



Proposal for PhD topic: Two Neutron Correlation Study in Photofission of Actinides

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Abstract

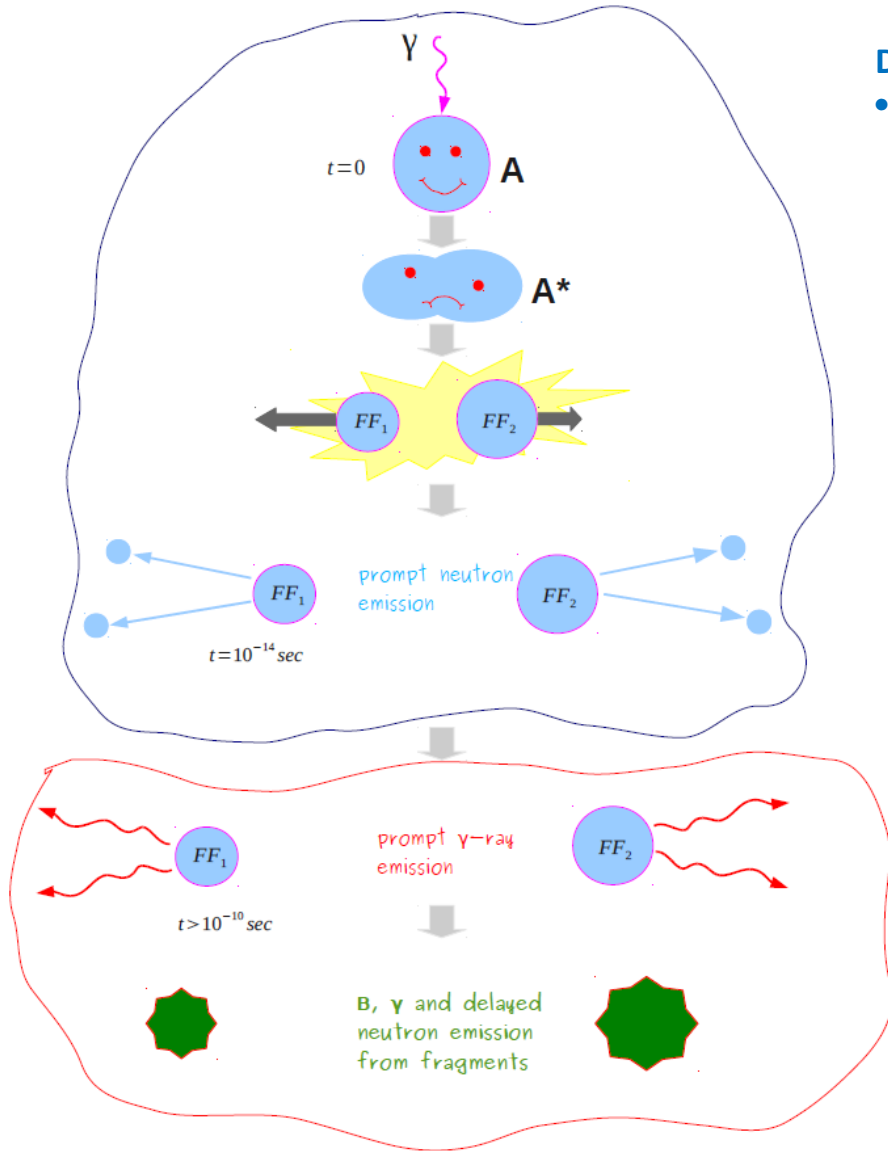
It is well known that two fission fragments (FF's) are emitted essentially back to back in the laboratory frame. That can be used widely in many applications as a unique signature of fissionable materials. However, such fission fragments are difficult to detect. The energy and angular distributions of neutrons, on the other hand, are easy to measure, and that distribution will carry information about the fission fragment's energy and angular spectra, as well as the neutron spectra in the fission fragment rest frame.

We propose to investigate the two neutron correlation yield resulting from two FF's as a function of different targets, the angle between the two neutrons and the neutron energies. The preliminary calculation of the two neutron correlation shows a huge asymmetry effect: many more neutrons are emitted anti-parallel to each other than parallel to each other. That asymmetry becomes even more if the energy cut on each neutron is done. This study will potentially permit a new technique for actinide detection for homeland security and safeguards applications as well as improve our knowledge of correlated neutron emission.

Outline

1. Statement of the physics problems
 - 1.1. Simple summary of fission physics
 - 1.2. Idea of $2n$ correlations
2. Brief review of what has been done
3. Our experimental set-up
4. Expected results
 - 4.1 Asymmetry calculation
 - 4.2 Count rate calculation
 - 4.3 Beam time calculation
5. Summary, conclusion

Simple summary of Fission Physics (1)



Discuss:

- up to the time scale of about $10^{-14} - 10^{-13}$ sec, when the prompt neutrons are emitted from the fully accelerated fragments

Assume:

- no “scission” neutrons
- all neutrons are emitted from fully accelerated fragments

Ignore:

- Time scale greater than $t > 10^{-10}$ sec, when prompt gammas and delayed β, γ and n are emitted

Figure 1.1: Schematic representation of the fission process. Neutrons are emitted from fully accelerated fission fragments. The time scale gives the orders of magnitude only [26].

Simple summary of Fission Physics (2)

It has long been known that the photofission reaction with a heavy nucleus goes through the intermediate compound nucleus. That intermediate nucleus is in an excited state followed by the emission of two fission fragments:



$$TKE = T_1 + T_2 = 165 \text{ MeV}$$

$$\frac{T_1}{T_2} = \frac{m_2}{m_1}$$

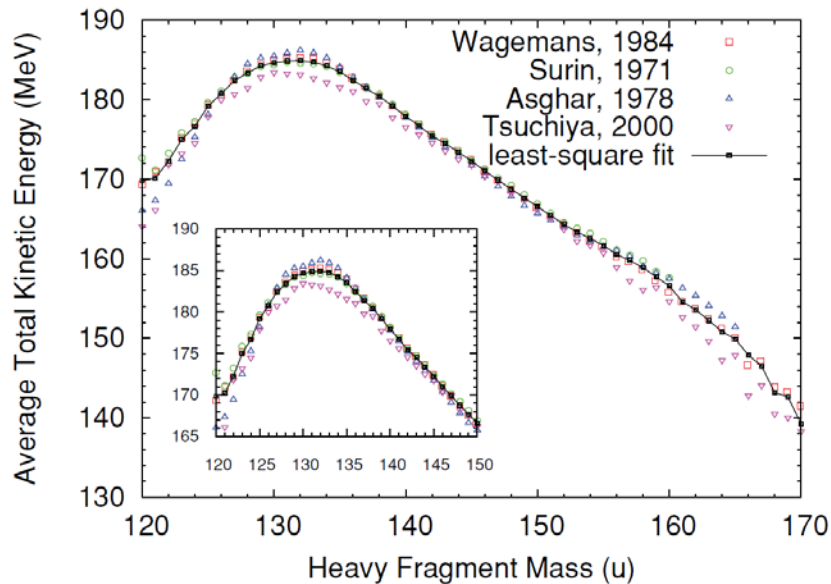


Figure 1.2: The average TKE as a function of the heavy fragment mass. The solid line is the result of a least-square fitting of the experimental data sets [17].

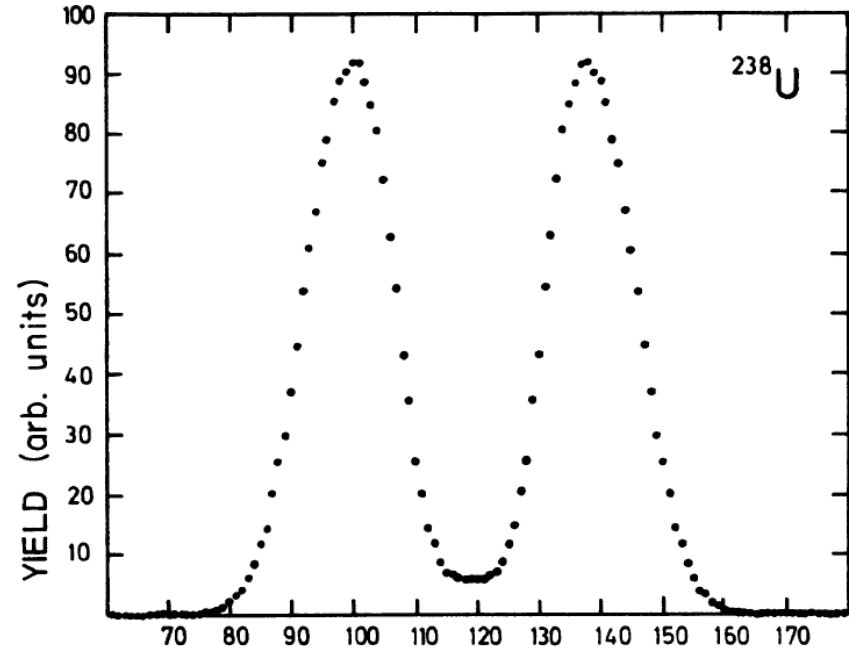


Figure 1.3: Integrated fission fragments yield versus fragment mass for the photofission of ^{238}U with 25-MeV bremsstrahlung [14].

Simple summary of Fission Physics (3)

The angular distribution of individual FF's:

- Aage Bohr's fission channel concept [5]
- Ratzek et al. [11]

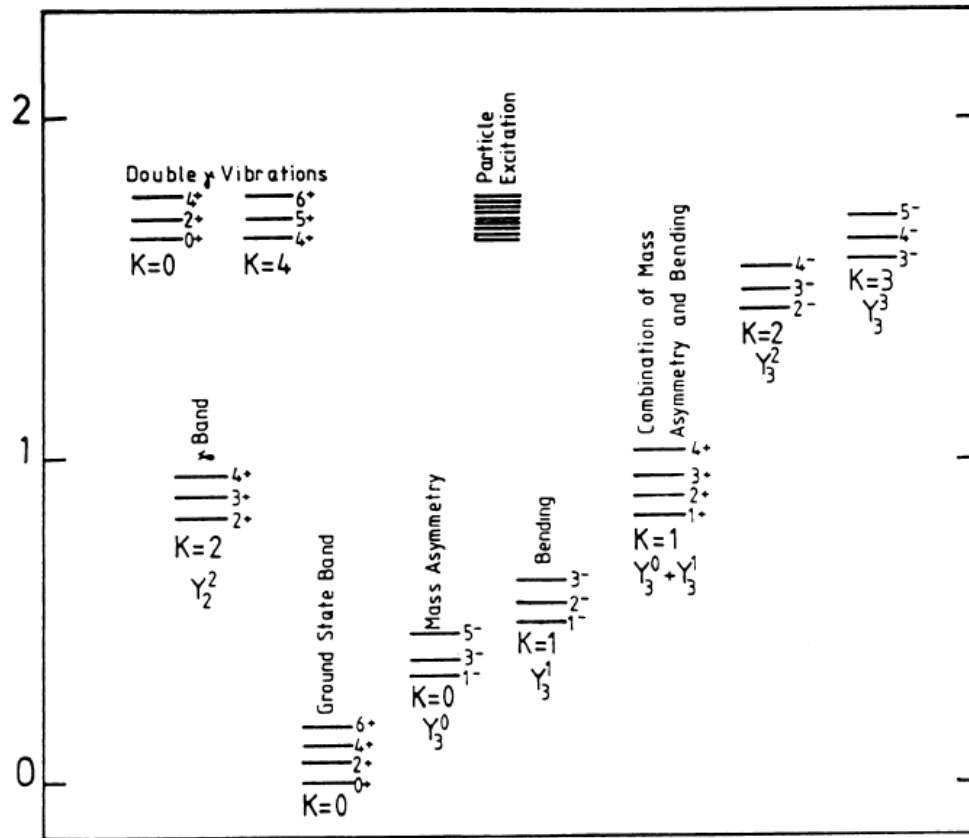
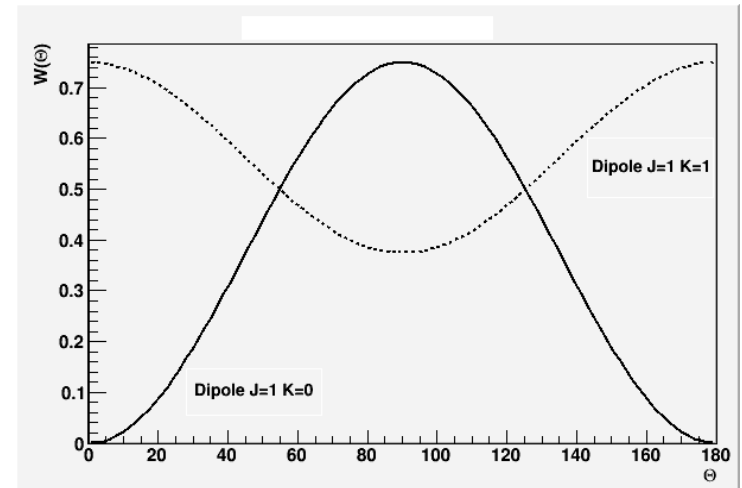


Figure 1.3: Spectrum of theoretically expected low lying collective transition states for even-even nucleus at saddle point [11].

- even-even nucleus $J^\pi = 0^+$
- electric dipole (E1) transition

$$W(\Theta) = A_0 + A_2 P_2(\cos(\Theta))$$



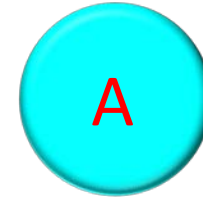
where

- A_0 and A_2 depend on the transition state (J,K)
- K is the projection of the total spin J on the symmetry axis of the deformed nucleus
- For J=1, K=0, we have $A_0 = \frac{1}{2}$, $A_2 = -\frac{1}{2}$
- For J=1, K=1, we have $A_0 = \frac{1}{2}$, $A_2 = \frac{1}{4}$
- $P_2(\cos(\Theta)) = \frac{1}{2}(2 - 3\sin^2(\Theta))$

Simple summary of Fission Physics (4)

1) Simple kinematics of reaction:

- If $E_v < 20$ MeV \rightarrow the excited nucleus A^* is almost in rest
- FF1 and FF2 are flying in opposite direction in LAB

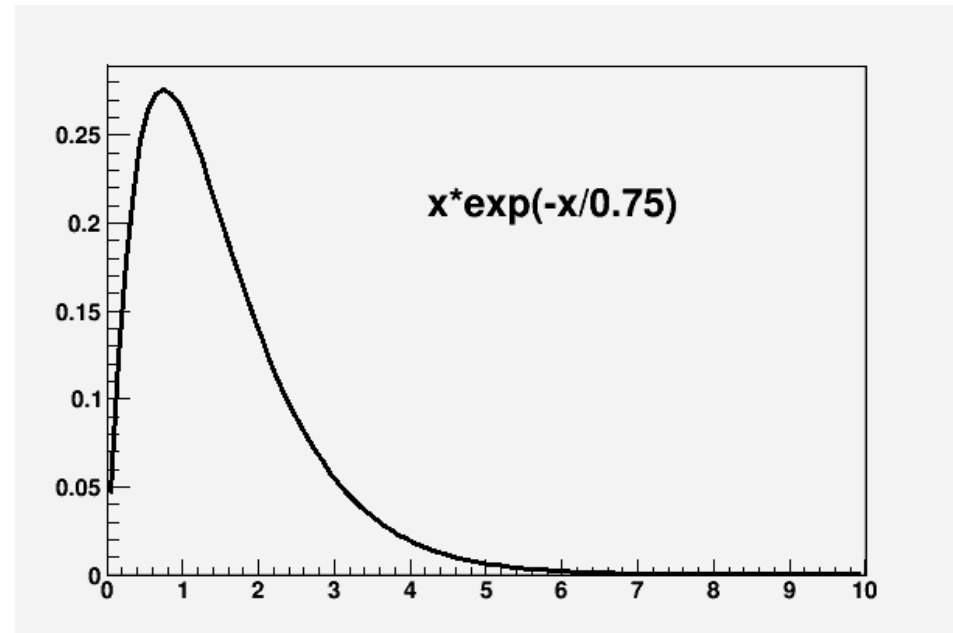


I am so heavy,
I am in rest

2) Some facts about prompt neutrons:

- All prompt neutrons are emitted from fully accelerated fragments
- There are no “scission” neutrons
- The light fragments will emit more neutrons.
- Isotropic angular distribution in the CM frame
- Evaporation spectrum of neutron given by Maxwell:

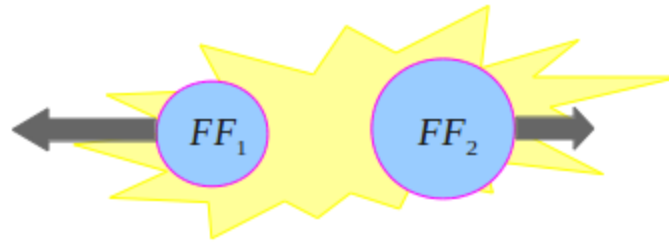
$$\rho(\varepsilon) = \varepsilon \exp\left(-\frac{\varepsilon}{T}\right)$$



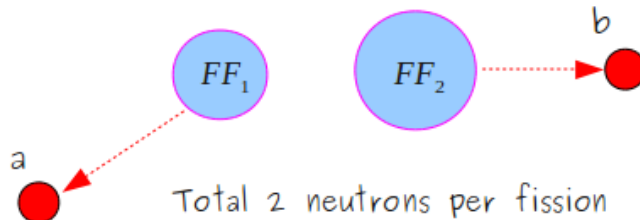
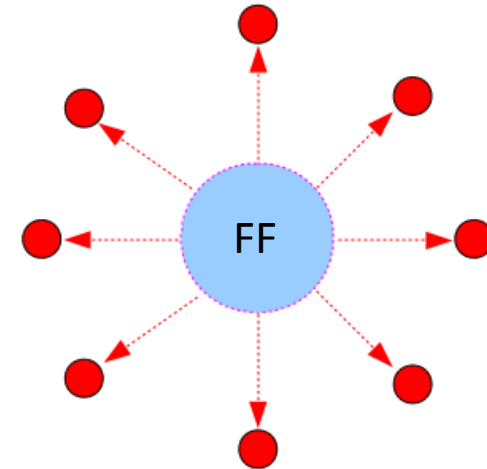
Simple summary of Fission Physics (5)

Below is a short summary of the photofission reaction mechanisms discussed above which will be used in the following to discuss the idea of the proposed two neutron correlation:

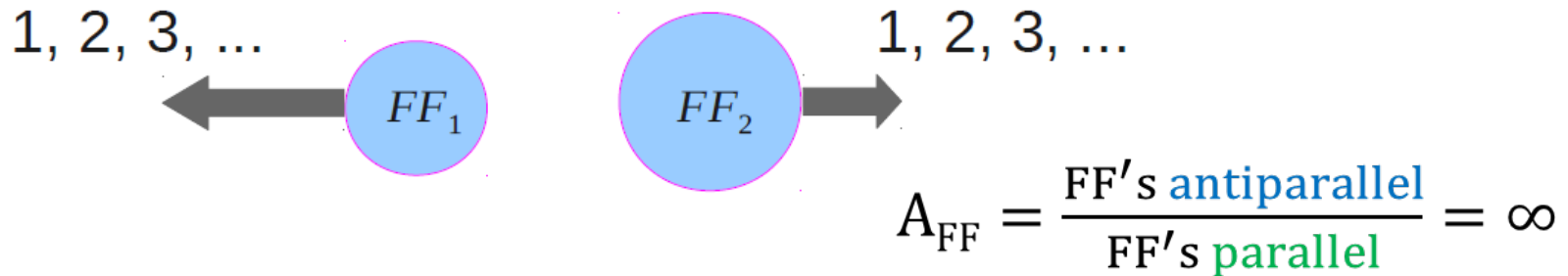
- two fission fragments recoil essentially back to back.



- the angular distribution of the prompt neutrons is isotropic in the center-of-mass of the FF's with a statistical energy distribution.
- each fully accelerated FF emits only one neutron.



Idea of 2n correlation (1)



But:

- The fission fragments are very difficult to detect.
- Neutrons emitted by FF's will fly outside of the target and could be easily detected.
- Whether or not the asymmetry of fission fragments A_{FF} is manifest in the correlated angular distribution of prompt neutrons?

In order to answer this question, we propose to measure the two neutron angular and energy distributions with the ultimate goal of calculation the two neutron asymmetry:

$$Y_{2n}(\Theta_{2n}, E_a, E_b) \Rightarrow A_{2n} = \frac{2n's \text{ antiparallel}}{2n's \text{ parallel}}$$

Idea of 2n correlation (2)

1) Take a typical 1 MeV neutron in the center-of-mass frame of the fission fragment:

$$T = \frac{mc^2\beta^2}{2} \quad \beta = \sqrt{\frac{2 * 1 \text{ MeV}}{939 \text{ MeV}}} = 4.6\%$$

2) Take two fission fragments with typical mass numbers $A_1 = 95$ and $A_2 = 143$

$$\frac{T_1}{T_2} = \frac{m_2}{m_1} = 1.51 \quad \Rightarrow \quad T_1 = 99 \text{ MeV} \quad \Rightarrow \quad \beta_1 = \sqrt{\frac{2 * 99 \text{ MeV}}{95,000 \text{ MeV}}} = 4.6\%$$
$$T_1 + T_2 = 165 \text{ MeV} \quad T_2 = 66 \text{ MeV} \quad \Rightarrow \quad \beta_2 = \sqrt{\frac{2 * 66 \text{ MeV}}{143,000 \text{ MeV}}} = 3.0\%$$

3) The expected 2n correlation asymmetry could be thought of as:

$$A_{2n} = A_{FF} \times W_n \times W_{scat} = \text{LARGE}$$

- A_{FF} - asymmetry of two fission fragments
- W_n - washing effect due to isotropic angular distribution of neutrons in the FF rest frame
- W_{scat} - washing effect due to neutron multiple scattering effect inside the target and surrounding materials

Review of what has been done (1)

- 1956, Winhold and Halpern [9].

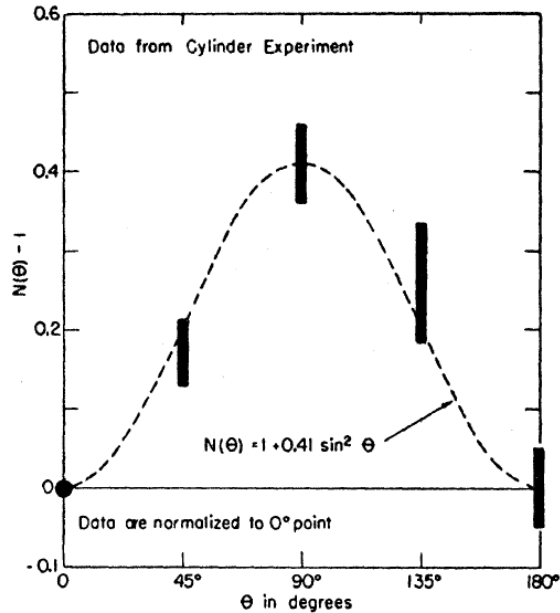


Figure 2.1: The angular distribution, $N(\theta)$, of fission fragments from Th^{232} caught at the angles θ to the x-ray beam.

- **Motivation:**
Goldhaber – Teller oscillation

- **Angular distribution has the form**

$$a + b \sin^2 \theta$$

- **b/a depends on:**

1. Energy of the photons
2. Fissionable target
3. Fission fragment observed

anisotropic photofission is due solely to photons with in about 3 MeV of the fission threshold
(could be explained by A. Bohr model)

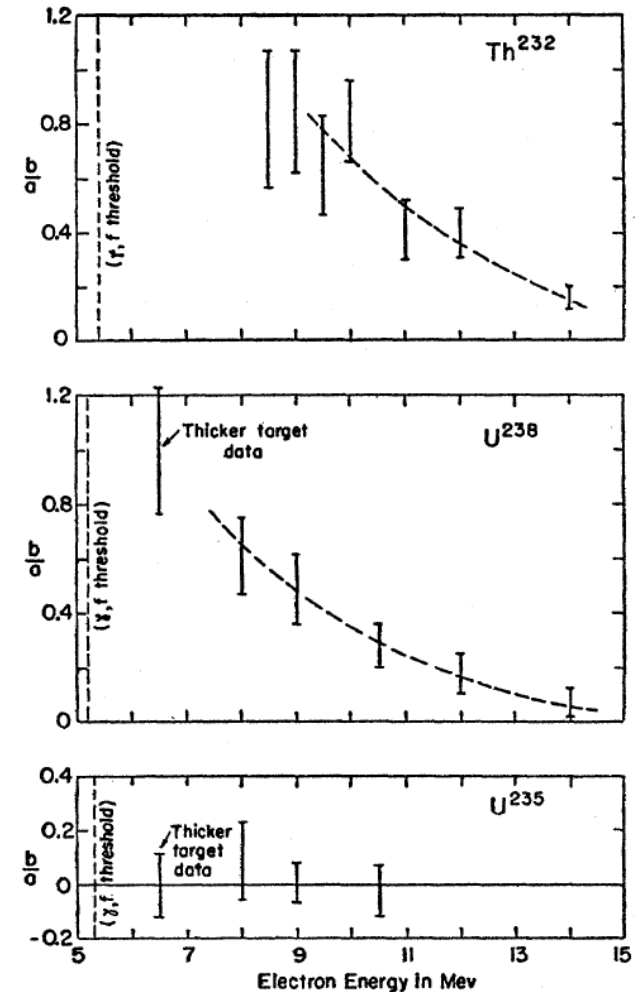


Figure 2.2: The anisotropy in the photofission of three targets. The angular distributions were all assumed to be the form $a + b \sin^2 \theta$ [9].

Review of what has been done (2)

- 1962, Bowman *et al.* [10]: ^{252}Cf spontaneous fission, TOF to measure neutrons in coincidence with FF's

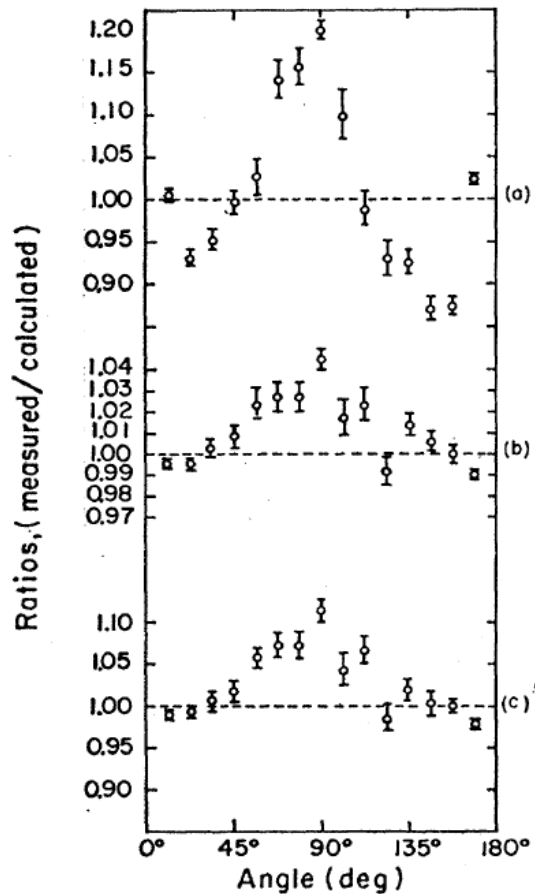
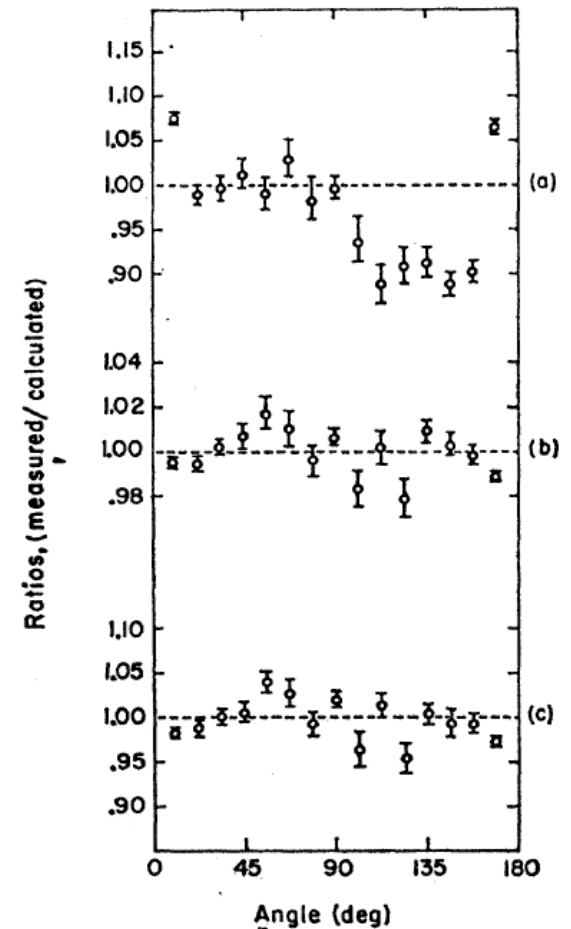


Figure 2.3: The ratio of measured to calculated values for

- (a) numbers of neutrons,
- (b) average velocities
- (c) average energies

as a function of angles.

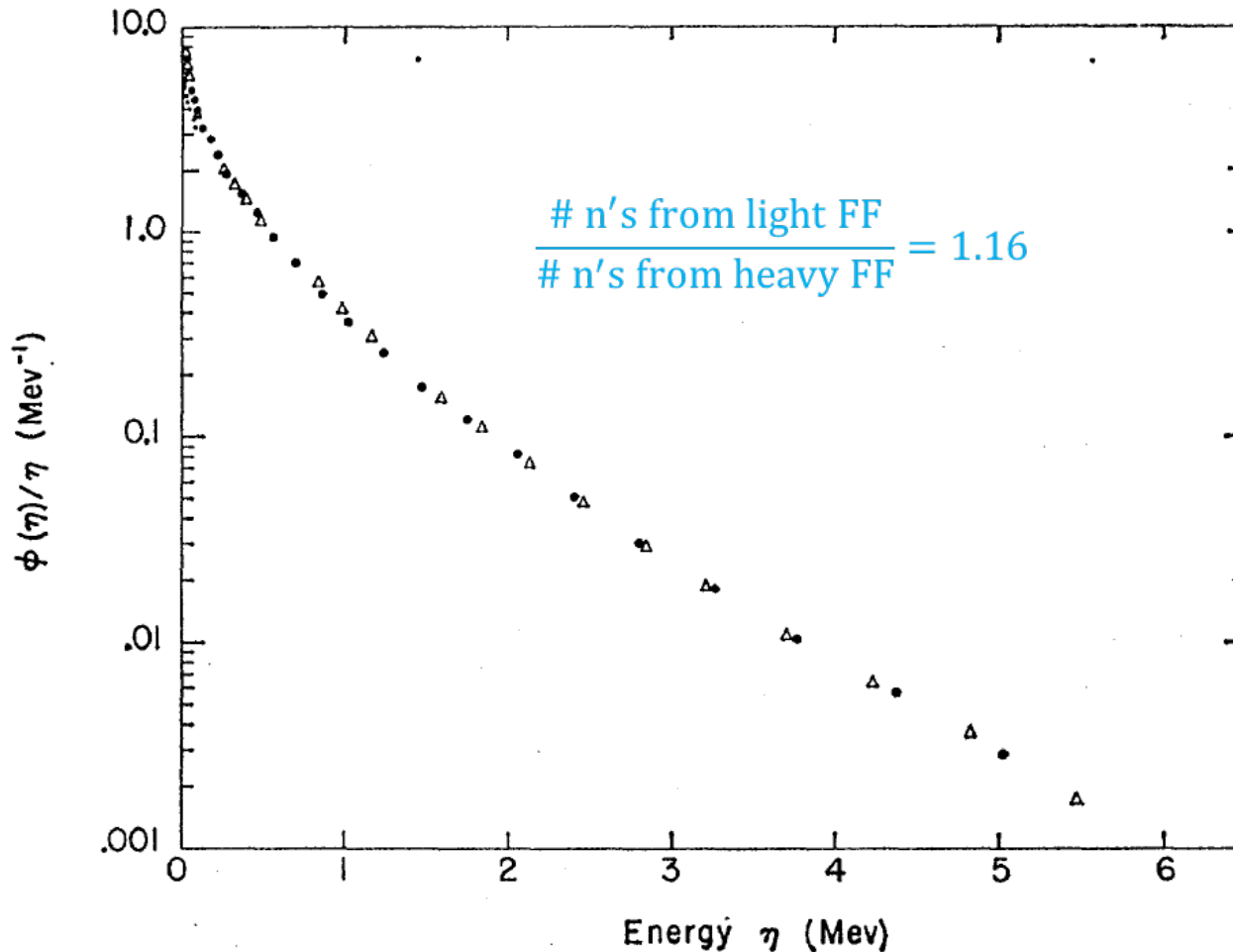


no "scission" neutrons

10% of "scission" neutrons

Review of what has been done (3)

- Bowman *et al.* [10]. Continue...



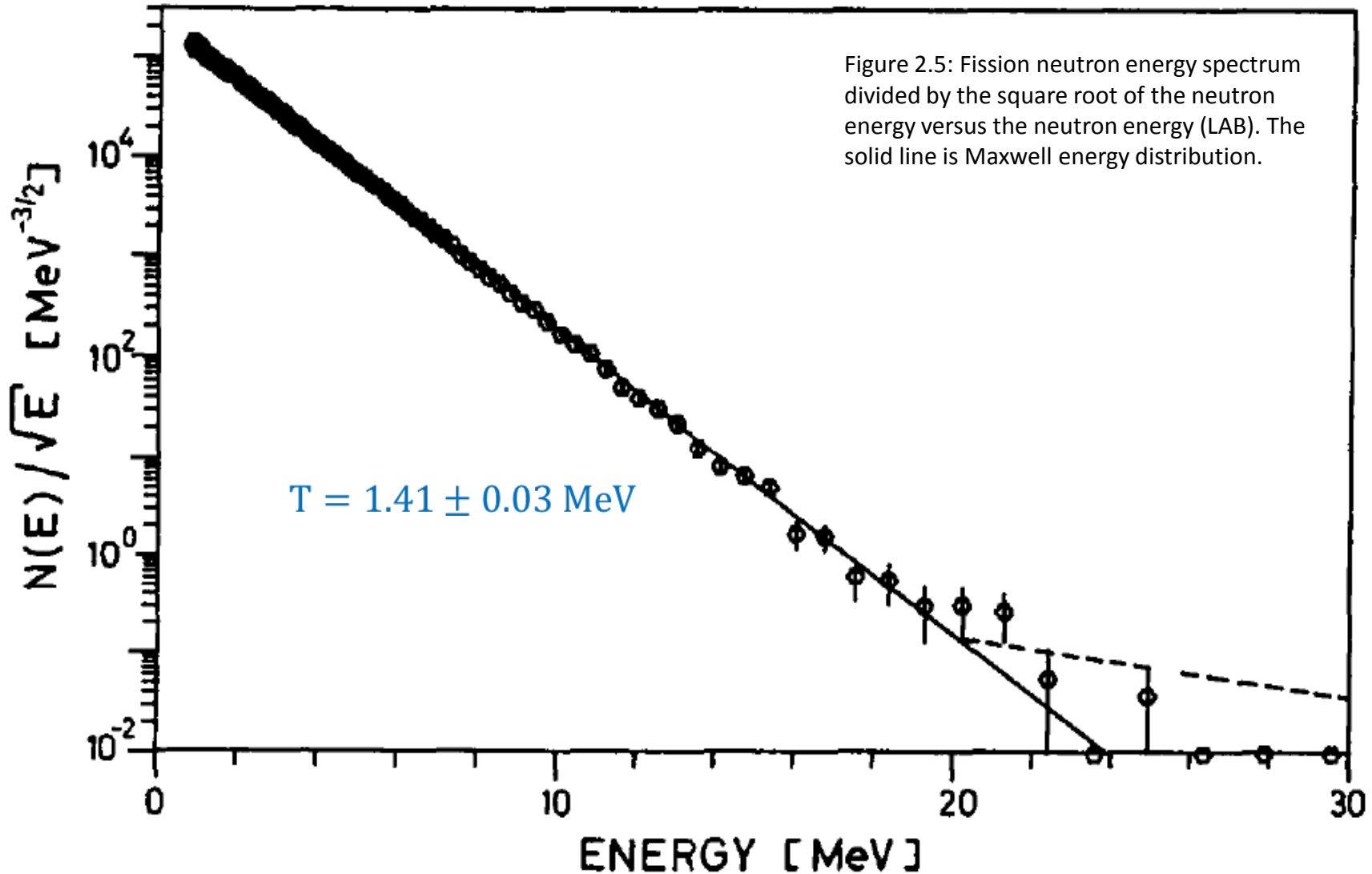
The results can be explained well by assumption of isotropic evaporation of neutrons from the fully accelerated fragments.

Figure 2.4: The center-of-mass neutron energy spectrum $\Phi(\eta)$ (CM) divided by η . The large dots - the neutrons emitted in the direction of the light fragments and the triangles - the neutrons emitted in the direction of the heavy fragments. The curve for light fragments was reduced by the factor 1.16. Light – 11.25, heavy – 168.75 deg

Review of what has been done (4)

- 1988, Budtz-Jorgensen and Knitter [12]

^{252}Cf spontaneous fission, TOF techniques



Review of what has been done (5)

- Budtz-Jorgensen and Knitter [12]. Continue...

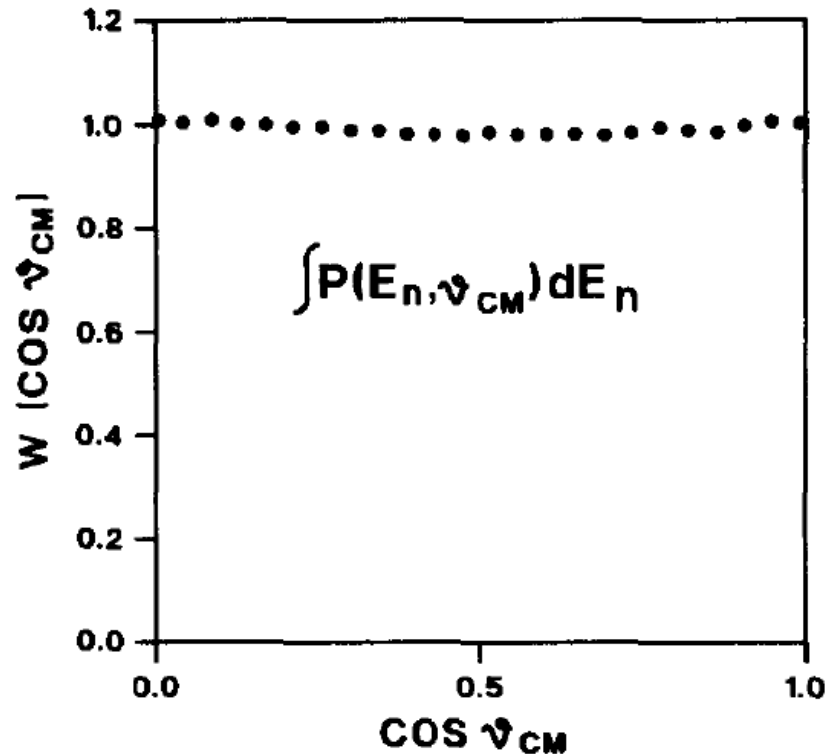


Figure 2.6: Fission neutron angular distribution in the fragment center-of-mass system integrated over all neutron energies

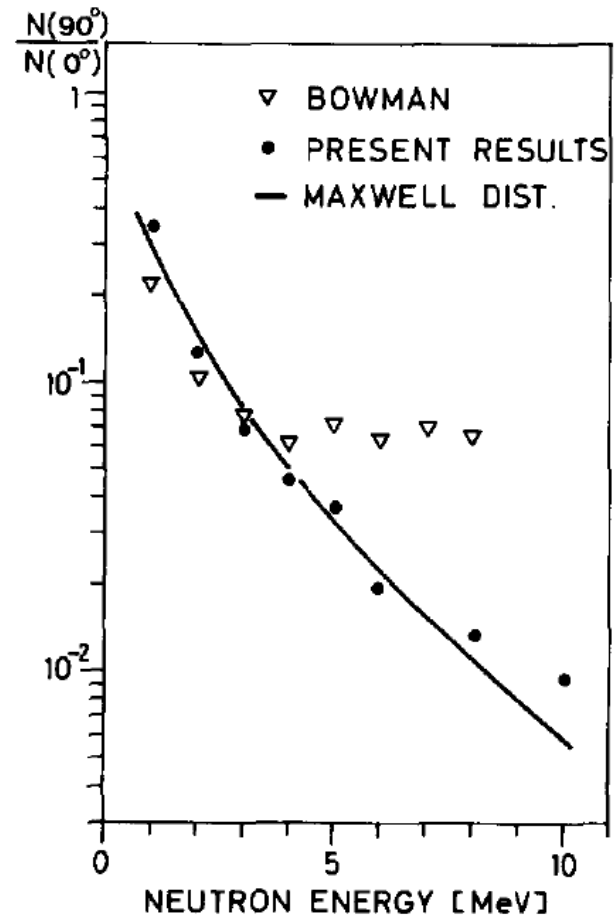


Figure 2.7: Fission neutron intensity ratio $N(90^\circ)/N(0^\circ)$ is plotted versus the fission neutron energy. The solid line is a theoretical line calculated with the assumption that there are no 'scission' neutrons and is in good agreement with the Budtz-Jorgensen measurements.

Experimental set-up (1)

We plan to use the HRRL LINAC to construct the beamline to produce the bremsstrahlung photons:

- 20 ns pulse width
- 10-80 mA peak current
- hopefully 1000 Hz beam pulse repetition rate

When electrons strike the radiator, that results in the bremsstrahlung radiation in the forward with respect to the beam direction. The typical energy spectrum of bremsstrahlung photons for the 7 MeV endpoint energy is shown in Fig 4.6.

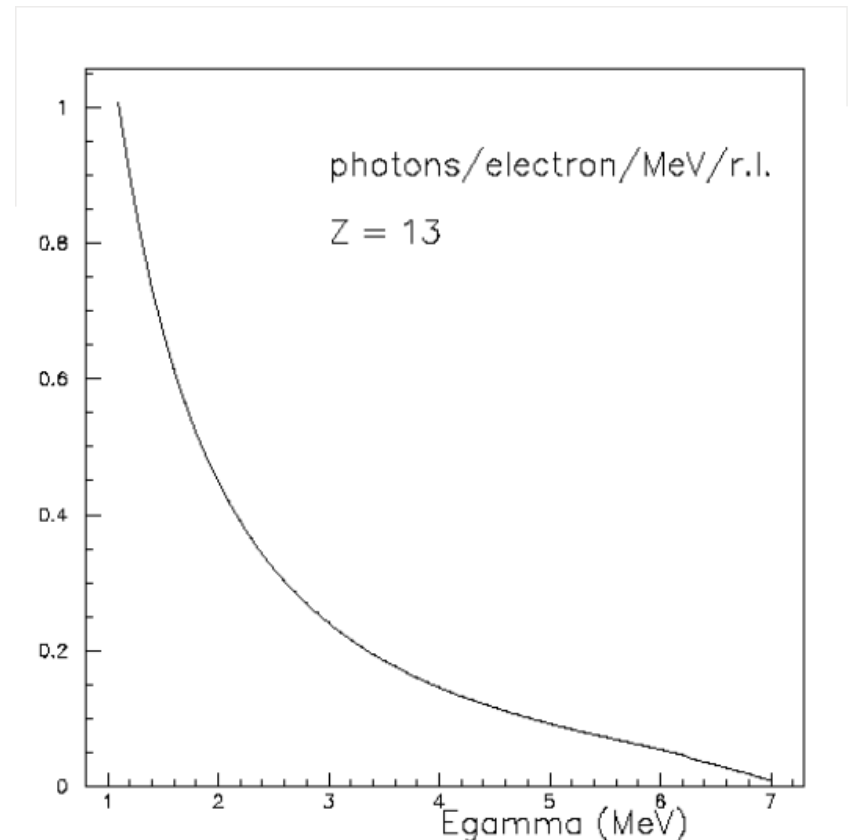
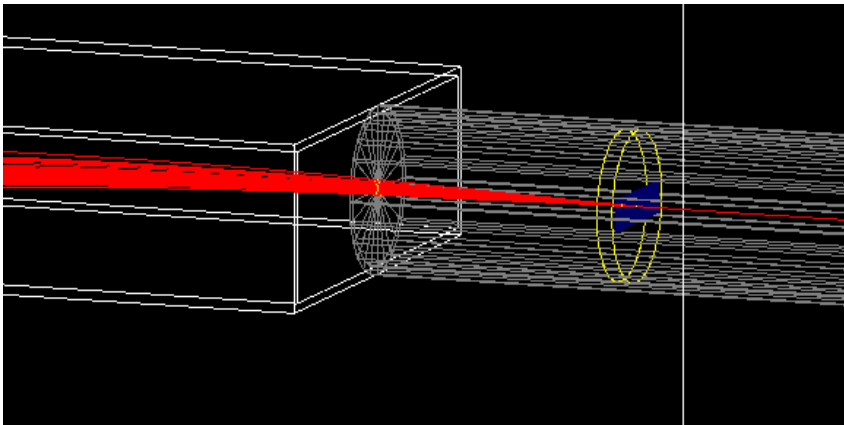


Figure 4.6: Bremsstrahlung spectrum of photons produced by 7 MeV electrons hitting the Al radiator (Owens and Matthews)

Experimental set-up (2)

- The time of flight (TOF) technique will be used to identify neutrons and to measure their energy, with the start signal coming from the accelerator beam pulse.

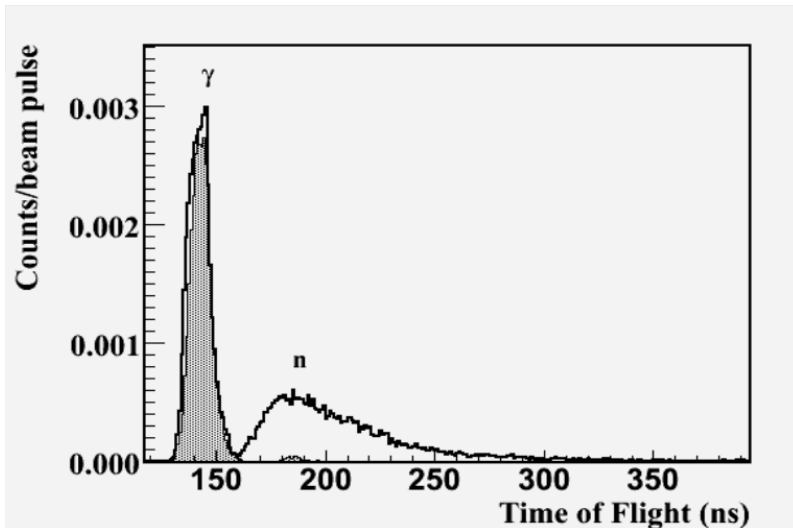


Figure 3.1: Typical TOF spectrum from photodisintegration of deuteron measured from previous HRRL runs. The distance from target to detector is about 2 m. The spectrum illustrates the ability to distinguish gammas peak from neutrons one.

Take a typical 1 MeV neutron located 1 m away from target:

$$\text{TOF}(n's) = \frac{1 \text{ m}}{0.05 \times 3 \cdot 10^8 \text{ m/s}} \approx 67 \text{ ns} \quad (1)$$

$$\text{TOF}(\gamma's) \approx 3.3 \text{ ns} \quad (2)$$

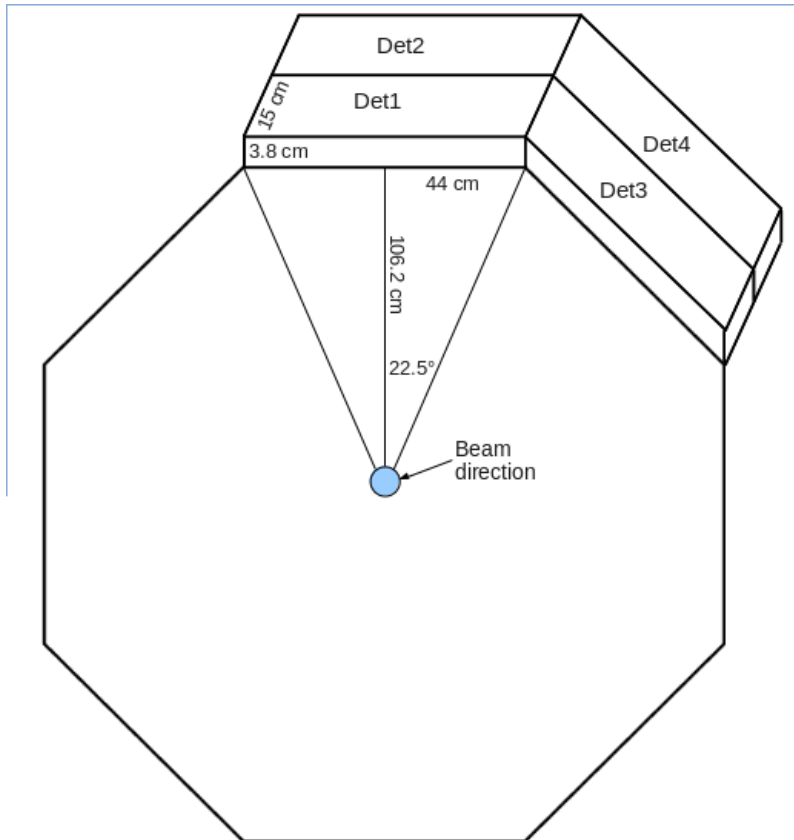
$\text{TOF}(n's) \gg \text{TOF}(\gamma')$ so we can distinguish n's from γ 's

Error in n energy will depend on the LINAC pulse width

$$\frac{\delta E}{E} = \frac{\delta t}{t} = \frac{20 \text{ ns}}{67 \text{ ns}} \approx 30\%$$

Experimental set-up (3)

There are 16 plastic scintillators with area of $15 \text{ cm} \times 75.8 \text{ cm} = 0.114 \text{ m}^2$



To maximize the $2n$ correlation yield such plastic scintillators will be placed at the angle of 90 degree with respect to the beam surrounding the target. Further thinking and calculation about the detector location should be done but, in principal, that will allow almost 2π cover as can be seen from Fig 3.2.

Figure 3.2: Possible detector geometry to measure the two neutron correlation yield. Total 16 neutron detectors are placed at the angle of 90 degree with respect to the beam. The detector size is $15 \text{ cm} \times 75.8 \text{ cm} \times 3.8 \text{ cm}$.

Experimental set-up (4)

To find the angle between two neutrons we need to find the position y where the neutron hits the detector

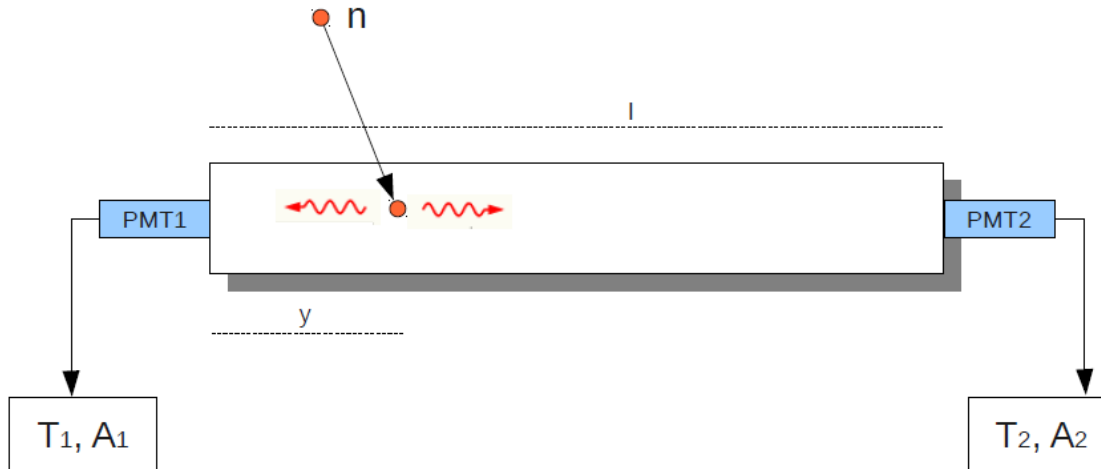


Figure 3.3: Neutron detector with two PMT's attached to both ends. Neutron n hits the detector at distance y from left PMT. The amplitude signals A_1 , A_2 and TOF signals T_1 , T_2 are measured from PMT1 and PMT2 correspondingly.

Amplitude method:

$$A_1 = I_0 e^{-\alpha y} \quad A_2 = I_0 e^{-\alpha(l-y)}$$

$$y = \frac{l}{2} - \frac{1}{2\alpha} \ln \frac{A_1}{A_2}$$

TOF method:

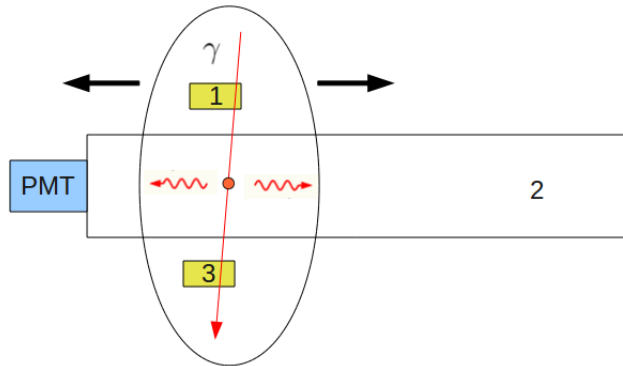
$$T_1 = \frac{L}{c} + \frac{yn}{c} \quad T_2 = \frac{L}{c} + \frac{(l-y)v}{c}$$

$$y = \frac{c}{2n} (T_1 - T_2) + \frac{l}{2}$$

- Both techniques can be used to calculate the position where the neutron hits the detector.
- However the last method looks more simple and preferable

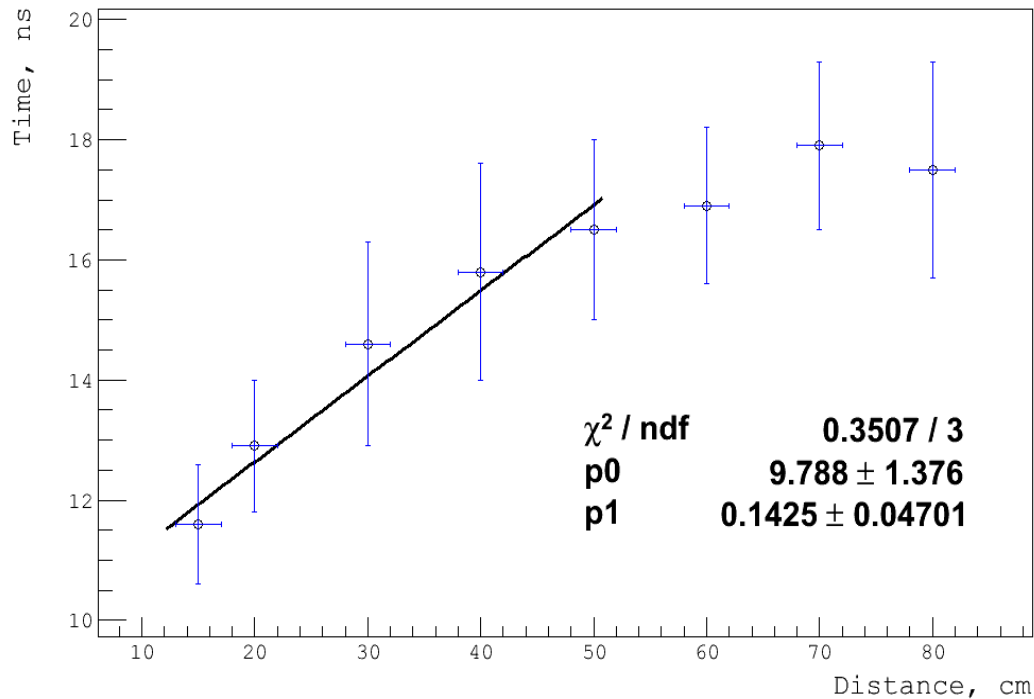
Experimental set-up (5)

Preliminary TOF measurements with 1 PMT attached to the end of the detector



Two small plastic detectors 1 and 2 were placed above and under the "big" plastic detector and were moved along the "big" one. The triple coincidence between detectors 1, 2 and 3 from the cosmic ray was used as a start signal to measure the time as a function of distance.

Time vs. distance (total 30 events)



- The results show the ability to identify the source position as a function of measured TOF.

Figure 3.5: TOF measurements with 1 PMT attached to the end of detector.

Expected results. Asymmetry calculation (1)

To estimate the expected asymmetry in 2n correlations, a Monte-Carlo simulation was performed:

Assumption:

- ^{238}U with $J = 1$ and $K = 0$ is used as the fissionable target
- $85 < A_{\text{light}} < 105$ and $130 < A_{\text{heavy}} < 150$
- TKE = 165 MeV
- Each fission fragment emits one neutron. Two neutrons per fission.
- Neutrons are emitted isotropically in the CM of fully accelerated fission fragments with the energy distribution given by:

$$N(E) = \sqrt{E} \exp\left(-\frac{E}{0.75}\right)$$

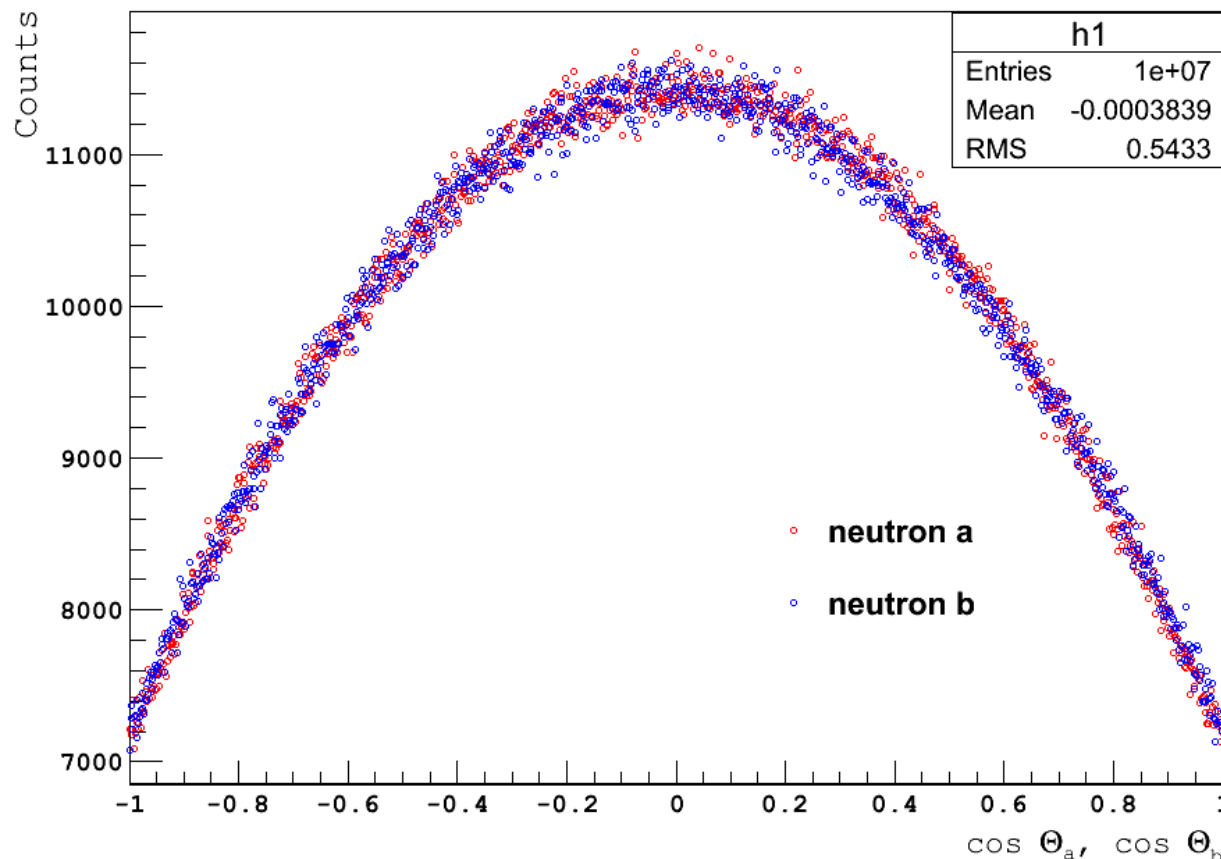
- Two fission fragment are back to back. The fission fragment angular distribution is sampled according to:

$$W(\Theta) = \frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} (2 - 3\sin^2\Theta) \right) = \frac{3}{4} \sin^2\Theta$$

Expected results. Asymmetry calculation (2)

After both angular and energy distributions of neutrons and FF's were sampled using the assumptions above, neutrons were boosted from the fission fragments rest frame into the laboratory frame.

Cos Θ of neutrons a and b in LAB frame.

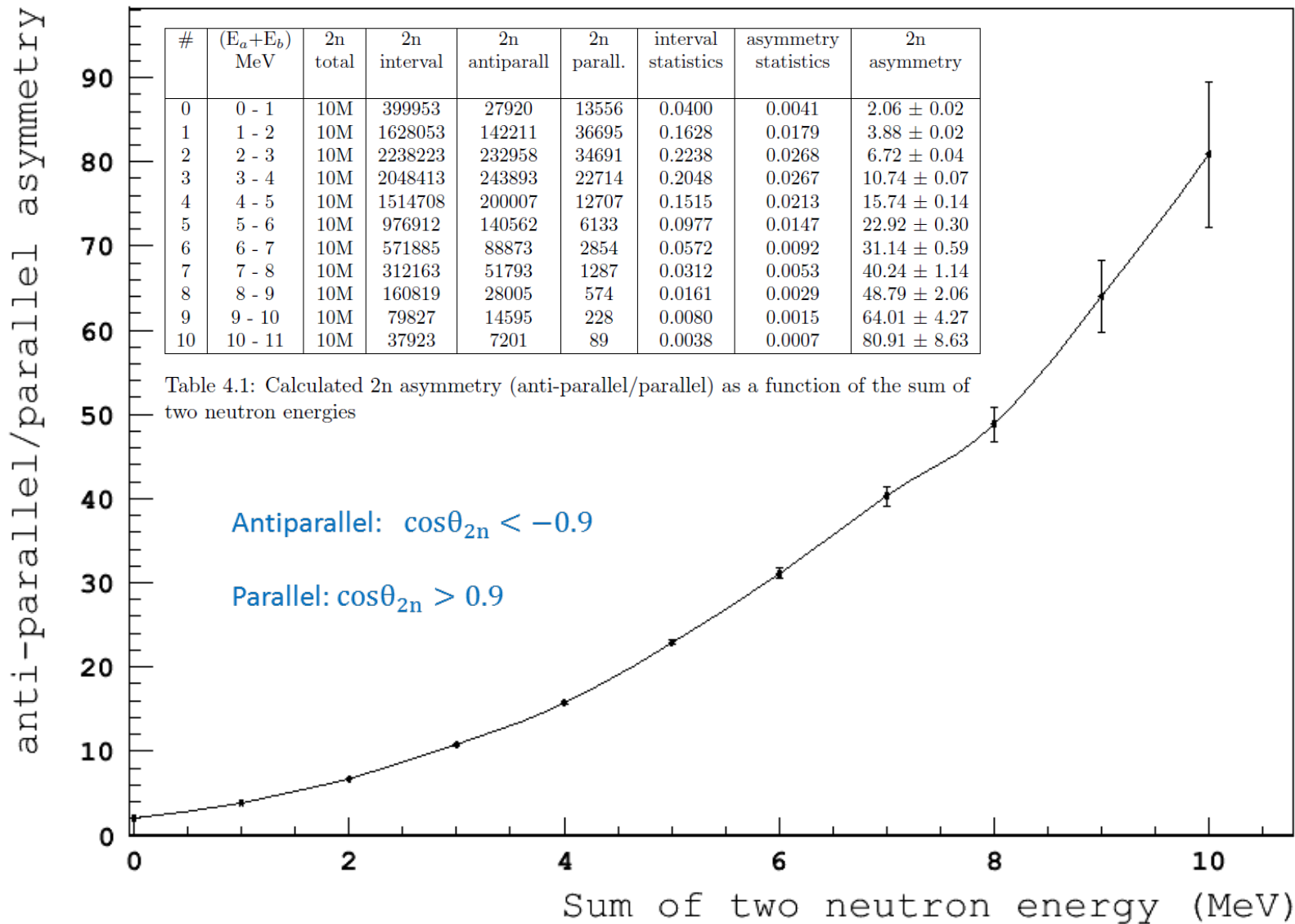


- angular distributions of both neutrons a and b look statistically similar
- angular distribution is strongly anisotropic
- the angular distribution of the FF's is strongly manifested in the angular distribution of prompt neutrons in laboratory frame.

Figure 4.1: Angular distribution of prompt neutrons a (red) and b (blue) emitted by two fission fragments as seen in laboratory frame

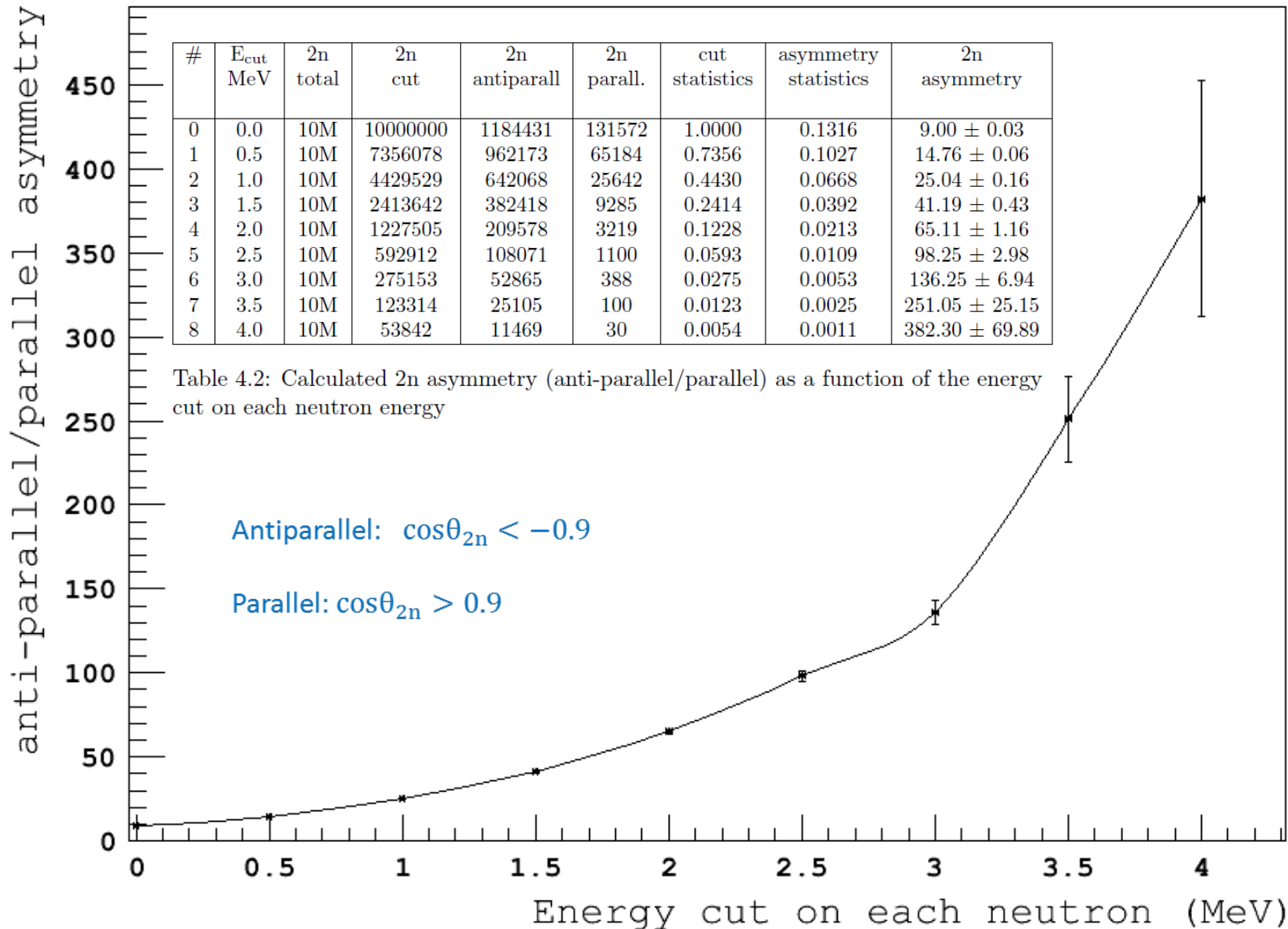
Expected results. Asymmetry calculation (3)

Two Neutrons Asymmetry vs. Energy



Expected results. Asymmetry calculation (4)

Two Neutrons Asymmetry vs. Energy



Expected results. Asymmetry calculation (5)

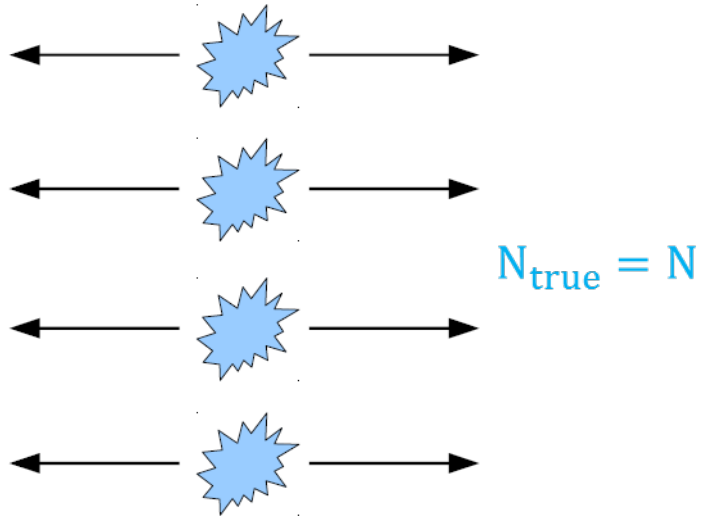
What can be done extra to improve calculation:

- Use realistic FF's mass distribution
- Use realistic multiplicity value
- Use separate nuclear temperature T for each FF
- Use realistic TKE
- Calculate the neutron multiple scattering effect inside the target.
- That all can be done later, however, the results of simulation show the huge asymmetry effect in $2n$ correlation.
- That will potentially permit a new technique for actinide detection for homeland security and safeguards applications.
- Very interesting physics can be done here.

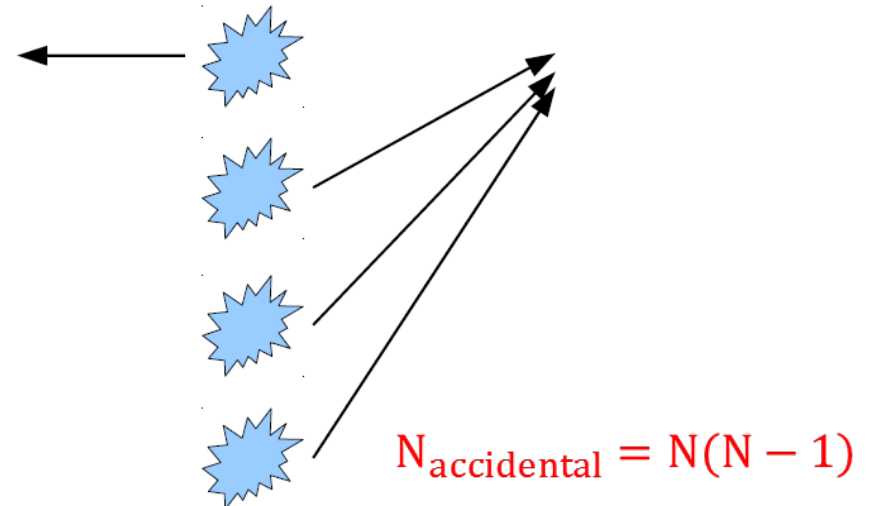
Expected results. Count rate calculation (1)

Assume we have N fission events per beam pulse.

True coincidence - between two neutrons coming from the same fission event



Accidental coincidence - between two neutrons coming from the different fission events



$$\frac{N_{\text{true}}}{N_{\text{accidental}} + N_{\text{true}}} = \frac{N}{N(N - 1) + N} = \frac{1}{N}$$

We need to design the experiment in such a way that the following condition is satisfied:

$$N = \frac{1 \text{ fission}}{\text{pulse}}$$

Expected results. Count rate calculation (2)

Beam parameter:

- Pulse width $\tau = 20$ ns
- Peak current $I = 20$ mA
- Energy

$$N_{e^-} = 20 \cdot 10^{-3} \frac{\text{C}}{\text{sec}} \times \frac{1 e^-}{1.6 \cdot 10^{-19} \text{C}} \times 20 \text{ ns} = 2.5 \cdot 10^9 \frac{e^-}{\text{pulse}}$$

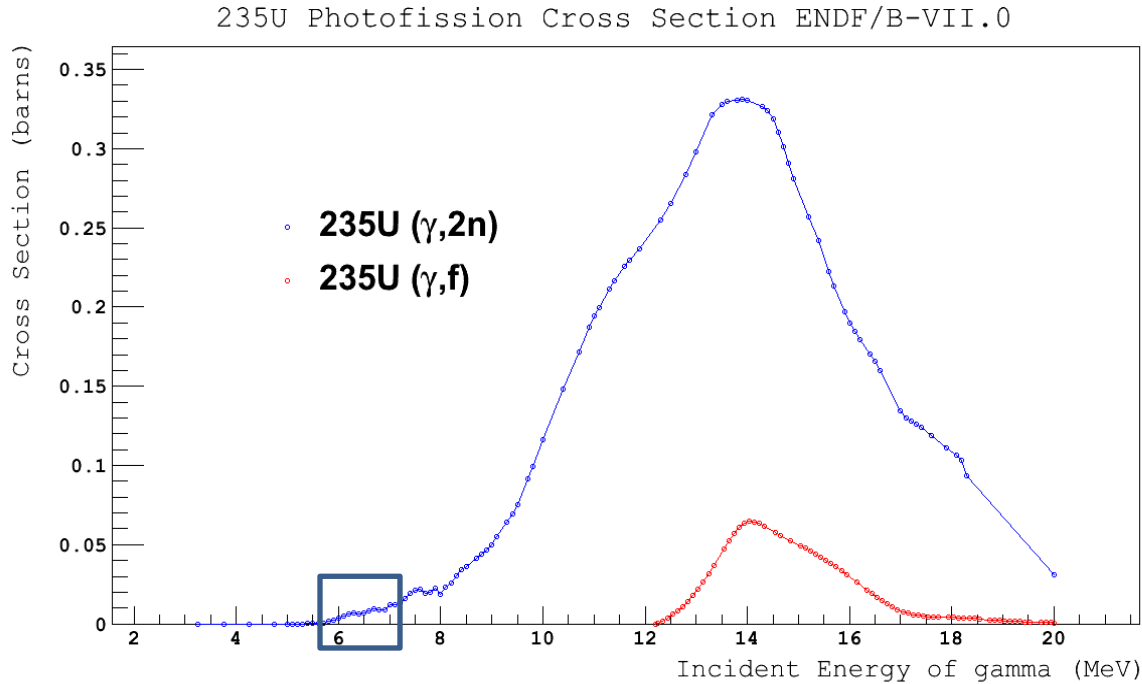


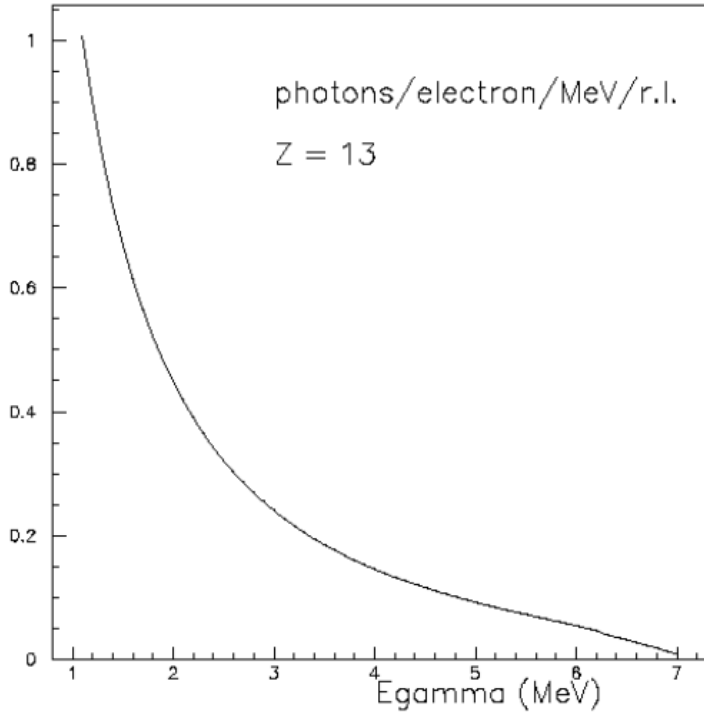
Figure 4.5: ^{235}U photofission cross section taken from ENDF/B-VII.0 [25]

Optimal energy of incident gammas would be about 6-7 MeV:



- (γ, f) cross-section is low
- no “2n knockout”
- study the pure (γ, f)

Expected results. Count rate calculation (3)



Bremsstrahlung out of radiator:

$$N_{\gamma'} = N_{e^-} \times 0.05 \frac{\text{photons}}{e^- \text{ MeV r.l.}} \times 1 \text{ MeV} \times 10^{-3} \text{ r.l.} = 1.25 \cdot 10^5 \frac{\gamma' \text{ s}}{\text{pulse}}$$

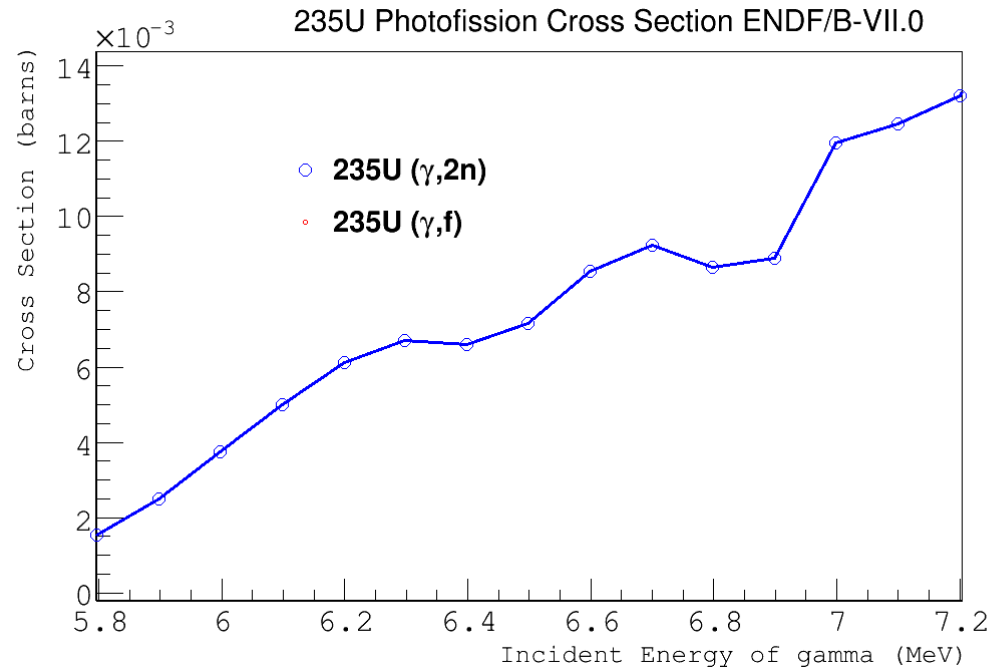
Bremsstrahlung after collimation:

$$N_{\gamma'} = 1.25 \cdot 10^5 \frac{\gamma' \text{ s}}{\text{pulse}} \times 50\% = 6.25 \cdot 10^4 \frac{\gamma' \text{ s}}{\text{pulse}}$$

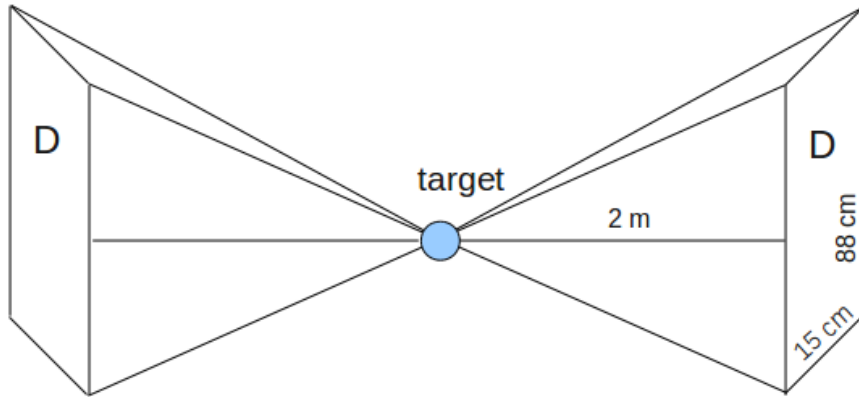
$$N_{\text{target}} = N_{\gamma} \times t \times \sigma = \frac{1 \text{ fission}}{\text{pulse}}$$

$$\Rightarrow t = 2.29 \cdot 10^{21} \frac{\text{atoms}}{\text{cm}^2}$$

$$\Rightarrow t = 470 \mu\text{m}$$



Expected results. Beam time calculation



$$\Omega = \frac{S}{r^2} = 3.3 \cdot 10^{-2} \text{ rad}$$

Figure 4.7: Two detector geometry located 2 m away from target

The count rate for two neutron detectors, located 2 m away from the target

$$N \left[\frac{\text{counts}}{\text{sec}} \right] = \frac{1 \text{ fission}}{\text{pulse}} \times N_G^2 \times N_{\text{intr}}^2 \times N_{\text{cut}} \times 2.2 \frac{\text{neutrons}}{\text{pulse}} \times 10^3 \text{ Hz} = 4 \cdot 10^{-4} \frac{\text{counts}}{\text{sec}}$$

- N_G is the geometrical detector efficiency, Ω
- N_{intr} is the intrinsic detector efficiency, assume 25%
- N_{cut} is neutron energy cut efficiency, for 1 MeV – 44%
- 2.2 is the average number neutron per pulse
- 10^3 Hz is HRRL repetition rate

16 neutron detectors (factor 64): $\Rightarrow N_{16 \text{ det}} = 2.6 \cdot 10^{-2} \frac{\text{counts}}{\text{sec}}$

Run 1 day: $\Rightarrow N_{\text{day}} = N_{16 \text{ det}} \times 28,800 \text{ sec} \approx 750 \frac{\text{counts}}{\text{day}}$

Summary, conclusion

- A huge asymmetry effect: many more neutrons are emitted anti-parallel to each other than parallel.
- There are a total of 16 “big” plastic detectors
 - ⇒ 750 counts per day with HRRL
 - ⇒ couple days and we are done
- Potential for a new technique for actinide detection for homeland security and safeguards applications
- Improve our knowledge of correlated neutron emission.
- **A lot of interesting work can be done here**

Bibliography (1)

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#	($E_a + E_b$) MeV	2n total	2n interval	2n antiparall	2n parall.	interval statistics	asymmetry statistics	2n asymmetry
0	0 - 1	10M	399953	27920	13556	0.0400	0.0041	2.06 ± 0.02
1	1 - 2	10M	1628053	142211	36695	0.1628	0.0179	3.88 ± 0.02
2	2 - 3	10M	2238223	232958	34691	0.2238	0.0268	6.72 ± 0.04
3	3 - 4	10M	2048413	243893	22714	0.2048	0.0267	10.74 ± 0.07
4	4 - 5	10M	1514708	200007	12707	0.1515	0.0213	15.74 ± 0.14
5	5 - 6	10M	976912	140562	6133	0.0977	0.0147	22.92 ± 0.30
6	6 - 7	10M	571885	88873	2854	0.0572	0.0092	31.14 ± 0.59
7	7 - 8	10M	312163	51793	1287	0.0312	0.0053	40.24 ± 1.14
8	8 - 9	10M	160819	28005	574	0.0161	0.0029	48.79 ± 2.06
9	9 - 10	10M	79827	14595	228	0.0080	0.0015	64.01 ± 4.27
10	10 - 11	10M	37923	7201	89	0.0038	0.0007	80.91 ± 8.63

Table 4.1: Calculated 2n asymmetry (anti-parallel/parallel) as a function of the sum of two neutron energies

#	E_{cut} MeV	2n total	2n cut	2n antiparall	2n parall.	cut statistics	asymmetry statistics	2n asymmetry
0	0.0	10M	10000000	1184431	131572	1.0000	0.1316	9.00 ± 0.03
1	0.5	10M	7356078	962173	65184	0.7356	0.1027	14.76 ± 0.06
2	1.0	10M	4429529	642068	25642	0.4430	0.0668	25.04 ± 0.16
3	1.5	10M	2413642	382418	9285	0.2414	0.0392	41.19 ± 0.43
4	2.0	10M	1227505	209578	3219	0.1228	0.0213	65.11 ± 1.16
5	2.5	10M	592912	108071	1100	0.0593	0.0109	98.25 ± 2.98
6	3.0	10M	275153	52865	388	0.0275	0.0053	136.25 ± 6.94
7	3.5	10M	123314	25105	100	0.0123	0.0025	251.05 ± 25.15
8	4.0	10M	53842	11469	30	0.0054	0.0011	382.30 ± 69.89

Table 4.2: Calculated 2n asymmetry (anti-parallel/parallel) as a function of the energy cut on each neutron energy

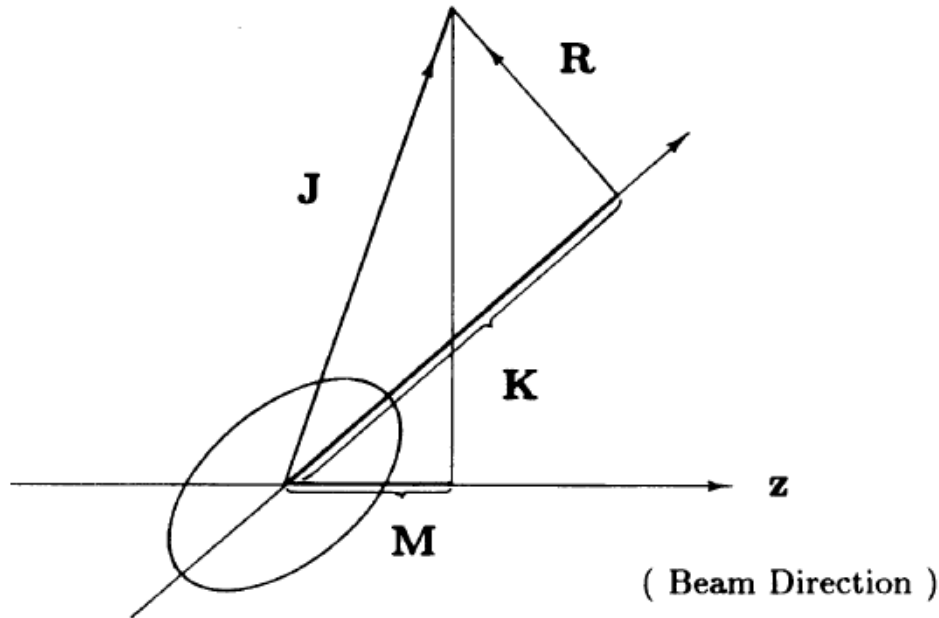


FIGURE 3. Coupling scheme of angular momenta for a deformed nucleus: J = total angular momentum, R = rotational angular momentum, K = projection of J on the symmetry axis, and M = projection of J on the quantization axis z (beam axis).

$$W_{M,K}^J(\theta) = \frac{(2J + 1)}{2} \cdot |d_{M,K}^J(\theta)|^2$$

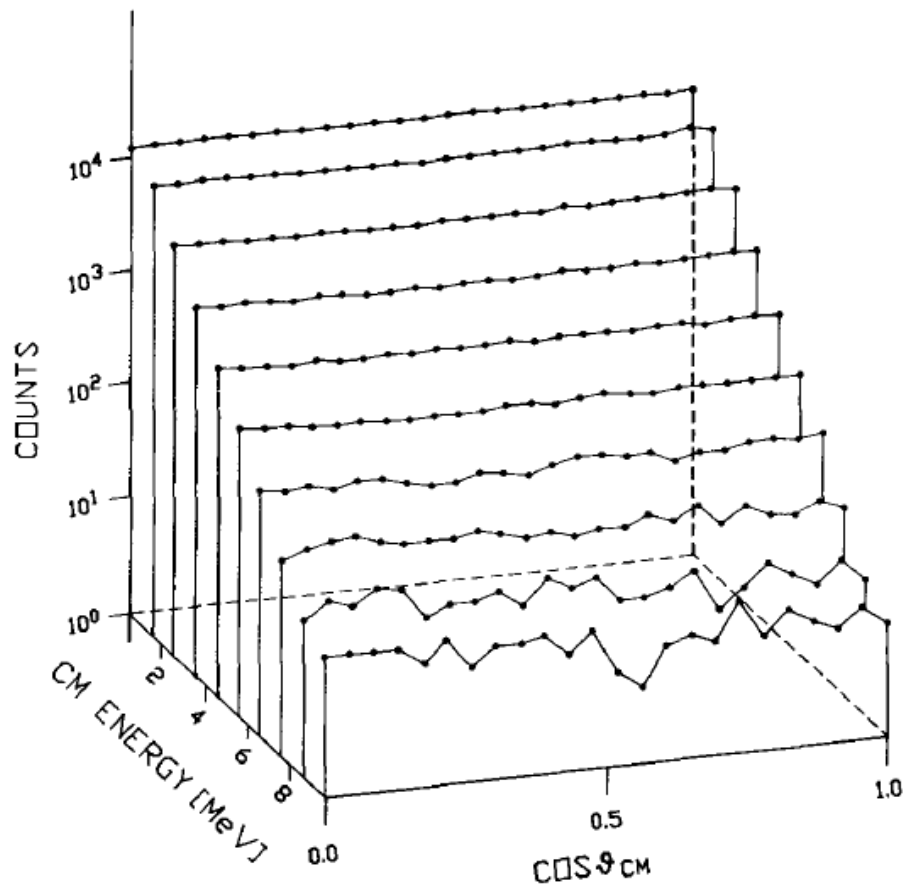


Fig. 9. Fission neutron angular distribution $0 \leq \cos \theta_{c.m.} \leq 1$ as a function of fragment center-of-mass fission neutron energy.

3.2.3. *The fission neutron spectrum in the center-of-mass system.* The neutron energy η in the center-of-mass system of the fragment was evaluated event by event using the procedures described in the beginning of sect. 3.2. Fig. 16 displays the η spectrum integrated over all fragments. According to standard nuclear evaporation theory the center-of-mass neutron energy spectrum corresponding to a fixed residual nuclear temperature T is given approximately by Weisskopf²³⁾

$$\Phi(\mu) = \frac{\eta}{T} \exp(-\eta/T). \quad (7)$$

The evaporation spectrum for neutrons emitted in a cascade process is slightly modified and Le Couteur and Lang²⁴⁾ obtained:

$$\Phi(\eta) = \text{const } \eta^\lambda \exp(-\eta/T_{\text{eff}}), \quad (8)$$

is ≈ 2 MeV and varies only little with the fissioning nuclei in the actinide region. The simplest and most commonly used approximations for the fission neutron spectra in the laboratory reference system are Maxwellian distributions as presented by Terrell:¹⁴⁻¹⁶

$$N_M(E) = \frac{2 \cdot E^{1/2}}{\pi^{1/2} \cdot T_M^{3/2}} \cdot \exp\left(-\frac{E}{T_M}\right) \quad (6)$$

where T_M is the only parameter characterizing the distribution. The average neutron energy is given by $\bar{E} = 3/2 \cdot T_M$.

If for the shape of the neutron evaporation spectrum a Maxwell distribution is assumed, $N(E) \sim \sqrt{E} \cdot \exp(-E/T_e)$, where T_e is the temperature of the nucleus after the evaporation of one neutron, and furthermore if it is assumed that all fragments have the same kinetic energy per nucleon E_f , then the laboratory neutron spectrum shape is a Watt spectrum:¹⁷

$$N_w(E) = \frac{2 \cdot A^{3/2}}{(\pi \cdot B^{1/2})} \cdot \exp\left(-\frac{B}{4 \cdot A}\right) \cdot \exp(-A \cdot E) \cdot \sinh(B \cdot E)^{1/2} \quad (7)$$

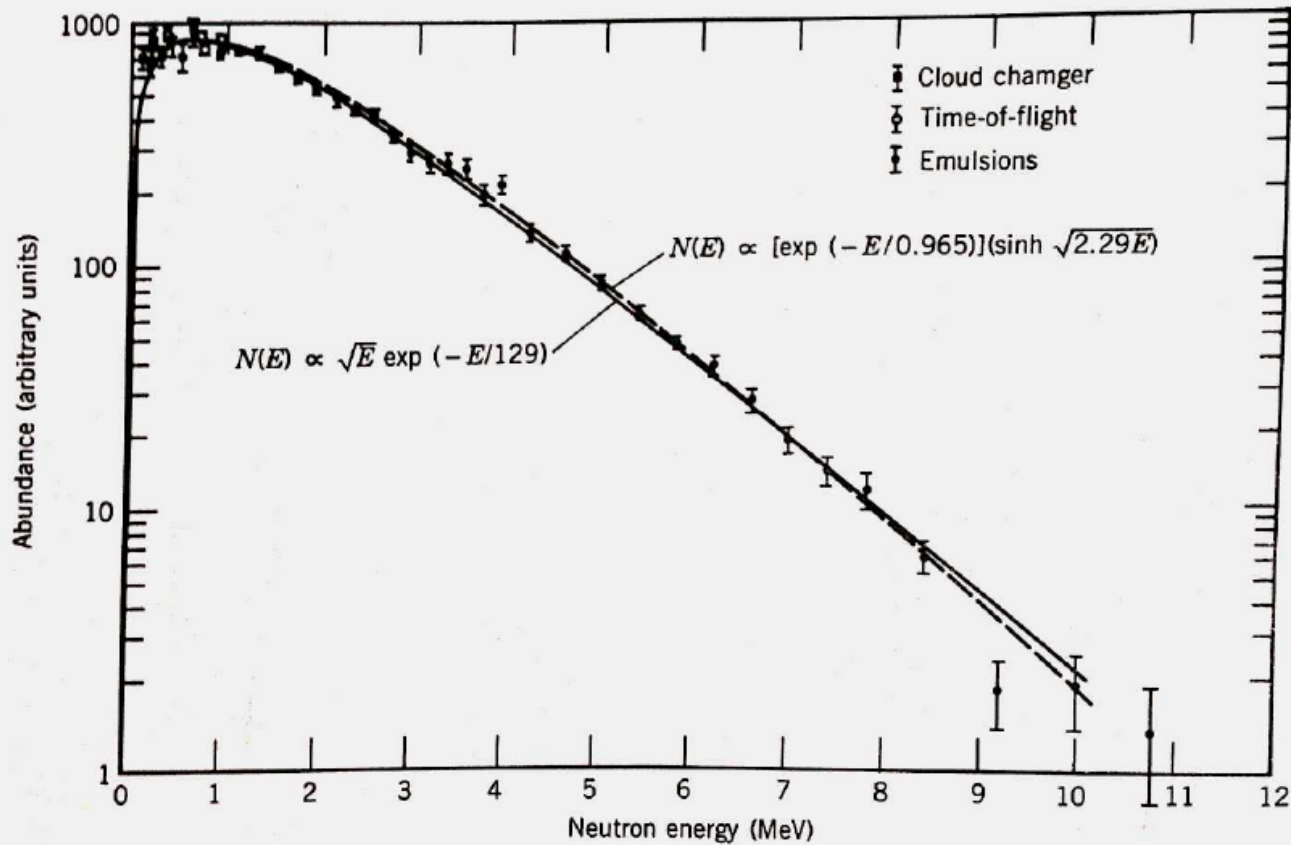
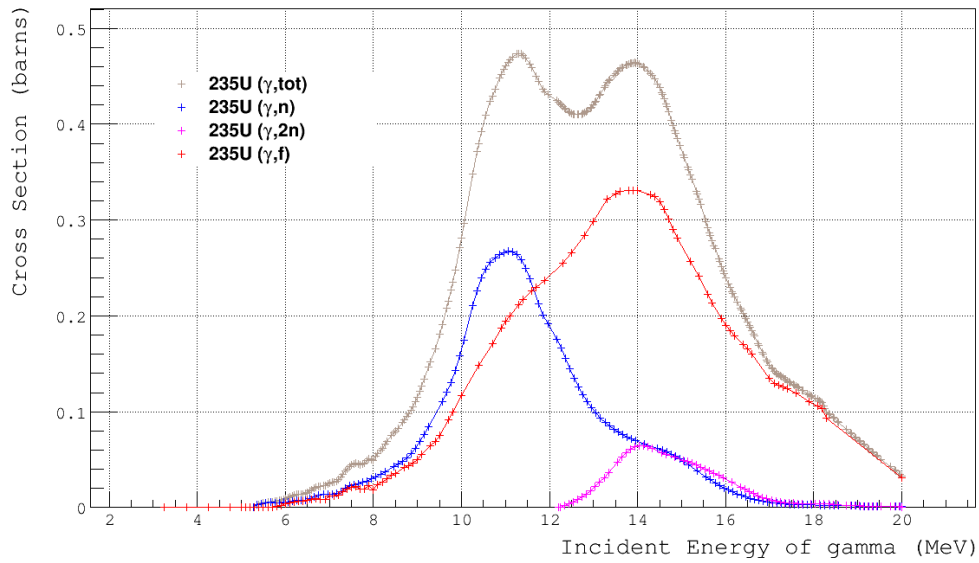
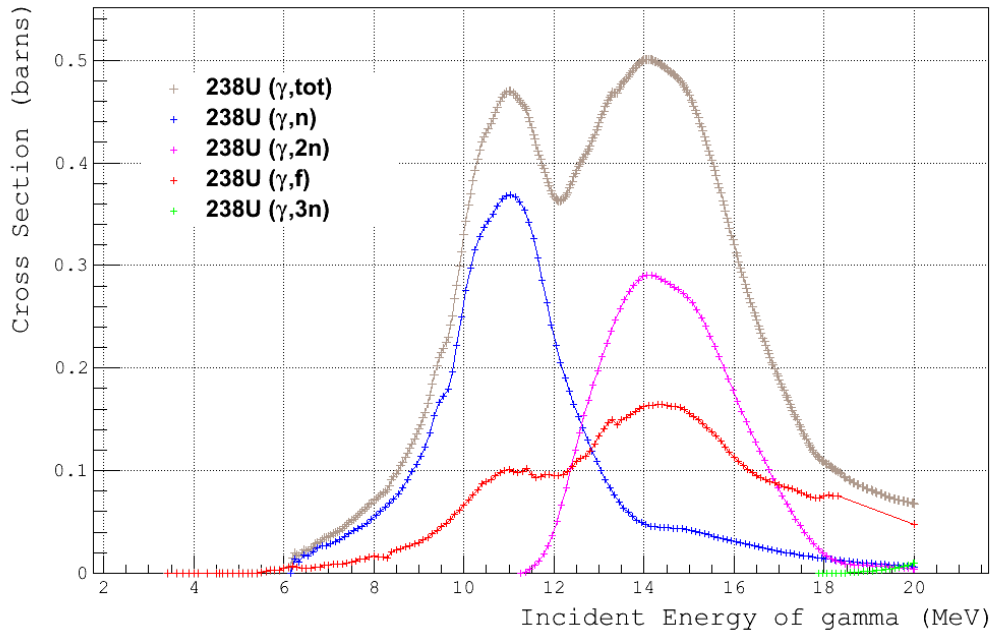


Figure 13.13 Energy spectrum of neutrons emitted in the thermal-neutron fission of ^{235}U . From R. B. Leachman, in *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy*, Vol. 2 (New York: United Nations, 1956), p. 193.

235U Photofission Cross Section ENDF/B-VII.0



238U Photofission Cross Section ENDF/B-VII.0



We want one fission per pulse. That can be found by adjusting the target thickness from the equation below:

$$\frac{1 \text{ fission}}{\text{pulse}} = N_\gamma \times t \times \sigma \quad (4.8)$$

where t is the target thickness in atoms/cm² and the σ is the $(\gamma, 2n)$ photo-nuclear cross section and is about 7 mb/atom in the 6-7 MeV energy range as can be seen from Fig 4.5 above. The thickness becomes:

$$t \left[\frac{\text{atoms}}{\text{cm}^2} \right] = \frac{1 \frac{\text{fission}}{\text{pulse}}}{6.25 \cdot 10^4 \frac{\gamma/s}{\text{pulse}} \times 7 \frac{\text{mb}}{\text{atom}}} = 2.29 \cdot 10^{21} \frac{\text{atoms}}{\text{cm}^2} \quad (4.9)$$

and could be converted into cm as follows:

$$t [cm] = \frac{t \cdot M}{\rho \cdot N_A} = \frac{2.29 \cdot 10^{21} \frac{\text{atoms}}{\text{cm}^2} \times 235.04 \frac{\text{g}}{\text{mol}}}{19.1 \frac{\text{g}}{\text{cm}^3} \times 6.02 \cdot 10^{23} \frac{\text{atoms}}{\text{mol}}} = 470 \mu\text{m} \quad (4.10)$$

where M is the molar mass, ρ is the density of ²³⁵U and N_A is the Avogadro number.