Reconstruction of electron spectra from depth doses with adaptive regularization

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Electron spectral reconstruction of medical accelerators from measured depth doses is a practical method for providing the input initial phase space distribution at the patient surface that is required by Monte Carlo treatment planning systems. The posed inverse problem of spectral reconstruction is ill conditioned and this may lead to nonphysical oscillations in the reconstructed spectra. Use of a variational method of solution with a regularization technique removes the oscillations but tends to smooth the sharp (deltalike) energy peak that is a common feature in electron spectra generated by medical accelerators. Because the sharp peak contains a large percentage of the electrons in the spectrum, an accurate estimate of the peak width, height and position is critical to the success of the technique for spectrum reconstruction with regularization. We propose use of an adaptive regularization term as a special form of the general Tichonov regularization function. The variational method with the adaptive regularization term is applied to reconstruct electron spectra for the 6, 9, and 18 MeV electron beams of a Varian Clinac 2100C accelerator and proves to be a very simple, effective and accurate approach. Results using this variational method with adaptive regularization almost perfectly reconstruct electron spectra from depth dose distributions. © *2006 American Association of Physicists in Medicine*. [DOI: 10.1118/1.2161404]

Key words: spectral reconstruction, quadratic objective function, regularization, ill-conditioned problem

I. INTRODUCTION

Monte Carlo simulation provides an accurate electron beam dose calculation method that may become routinely available in commercial treatment planning systems. However, Monte Carlo simulation usually requires the phase space distribution of the electrons at the patient surface, or some level above it, in explicit form. Reconstruction of this phase space distribution from depth dose curves and lateral dose profiles is a fast and practical alternative to measuring the electron spectra by a spectrometer or detailed Monte Carlo simulation of the accelerator treatment head.[1,2] Electron spectral reconstruction from depth dose curves requires solving an integral Fredholm equation of the first kind. Various solution methods and techniques for this reconstruction have been previously proposed and investigated.[3–8]

There are physical and mathematical limitations in the reconstruction of electron spectra from depth-dose distributions. Physical limitations are associated with the bremsstrahlung contamination and significant angular spread of the low-energy component of electron spectra. Mathematical limitations are associated with the ill-conditioned state of the reconstruction problem and the specific form of the electron spectrum that consists of a deltalike peak and flat low-energy part. This requires solving a modified or regularized reconstruction problem.

In this article, we address the mathematical limitations and present a regularized reconstruction method which is able to produce energy spectra under assumptions that bremsstrahlung extraction is done and broad angular spread is neglected. A major problem associated with application of a regularization technique is that it smoothes not only nonphysical but also physical peaks. So, to solve the problem more efficiently, the physical peaks must be separated from the regularization effect. Previously, this was achieved by splitting the reconstruction problem into two parts where the low-energy part is regularized and the peak not.[8] While any analytical or numerical method could be selected for the peak, a simple Gaussian function was selected for practical reasons. This previous approach has certain disadvantages associated with the difficulty of joining two spectral parts and the compromised accuracy of the peak reconstruction due to the use of a simple Gaussian function to model it.

We propose use of the variational method with a new reconstruction technique where the regularization effect in the peak region is reduced using a weighting function. We call this technique an adaptive regularization. This technique is a major improvement because the reconstruction is solved as a single problem without any compromise in accuracy.

This article is organized as follows. In Sec. II we describe the variational method with the adaptive regularization technique together with some computational details. In Sec. III we present the results of the new procedure applied to 6, 9, and 18 MeV electron beams of a Varian Clinac 2100C accelerator. Conclusions are provided in Sec. IV.

II. A VARIATIONAL METHOD WITH ADAPTIVE REGULARIZATION

Accurate reconstruction of electron spectra from measured depth-dose distributions produced by medical accelerators requires (1) subtraction of treatment head bremsstrahlung, (2) correction for initial angular spread, and (3) a mathematical algorithm that is able to take into account the specific form of electron spectra with a sharp peak and flat low-energy part. The effective algorithms to estimate the dose contribution of treatment head bremsstrahlung to electron beam dose distributions in patients have been previously developed by Faddegon and Bleves[4] and Deng *et al.*[5] The influence of the initial angular spread of incident electrons on the reconstructed spectra was estimated and discussed in our previous article.[9] Therefore, we restrict our discussion to spectral reconstruction based only on depth doses of monodirectional and normally incident electrons on a semi-infinite water medium and hence do not consider the initial angular spread of the electron beam or bremsstrahlung photons coming from treatment head. This restriction adequately serves our purpose to illustrate the essence of the proposed adaptive regularization technique. The depth-dose distributions considered in this article were calculated using electron spectra presented by Rogers and Ding and the one-dimensional discrete-ordinates transport code CEPXS/ONEDANT.

The reconstruction of an electron spectrum from a measured central axis depth dose, $D(z)$, is based on numerical solution of an integral Fredholm equation of the first kind,

$$D(z) = \int_0^{E_{\max}} f(E) d(z,E) dE, \tag{1}$$

where $D(z)$ is the measured central axis depth dose in a water phantom, $f(E)$ is the electron spectrum, and $d(z,E)$ is the depth dose distribution for a monoenergetic beam of energy E. It is the purpose of the reconstruction process to obtain an accurate estimate of $f(E)$.

As illustrated in Fig. 2 of our previous article,[8] the ill-conditioned state of the integral Fredholm equation causes very small perturbations in the depth dose distribution to often produce large high frequency perturbations in the reconstructed spectrum. To correct for the ill-conditioned property of the equation, a variational method with regularization is required to suppress the fluctuation and achieve stability in the reconstructed spectrum. A spatial Tichonov regularization function was previously included into the least squares objective function of an optimization procedure.[10] The general form of the optimization was as follows:

$$\min \Theta_m[f(E)], \tag{2}$$

$$\text{subject to } f(E) \geqslant 0, \tag{3}$$

$$\Theta_m[f(E)] = \int_0^{z_{\max}} (\bar{D}(z) - D(z))^2 \, dz + \alpha \Omega_m[f(E)], \tag{4}$$

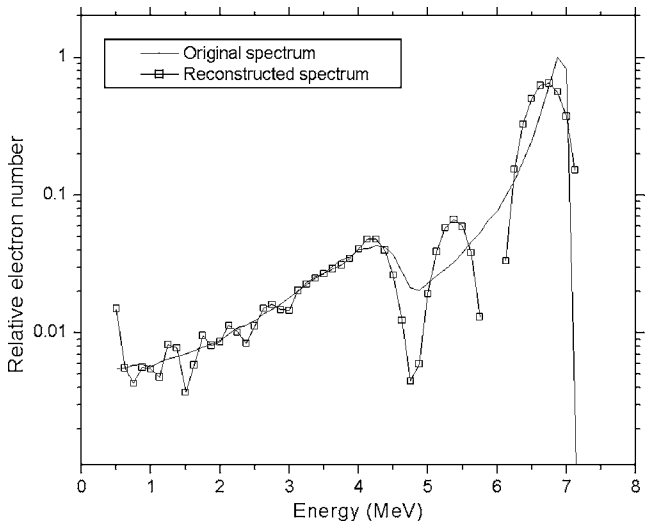

FIG. 1. Comparison of the incident spectrum for the 6 Mev beam and the reconstructed spectrum with weighting function $p_k(E) \equiv 1$.

$$\Omega_m[f(E)] = \int_0^{E_{\max}} dE \sum_{k=0}^{m} p_k(E) \left(\frac{d^k f(E)}{dE^k} \right)^2. \tag{5}$$

In the above equations, $\bar{D}(z)$ is the measured or simulated depth dose distribution caused by the incident electron beam, $D(z)$ is the depth dose obtained from Eq. (1) based on a known $f(E)$ at each optimization step, $\Omega_m[f(E)]$ is the regularization function, α is the regularization parameter, and $p_k(E)$ is a positive continuous weighting function. Numerical experiments show that regularization of the solution can be achieved in many cases using $m=1$ and the terms with $m > 0$ may lead to excessive smoothing of solutions. In our previous article,[8] we assumed $p_k(E) \equiv 1$ and found that this leads to low accuracy in the reconstructed spectral peak, as shown in Fig. 1. The peak region is smoothed out (penalized too much) while nonphysical fluctuations remain in the lower energy region. Applying the singular component decomposition technique proposed in our previous article[8] improved peak reconstruction accuracy but systematic errors in the position (central energy), width and height of reconstructed peak remained.

The high energy spectral peak has significantly larger first derivatives than the low-energy (tail) region and assuming $p_k(E) \equiv 1$ therefore causes the peak region to be penalized by the regularization function much more than the tail region. This is the reason the peak region is already smoothed out (penalized too much) in Fig. 1 while there are still some nonphysical fluctuations in the lower energy region. We now propose a nonuniform weighting function $p_k(E)$ that is appropriate to counteract this effect. Since the high intensity [large $f(E)$] region has large values of first derivatives, we propose to use $p_1(E) = (1/f)^2$ as the weighting function for the first derivative term, thereby achieving a more even penalization (regularization) across the spectrum. Thus the first order regularization becomes

$$\int_0^{E_{\max}} dE\left(\frac{df(E)/dE}{f}\right)^2 = \int_0^{E_{\max}} dE\left(\frac{d\ln[f(E)]}{dE}\right)^2. \quad (6)$$

Another alteration we make to the general form for the Tichonov regularization function $\Omega_m[f(E)]$ is about the zero-order terms $\int_0^{E_{\max}} dE\, p_0(E) f(E)^2$. This zero-order term tends to reduce the spectrum $f(E)$ to zero and penalize the high intensity [large $f(E)$] region in the spectrum more than the low intensity region. Again we propose to use $p_0(E) = (1/f)^2$ as the weighting function for the zero-order regularization term, which makes the term a constant value $\int_0^{E_{\max}} dE = E_{\max}$. Thus it will have no effect in the optimization problem outlined in Eqs. (2)–(5) and can be eliminated in the formalism.

We now rewrite the optimization problem as follows:

$$\min \Theta[f(E)], \quad (7)$$

$$\text{subject to } f(E) \geqslant 0, \quad (8)$$

$$\Theta[f(E)] = \int_0^{z_{\max}} (\bar{D}(z) - D(z))^2\, dz + \alpha\Omega[f(E)], \quad (9)$$

$$\Omega[f(E)] = \int_0^{E_{\max}} dE\left(\frac{d\ln[f(E)]}{dE}\right)^2. \quad (10)$$

The regularization parameter α defines the relative importance of the least squared term of the depth dose difference and the regularization term. In this article it is selected by visual control and the smallest value of α that can remove the nonphysical fluctuation is chosen. We found that α values in the range from 1 to 5×10^{-7} were small enough for all cases studied.

The minimization of the objective function is achieved using the L-BFGS-B code package that was developed at the Optimization Technology Center, a joint venture of Argonne National Laboratory and Northwestern University.[11] The code is a limited memory version of the quasi-Newton method and only requires as input an objective function and its derivatives with respect to spectral weight. It is available by download from the following web site http://www-fp.mcs.anl.gov/otc/Tools/LBFGSB/

We applied the proposed adaptive regularization technique [Eqs. (7)–(10)] to the reconstruction of electron spectra for the 6, 9, and 18 MeV electron beams from a Varian Clinac 2100C accelerator.[12] The depth doses $d(z,E)$ for monoenergetic electron beams are calculated in parallel broad beam geometry by the standard discrete ordinates package CEPXS/ONELD-1.0.[13–15] As illustrated in Fig. 4 of our previous article,[8] the agreement between the EGS4 Monte Carlo code and the CEPXS/ONELD-1.0 discrete ordinates package predictions of depth doses for 10 MeV and 20 MeV electron beams is excellent. The more clinically realistic polyenergetic depth doses are obtained by combining the depth doses for individual energies based on the spectra calculated by Monte Carlo treatment head simulation.[12] Energy over the range from 0.5 MeV to $E_{\max}$ was discretized into energy bins of 0.125 MeV for 6 and 9 MeV beams and energy bins of 0.25 MeV for the 18 MeV beam. $E_{\max}$ is the maximum energy obtained from the Monte Carlo treatment head simulation for electron beams of different nominal energies. To accommodate for possible shifts in the peak positions, we expanded the energy grids to include two more energy points beyond $E_{\max}$. Practically $E_{\max}$ may be derived from measured depth dose curves and energy-range relationships. The central axis depth dose was sampled at intervals of 0.1 cm for the 6 and 9 MeV beams, and 0.2 cm for the 18 MeV beam. For all nominal beam energies the initial spectrum was set to $f(E_n) \equiv 1.0$ for all $n = 1, 2, 3, \ldots$.

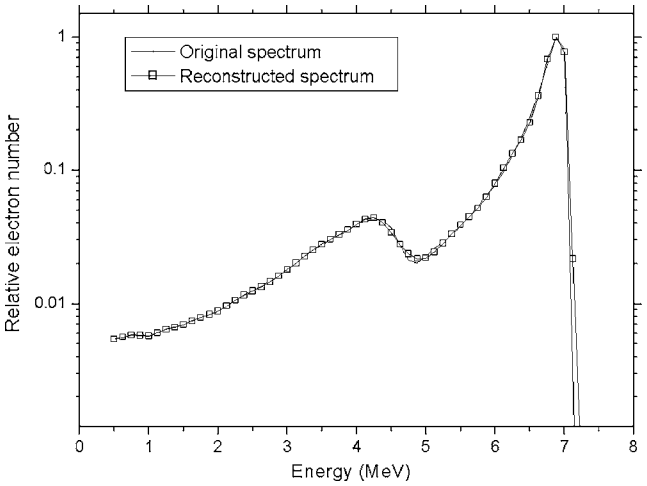

FIG. 2. Comparison of the incident and the reconstructed spectra for the 6 MeV electron beams from a Varian Clinac 2100C. The ordinate is a logarithmic scale and weighting function $p_1(E) = (1/f)^2$.

The higher energy side of the spectral peak is much steeper than the lower energy side and to reconstruct this feature more accurately, we fixed the spectral weight of the last energy point to be a very small value, e.g., 10^{-15} to 10^{-10}. This small value is very effective in reproducing the steep gradient on the high energy side of the peak but the actual value within the above range quoted does not make a significant difference in the overall reconstructed spectral shape.

III. RESULTS AND DISCUSSION

Figures 2–4 show the reconstructed spectra for 6, 9, and 18 MeV electron beams in logarithmic scale. The regularization parameter α used was 5×10^{-7}, 5×10^{-7}, and 1×10^{-7} for the 6, 9, and 18 MeV beams, respectively. For all these beam energies, both the low-energy tail and the high-energy peak are accurately reconstructed. The low-energy secondary peaks for all three energies due to electron scattering from the applicator are reproduced faithfully. Figure 5 is the linear scale graph of the same reconstructed 18 MeV spectrum corresponding to that in Fig. 4. This figure demonstrates far superior accuracy of the position, width and height of the reconstructed peak over the previous method of singular component decomposition.[8] Figure 6 shows that the depth dose based on the Monte Carlo treatment head simulated spectrum and the depth dose based on the reconstructed

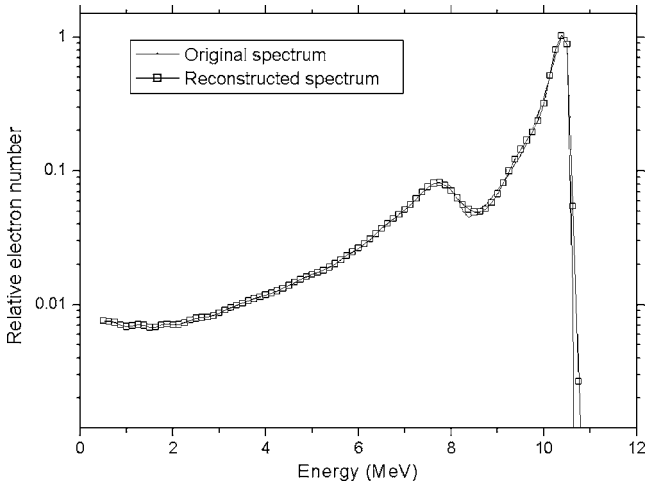

FIG. 3. Comparison of the incident and the reconstructed spectra for the 9 MeV electron beams from a Varian Clinac 2100C. The ordinate is a logarithmic scale and weighting function $p_1(E)=(1/f)^2$.

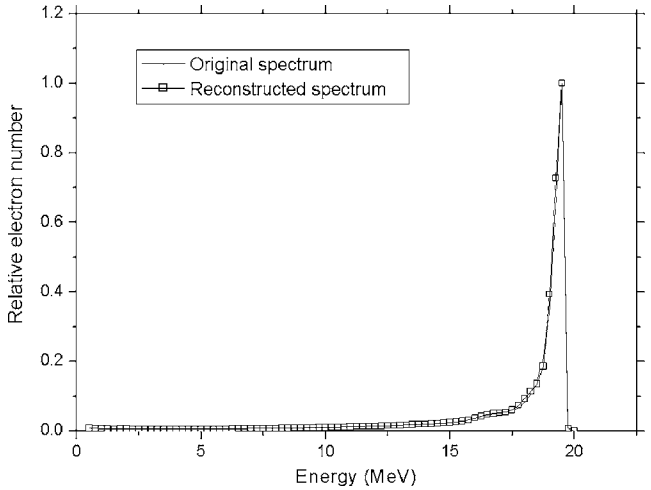

FIG. 5. Comparison of the incident and the reconstructed spectra for the 18 MeV electron beam from a Varian Clinac 2100C. The ordinate is a linear scale and weighting function $p_1(E)=(1/f)^2$.

spectrum are in perfect agreement. Figure 7 explicitly shows the difference of the above-mentioned two depth doses. The dose difference is bigger at the medium depths than at the shallow depths and near the maximum depths. For all depths, the difference is less than 5×10^{-6} of the maximum dose which is unity.

To demonstrate the stability of this optimization process in the presence of dose measurement errors, a 6 MeV depth dose purposely deviated from the depth dose previously used in Fig. 3 was applied to the reconstruction procedure. This deviation in depth dose is shown in Fig. 8. Doses in the shallow depth region and the fall-off region were purposely increased and decreased, respectively. The reconstructed spectrum based on this depth dose curve is shown in Fig. 9 (regularization parameter α of 1×10^{-5}). Compared to the original exact spectrum obtained by Monte Carlo treatment head simulation, the low-energy spectral values are increased while the spectral peak's position is shifted to the lower energy side. This change in the reconstructed spectrum is a reasonable and predictable response to the depth dose deviations invoked and shows the potential for translation of depth dose measurement errors to spectrum reconstruction errors.

The optimization code applied was compiled by the G77 Fortran compiler from the Free Software Foundation (GNU) and run on a 3 GHz personal computer running Windows XP Professional operating system. For both 6 and 9 MeV electron beams with the aforementioned discretization parameters, the computing time was less than 30 minutes for the search process to converge. For the 18 MeV beam and using the same energy bin (0.125 MeV) and depth interval (0.1 cm), the number of search steps increases significantly resulting in a much larger computing time of over 10 hours. This dramatic increase in computing time is due to the adaptive regularization terms (logarithmic terms) making the objective function phase space configuration significantly more

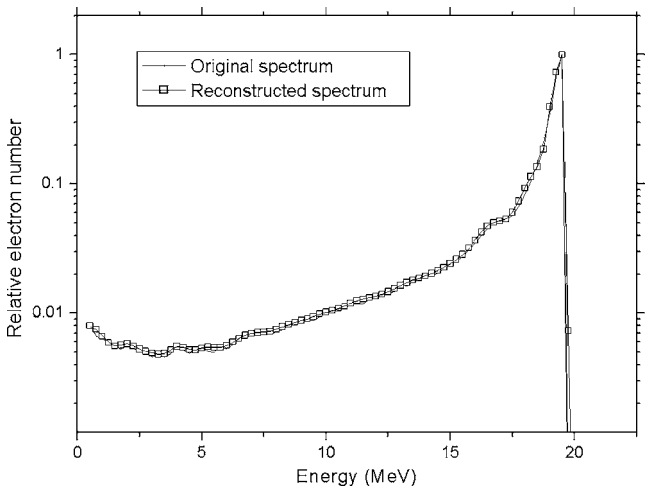

FIG. 4. Comparison of the incident and the reconstructed spectra for the 18 MeV electron beams from a Varian Clinac 2100C. The ordinate is a logarithmic scale and weighting function $p_1(E)=(1/f)^2$.

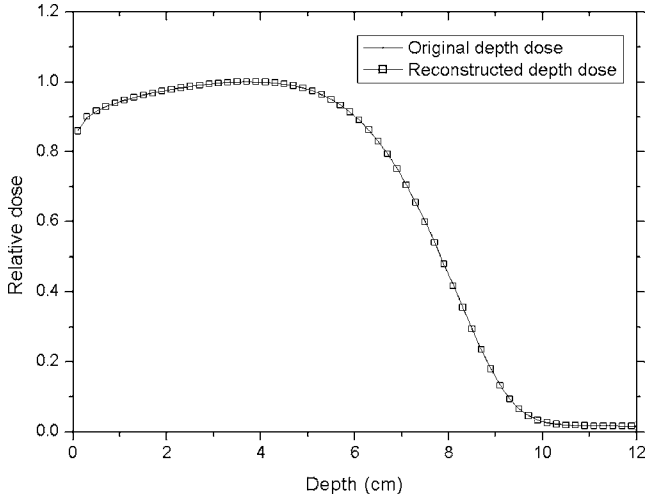

FIG. 6. Comparison of depth dose curves calculated using an exact Monte Carlo spectrum and the reconstructed spectrum for the 18 MeV electron beam from a Varian Clinac 2100C. The spectra are shown in Fig. 4.

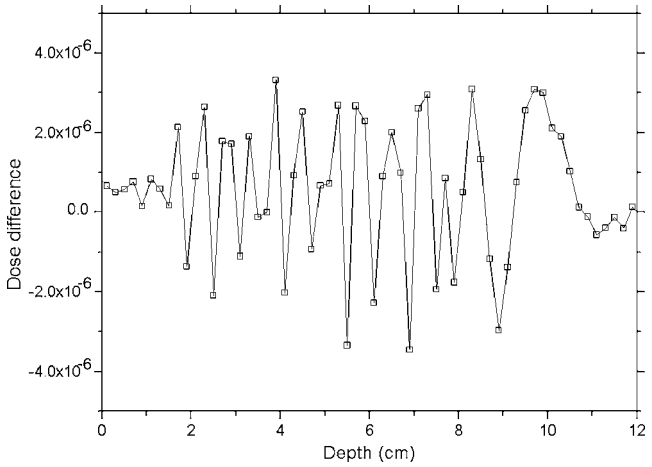

FIG. 7. Depth dose difference of the two curves in Fig. 6 for the 18 MeV electron beam. The dose difference is bigger in the middle than on both ends. For all depths, the difference is less than 5×10^{-6} of the maximum dose which is unity.

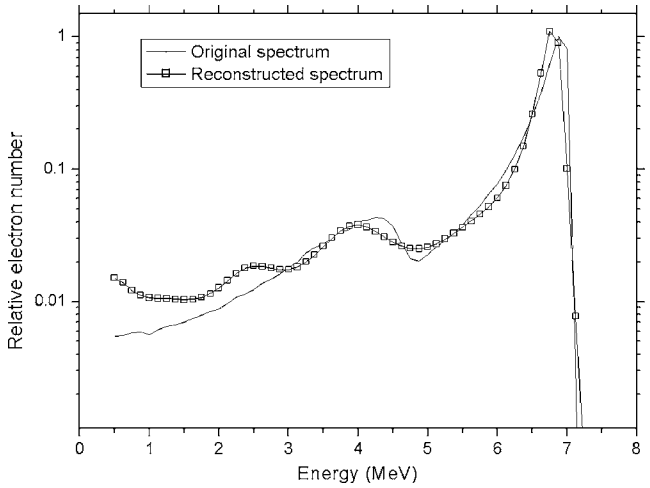

FIG. 9. Comparison of the original and the reconstructed spectra for the 6 MeV electron beam from a Varian Clinac 2100C. The reconstructed spectrum is based on the purposely deviated depth dose curve in Fig. 8 and weighting function $p_1(E)=(1/f)^2$.

complicated when the number of optimization variables is large. If a larger energy bin size of 0.25 MeV and depth interval of 0.2 cm are used for the 18 MeV electron beam, the number of variables reduces to that for the 9 MeV beam and the search process converges in a similar computing time and with the same accuracy as for reconstruction of the 9 MeV beam.

IV. CONCLUSION

The algorithm presented in this article is based on the variational method with adaptive regularization and is useful for reconstructing electron spectra from central axis depth doses. The accuracy in the reconstructed spectra, especially the position, width and height of the peak, has been significantly improved over the previously published method of singular component decomposition.[8]

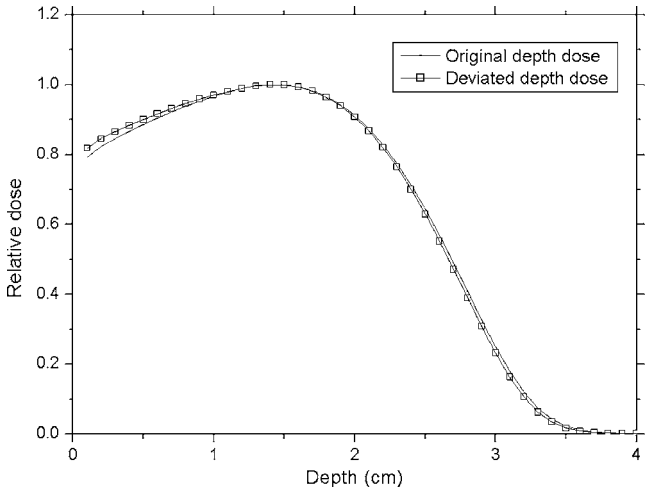

FIG. 8. Original depth dose curve for a 6 MeV electron beam from a Varian Clinac 2100C calculated from the exact Monte Carlo spectrum and a depth dose curve purposely deviated from the original with increased doses at shallow depths and decreased doses in the fall-off region.

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