MECHANICS OF MATERIALS 2

An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Materials

THIRD EDITION

E. J. HEARN

PhD; BSc(Eng) Hons; CEng; FIMechE; FIProdE; FIDiagE

University of Warwick
United Kingdom

BUTTERWORTH
HEINEMANN

Similarly, the **maximum deflections** in each case are given by the following equations:
For *uniformly distributed loads*,

$$y_{max} = k_2 \frac{qR^4}{Et^3} \tag{7.49}$$

For *loads concentrated around the central hole*,

$$y_{max} = k_2 \frac{FR^2}{Et^3} \tag{7.50}$$

The values of the factors k_1 and k_2 for the loading cases of Fig. 7.10 are given in Table 7.2, assuming a Poisson's ratio ν of 0.3.

Table 7.2. Coefficients k_1 and k_2 for the eight cases shown in Fig. 7.10[a].

$\frac{R}{R_1}$	1.25		1.5		2		3		4		5	
Case	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
1	1.10	0.341	1.26	0.519	1.48	0.672	1.88	0.734	2.17	0.724	2.34	0.704
2	0.66	0.202	1.19	0.491	2.04	0.902	3.34	1.220	4.30	1.300	5.10	1.310
3	0.135	0.00231	0.410	0.0183	1.04	0.0938	2.15	0.293	2.99	0.448	3.69	0.564
4	0.122	0.00343	0.336	0.0313	0.74	0.1250	1.21	0.291	1.45	0.417	1.59	0.492
5	0.090	0.00077	0.273	0.0062	0.71	0.0329	1.54	0.110	2.23	0.179	2.80	0.234
6	0.115	0.00129	0.220	0.0064	0.405	0.0237	0.703	0.062	0.933	0.092	1.13	0.114
7	0.592	0.184	0.976	0.414	1.440	0.664	1.880	0.824	2.08	0.830	2.19	0.813
8	0.227	0.00510	0.428	0.0249	0.753	0.0877	1.205	0.209	1.514	0.293	1.745	0.350

[a] S. Timoshenko, *Strength of Materials*, Part II, *Advanced Theory and Problems*, Van Nostrand, p. 113.

B. BENDING OF RECTANGULAR PLATES

The theory of bending of rectangular plates is beyond the scope of this text and will not be introduced here. The standard formulae obtained from the theory,† however, may be presented in simple form and are relatively easy to apply. The results for the two most frequently used loading conditions are therefore summarised below.

7.15. Rectangular plates with simply supported edges carrying uniformly distributed loads

For a rectangular plate length d, shorter side b and thickness t, the *maximum deflection* is found to occur at the centre of the plate and given by

$$y_{max} = \alpha \frac{qb^4}{Et^3} \tag{7.51}$$

the value of the factor α depending on the ratio d/b and given in Table 7.3.

† S. Timoshenko, *Theory of Plates and Shells*, 2nd edn., McGraw-Hill, New York, 1959.

Table 7.3. Constants for uniformly loaded rectangular plates with simply supported edges[(a)].

d/b	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
α	0.0443	0.0530	0.0616	0.0697	0.0770	0.0843	0.0906	0.0964
β_1	0.0479	0.0553	0.0626	0.0693	0.0753	0.0812	0.0862	0.0908
β_2	0.0479	0.0494	0.0501	0.0504	0.0506	0.0500	0.0493	0.0486
d/b	1.8	1.9	2.0	3.0	4.0	5.0	∞	
α	0.1017	0.1064	0.1106	0.1336	0.1400	0.1416	0.1422	
β_1	0.0948	0.0985	0.1017	0.1189	0.1235	0.1246	0.1250	
β_2	0.0479	0.0471	0.0464	0.0404	0.0384	0.0375	0.0375	

(a) S. Timoshenko, *Theory of Plates and Shells*, 2nd edn., McGraw-Hill, New York, 1959.

The *maximum bending moments*, per unit length, also occur at the centre of the plate and are given by

$$M_{XY_{\max}} = \beta_1 q b^2 \tag{7.52}$$

$$M_{YZ_{\max}} = \beta_2 q b^2 \tag{7.53}$$

the factors β_1 and β_2 being given in Table 7.4 for an assumed value of Poisson's ratio ν equal to 0.3.

It will be observed that for length ratios d/b in excess of 3 the values of the factors α, β_1, and β_2 remain practically constant as also will the corresponding maximum deflections and bending moments.

7.16. Rectangular plates with clamped edges carrying uniformly distributed loads

Here again the *maximum deflection* takes place at the centre of the plate, the value being given by an equation of similar form to eqn. (7.51) for the simply-supported edge case but with different values of α,

i.e.

$$y_{\max} = \alpha \frac{qb^4}{Et^3}$$

The *bending moment* equations are also similar in form, the *numerical maximum occurring at the middle of the longer side* and given by

$$M_{\max} = \beta q b^2$$

Typical values for α and β are given in Table 7.4. In this case values are practically constant for $d/b > 2$.

Table 7.4. Constants for uniformly loaded rectangular plates with clamped edges.[(a)]

d/b	1.00	1.25	1.50	1.75	2.00	∞
α	0.0138	0.0199	0.0240	0.0264	0.0277	0.0284
β	0.0513	0.0665	0.0757	0.0806	0.0829	0.0833

(a) S. Timoshenko, *Theory of Plates and Shells*, 2nd edn., McGraw-Hill, New York, 1959.