In this numerical model study, the angular distribution $\chi_n(\mu)$ was taken from Monte Carlo treatment head simulations[6] and approximated by simple Gaussian functions.

The electron energy spectra reconstructed with corrected

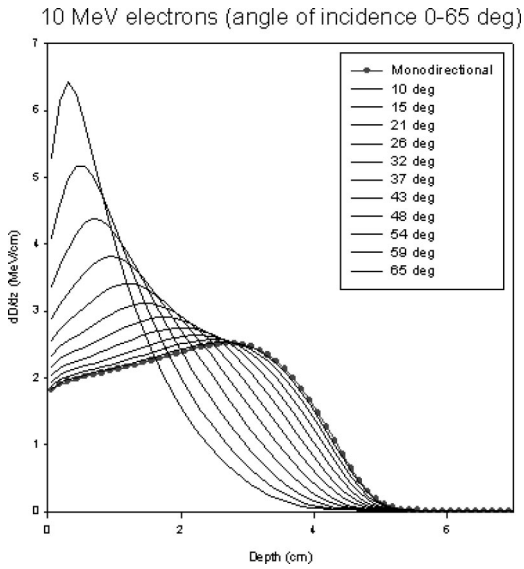

FIG. 4. Angular dependence of the energy deposition profile for 10 MeV electrons incident upon a water phantom.

monoenergetic depth-dose distributions are presented in Figs. 1 and 2 for a Varian Clinac 2100C and an Elekta-Philips SL75-20 accelerator. Angular correction improves the agreement between the known incident spectra and the reconstructed spectra in the low-energy region. The region of energy peak is practically not affected by angular correction.

IV. CONCLUSIONS

We have shown that the neglect of angular distribution in the electron beams of medical accelerators produces systematic errors in the low energy region of spectra reconstructed from central axis depth-dose distributions. Therefore, determination of the incident angular distribution and reconstruction of electron spectra from depth-dose distributions are related problems. The one-dimensional model described in this article is proposed to correct our reconstruction algorithm for electron incident angle. This model is effective and leads to good agreement with Monte Carlo data.

It should be noted that our database of monoenergetic depth-dose distributions was obtained in broad beam geometry and, therefore, broad beam conditions must be satisfied in measurements of the central axis depth-dose distributions. Any SSD can be used if an appropriate divergence correction is applied. Also, the angular correction in the reconstruction algorithm requires an estimate of the angular distribution.

The broad beam condition may be not valid for a small field size produced by collimation. Therefore, our reconstruction algorithm cannot be applied directly to small field sizes. Extrapolation to small field sizes of the energy spectra originally derived from larger field sizes may not be highly accurate because of different scattering conditions. This problem remains a limitation of the method until further work is performed.

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