

hep-ph/9907265
INRNE-TH 33/99
July 1999

Semi-inclusive π^\pm production – tests for independent fragmentation and for polarized quark densities

Ekaterina Christova

*Institute of Nuclear Research and Nuclear Energy,
Boul. Tzarigradsko Chaussee 72, Sofia 1784, Bulgaria
e-mail: echristo@inrne.bas.bg*

Elliot Leader

*Department of Physics
Birbeck Colledge, University of London
Malet street., London WC1E 7HX, England
e-mail: e.leader@physics.bbk.ac.uk*

Abstract

We show that measurements of semi-inclusive π^\pm asymmetries on p and n with polarized and unpolarized target and beams allow, without any knowledge of the polarized parton densities 1) to test independent fragmentation and SU(2) symmetry for the polarized sea, and 2) to determine separately the polarized strange and valence quarks densities, and the ratio of the fragmentation functions $D_s^{\pi^+\pi^-} / D_u^{\pi^+\pi^-}$.

1 Introduction

Recently measurements of semi-inclusive processes with polarised and unpolarised initial beams have been performed [1, 2, 3]. New, more precise data are expected soon. The goal of these experiments is to determine better the parton distribution functions, to distinguish between the quark and antiquark contributions and to understand better the fragmentation of quarks into hadrons. One of the basic theoretical elements assumed in these studies is the factorization of the process into quark production followed by independent quark fragmentation into hadrons. We shall refer to this as independent fragmentation. Some possibilities for a test of independent fragmentation have been considered [2, 4].

In this paper we consider $\pi^+ + \pi^-$ semi-inclusive deep inelastic lepton nucleon scattering with both polarized and unpolarized initial particles. We discuss different tests for independent fragmentation. Further, assuming it to be valid we formulate tests for the polarized sea and valence quark distribution functions, and for some general symmetries among them. Apart from independent fragmentation the tests formulated in this paper require no other assumptions. They are general and independent of the parametrization of the polarized quark densities and fragmentation functions. This is achieved by finding combinations of the cross sections on p and n for semi-inclusive ($\pi^+ + \pi^-$) production, both for polarized and unpolarized beams, which allow the use of the data on inclusive deep inelastic scattering (DIS) with polarized and unpolarized beam and target and thus to formulate tests that involve only directly measurable quantities. We work in the framework of the Parton Model in the current fragmentation region [5], which implies that we consider π^\pm mesons produced mainly in the forward direction.

2 Testing independent fragmentation

Consider the sum of the cross sections for producing π^+ and π^- in the semi-inclusive deep inelastic scattering

$$l + N \rightarrow l' + \pi + X \tag{1}$$

in both the unpolarized case, and when beam and target are longitudinally polarized.

We shall use the notation

$$\tilde{\sigma} \equiv \frac{x(P+l)^2}{4\pi\alpha^2} \left(\frac{2y^2}{1+(1-y)^2} \right) \frac{d^3\sigma}{dx dy dz} \quad (2)$$

and

$$\Delta\tilde{\sigma} \equiv \frac{x(P+l)^2}{4\pi\alpha^2} \left(\frac{y}{2-y} \right) \left[\frac{d^3\sigma_{+-}}{dx dy dz} - \frac{d^3\sigma_{++}}{dx dy dz} \right] \quad (3)$$

where P^μ and l^μ are the nucleon and lepton four momenta, and $\sigma_{\lambda\mu}$ refer to a lepton of helicity λ and a nucleon of helicity μ . The variables x, y, z are the usual DIS kinematic variables [4].

Assuming independent fragmentation, and using isospin and charge conjugation invariance for the fragmentation functions, i.e.

$$D_u^{\pi^+\pi^-} \equiv D_u^{\pi^+} + D_u^{\pi^-} = D_d^{\pi^+\pi^-} \quad (4)$$

and

$$D_s^{\pi^+\pi^-} = D_{\bar{s}}^{\pi^+\pi^-} \quad (5)$$

we obtain for the polarized case in lowest order of QCD:

$$\Delta\tilde{\sigma}_p^{\pi^+\pi^-} = \frac{1}{9} \left\{ [4(\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d})] D_u^{\pi^+\pi^-} + 2\Delta s D_s^{\pi^+\pi^-} \right\} \quad (6)$$

$$\Delta\tilde{\sigma}_n^{\pi^+\pi^-} = \frac{1}{9} \left\{ [4(\Delta d + \Delta\bar{d}) + (\Delta u + \Delta\bar{u})] D_u^{\pi^+\pi^-} + 2\Delta s D_s^{\pi^+\pi^-} \right\}. \quad (7)$$

We eliminate the s -quark contribution by the difference:

$$\Delta\tilde{\sigma}_p^{\pi^+\pi^-} - \Delta\tilde{\sigma}_n^{\pi^+\pi^-} = \frac{1}{3} \{ (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \} D_u^{\pi^+\pi^-}. \quad (8)$$

Analogously for the cross section of $(\pi^+ + \pi^-)$ for the unpolarized case we have:

$$\tilde{\sigma}_p^{\pi^+\pi^-} = \frac{1}{9} \left\{ [4(u + \bar{u}) + (d + \bar{d})] D_u^{\pi^+\pi^-} + 2s D_s^{\pi^+\pi^-} \right\} \quad (9)$$

$$\tilde{\sigma}_n^{\pi^+\pi^-} = \frac{1}{9} \left\{ [4(d + \bar{d}) + (u + \bar{u})] D_u^{\pi^+\pi^-} + 2s D_s^{\pi^+\pi^-} \right\}. \quad (10)$$

We again eliminate the s -quark contribution by the difference:

$$\tilde{\sigma}_p^{\pi^+\pi^-} - \tilde{\sigma}_n^{\pi^+\pi^-} = \frac{1}{3} \{ (u + \bar{u}) - (d + \bar{d}) \} D_u^{\pi^+\pi^-}. \quad (11)$$

Then we consider the asymmetry $\Delta R_{np}^{\pi^+\pi^-}$:

$$\Delta R_{np}^{\pi^+\pi^-}(x, z, Q^2) \equiv \frac{\Delta \tilde{\sigma}_p^{\pi^+\pi^-} - \Delta \tilde{\sigma}_n^{\pi^+\pi^-}}{\tilde{\sigma}_p^{\pi^+\pi^-} - \tilde{\sigma}_n^{\pi^+\pi^-}} = \quad (12)$$

$$= \frac{(\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})}{(u + \bar{u}) - (d + \bar{d})}(x, Q^2). \quad (13)$$

The l.h.s of (13) is, in principle, a function of x , z and Q^2 , but as a consequence of independent fragmentation, it should be in practice a function only of x and Q^2 .

Moreover, the r.h.s. of (13) can be expressed, without any assumptions about the sea quarks in terms of the measured quantities of inclusive DIS. Namely:

$$g_1^p - g_1^n = \frac{1}{6} [(\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})] \quad (14)$$

$$F_1^p - F_1^n = \frac{1}{6} [(u + \bar{u}) - (d + \bar{d})]. \quad (15)$$

We obtain

$$\Delta R_{np}^{\pi^+\pi^-}(x, z, Q^2) = \frac{(g_1^p - g_1^n)}{F_1^p - F_1^n}(x, Q^2). \quad (16)$$

Hence, a test of (16) would permit a test of independent fragmentation that does not require any knowledge of the parton distribution functions. The advantage as compared to previous tests of factorization with unpolarized [2] and polarized beam and targets [4] is that the r.h.s. of (13) is expressed directly in terms of measurable quantities.

One can formulate also an integrated version of (16). If we define

$$\Delta N_{p,n}^{\pi^+\pi^-} = \int_{z_1}^{z_2} dz \int_0^1 dx \Delta \tilde{\sigma}_{p,n}^{\pi^+\pi^-}, \quad (17)$$

$$N_{p,n}^{\pi^+\pi^-} = \int_{z_1}^{z_2} dz \int_0^1 dx \tilde{\sigma}_{p,n}^{\pi^+\pi^-}, \quad (18)$$

then analogous to (16) we obtain

$$\frac{\Delta N_p^{\pi^+\pi^-} - \Delta N_n^{\pi^+\pi^-}}{N_p^{\pi^+\pi^-} - N_n^{\pi^+\pi^-}} = \frac{g_A/g_V}{3S_G} \quad (19)$$

independent of z_1 and z_2 . In (19) we have used the Bjorken sum rule:

$$\int_0^1 dx [(\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})] = \frac{g_A}{g_V}, \quad (20)$$

and Gottfried sum rule:

$$S_G = \int_0^1 \frac{F_2^p - F_2^n}{x} dx = \frac{1}{3} + \frac{2}{3} \int_0^1 (\bar{u} - \bar{d}) dx. \quad (21)$$

The value of g_A/g_V is determined with a very good precision $g_A/g_V = 1.2573 \pm 0.0028$, and recent measurements of DIS give the following value for S_G [6]:

$$S_G = 0.235 \pm 0.026 \quad (22)$$

In the following we assume independent fragmentation and formulate some methods of determining the quark densities and fragmentation functions.

3 Measurement of the polarized strange quark densities

One of the main uncertainties, in determining the parton distribution functions from the data on DIS scattering, is the contribution of the s -quarks. Different assumptions have been made in fitting the data [7]. Here we shall show that if we consider the asymmetries on p and n independently, we can single out the Δs -quark contribution.

We suppose that the proton and neutron asymmetries

$$A_{1p}^+ \equiv A_{1p}^{\pi^+\pi^-} = \frac{\Delta\tilde{\sigma}_p^{\pi^+\pi^-}}{\tilde{\sigma}_p^{\pi^+\pi^-}} \quad (23)$$

$$A_{1n}^+ \equiv A_{1n}^{\pi^+\pi^-} = \frac{\Delta\tilde{\sigma}_n^{\pi^+\pi^-}}{\tilde{\sigma}_n^{\pi^+\pi^-}} \quad (24)$$

are known. Then, after some algebra from (14), (15), (23) and (24) we obtain expressions for both Δs and the ratio $D_s^{\pi^+\pi^-}/D_u^{\pi^+\pi^-}$ in terms of measured quantities only:

$$\frac{\Delta s}{s} = \frac{A_{1p}^+ A_{1n}^+ (F_1^n - F_1^p) + g_1^p A_{1n}^+ - g_1^n A_{1p}^+}{(g_1^p - g_1^n) - (A_{1p}^+ F_1^p - A_{1n}^+ F_1^n)} \quad (25)$$

$$\frac{D_s^{\pi^+\pi^-}}{D_u^{\pi^+\pi^-}} = 1 + \frac{9 [(g_1^p - g_1^n) - (A_{1p}^+ F_1^p - A_{1n}^+ F_1^n)]}{s(A_{1p}^+ - A_{1n}^+)} \quad (26)$$

The consistency of using (25) and (26) to determine $\Delta s/s$ and $D_s^{\pi^+\pi^-}/D_u^{\pi^+\pi^-}$ can be checked experimentally, since in (25) the l.h.s. should not depend upon z , and in (26) the l.h.s. should not depend upon x .

4 SU(2) symmetry for the sea quarks?

As it is well known, electromagnetic inclusive DIS cannot yield information on the sea quark (i.e. antiquark) densities, for the simple reason that only the combinations $\Delta q + \Delta \bar{q}$ occur in the expressions for the observables. For various reasons it is sometimes useful to parametrise separately the valence and sea-quark densities when analysing inclusive DIS. Usually, then, some simplifying assumption is made, e.g. an SU(2) or SU(3) symmetric polarized sea. Of course these assumptions cannot be tested in inclusive DIS and it is thus of interest to study the sea via semi-inclusive DIS.

We shall show how the data on the $(\pi^+ + \pi^-)$ and $(\pi^+ - \pi^-)$ asymmetries provide a general test of the SU(2) invariance of the polarized sea quarks. We shall *not* assume SU(2) invariance for the unpolarized sea.

Assuming SU(2) for the polarized sea, (13) equals

$$\Delta R_{np}^{\pi^+\pi^-}(x, z, Q^2) = \frac{\Delta u_V - \Delta d_V}{(u + \bar{u}) - (d + \bar{d})}(x, Q^2) \quad (27)$$

On the other hand, by a slight rewriting of the results of de'Florian et al. [4], the polarized valence quark densities can be obtained without any assumptions about the sea quarks from

$$\frac{\Delta \tilde{\sigma}_p^{\pi^+-\pi^-} - \Delta \tilde{\sigma}_n^{\pi^+-\pi^-}}{\tilde{\sigma}_p^{\pi^+-\pi^-} - \tilde{\sigma}_n^{\pi^+-\pi^-}} = \frac{\Delta u_V - \Delta d_V}{u_V - d_V}. \quad (28)$$

It follows that if SU(2) holds for the polarized sea, then

$$\frac{\Delta \tilde{\sigma}_p^{\pi^++\pi^-} - \Delta \tilde{\sigma}_n^{\pi^++\pi^-}}{\tilde{\sigma}_p^{\pi^++\pi^-} - \tilde{\sigma}_n^{\pi^++\pi^-}} = \left[\frac{u_V - d_V}{(u + \bar{u}) - (d + \bar{d})} \right] \frac{\Delta \tilde{\sigma}_p^{\pi^+-\pi^-} - \Delta \tilde{\sigma}_n^{\pi^+-\pi^-}}{\tilde{\sigma}_p^{\pi^+-\pi^-} - \tilde{\sigma}_n^{\pi^+-\pi^-}}. \quad (29)$$

Thus, without requiring any knowledge of the *polarized* densities we can test whether the polarized sea is SU(2) symmetric.

5 Measurements of the polarized valence quark densities

For completeness we give finally the expressions relating the polarized valence densities to the observables. We have:

$$\frac{\Delta u_V}{u_V}(x, Q^2) = \frac{4\Delta \tilde{\sigma}_p^{\pi^+-\pi^-} + \Delta \tilde{\sigma}_n^{\pi^+-\pi^-}}{4\tilde{\sigma}_p^{\pi^+-\pi^-} + \tilde{\sigma}_n^{\pi^+-\pi^-}}(x, z, Q^2) \quad (30)$$

and

$$\frac{\Delta d_V}{d_V}(x, Q^2) = \frac{\Delta \tilde{\sigma}_p^{\pi^+ - \pi^-} + 4\Delta \tilde{\sigma}_n^{\pi^+ - \pi^-}}{\tilde{\sigma}_p^{\pi^+ - \pi^-} + 4\tilde{\sigma}_n^{\pi^+ - \pi^-}}(x, z, Q^2). \quad (31)$$

In order to improve statistics one may prefer to replace the numerator and denominator in (30) and (31) by the relevant cross-section differences integrated over some range of z .

6 Conclusions

It will be difficult to obtain accurate information on the various sums and differences of π^\pm asymmetries in semi-inclusive DIS with both proton and neutron targets. Nevertheless very important information can be extracted. Without using any knowledge of the parton densities one can i) test factorization of the production and fragmentation of a quark, i.e. the concept of independent fragmentation, ii) determine the polarized strange quark densities Δs and the ratio of the fragmentation functions $D_s^{\pi^+ + \pi^-} / D_u^{\pi^+ + \pi^-}$, and iii) measure the valence polarization densities $\Delta u_V / du_V$ and $\Delta d_V / d_V$. Finally, using information on the unpolarized u and d quark densities yields a test of SU(2) symmetry of the polarized sea.

7 Acknowledgements

The authors are grateful to the UK Royal Society for a Collaborative Grant. E.C.'s work was supported by the Bulgarian National Science Foundation. E.L. is grateful for the hospitality of the Department of Physics and Astronomy, the Vrije University, Amsterdam, where part of this work was carried out, supported by the Foundation for Fundamental Research on Matter (FOM) and the Dutch Organization for Scientific Research (NWO).

8 Note added in proof

After submission of the paper our attention has been drawn to a paper by L. Frankfurt et al. [8], which addresses related issues.

References

- [1] M. Arneodo et al. (EMC collaboration), Z. Phys. **C31** (1986) 1, Nucl. Phys. **B321** (1989) 541
- [2] J.J. Aubert et al. (EMC collaboration) Phys. Lett. **B114** (1982) 373, Z. Phys. **C31** (1986) 175
- [3] B. Adela et al. (SMC), Phys. Lett. **B369** (1996) 93, Phys. Lett. **B420** (1998) 180,
W. Melnitchouk, J. Speth, A.W. Thomas, Phys. Lett. **B435** (1998) 420,
W. Melnitchouk, hep-ph 9906488
- [4] D. de Florian, L.N. Epele, H. Fanchiotti, C.A. Garcia Canal, S. Joffily, R. Sassot, Phys. Lett. **B389** (1996) 358
- [5] D. Graudenz , Nucl. Phys. **B432** (1994) 351
D. de Florian, C.A. Garcia Canal, R. Sassot, Nucl. Phys. **B470** (1996) 195
D. de Florian, O.A.Sampayo, R. Sassot, Phys. Rev. **D57** (1998) 5803
- [6] A. Doyle, talk given at ICHEP'98, Vancouver hep-ex/9812029
- [7] G. Altarelli, R.D. Ball, S. Forte, G. Ridolfi, Nucl. Phys. **B496** (1997) 337, Acta Phys. Polon. **B29** (1998) 1145
E. Leader, A. Sidorov, D. Stamenov Phys. Rev. **D58** (1998) 114028, Int. J. Mod. Phys. **A13** (1998) 5573
C. Bourrely, F. Buccella, O. Pisanti, P. Santorelli, J. Soffer, Prog. Theor. Phys. **99** (1998) 1017
- [8] L. Frankfurt et al, Phys. Lett. **B230** (1989) 141