

CRITICAL ANALYSIS OF SOME ENTROPY CHARACTERIZATIONS

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Abstract

I offer a critical revision of a number of prominent published characterizations of the Shannon's entropy and relative entropy

$$S(\mathbf{p}) = \sum_{k=1}^n p_k \ln p_k$$
$$S[p, q] = \int dx p(x) \ln \frac{p(x)}{q(x)}$$

as the unique functionals to be used for inductive reasoning in the context of Jaynes' principle of Maximum Entropy. In particular, Shannon's[1], Tikochinsky, Tishby and Levine (TTL's)[2], Shore and Johnson's[3], and Paris and Vencovska's[4] characterizations are shown to possess either logical or technical flaws. Moreover, Lesche's stability argument against Renyi's entropies[5] is refuted on the grounds of using inadequate criterion for continuity. The Karbelkar's[6] and Uffink's[7] objections to some of the above characterizations are also criticized, although their conclusion is shown to stand - namely, the narrowest class of entropies singled out by reasonable requirements either on them as functionals, or on the resulting inference process, is that of the Renyi's relative entropies.

References:

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