Simulations of Particle Interactions with Matter. Homework #1.

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1 Maxwell Boltzmann

Given the Maxwell-Boltzmann Distribution:

$$N(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \tag{1}$$

1.1 Show $< v >$

Show that

$$< v >= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^2 \frac{\Gamma(2)}{2} \tag{2}$$

Solution:

$$< v >= \int_0^\infty vN(v)dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 e^{-\frac{mv^2}{2kT}} dv \tag{3}$$

using table integral (`http://en.wikipedia.org/wiki/Lists_of_integrals`):

$$\int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2} \tag{4}$$

when $a > 0$ we can calculate integral 3 as

$$< v >= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2\left(\frac{m}{2kT}\right)^2} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^2 \frac{1}{2} \tag{5}$$

and using the fact that $\Gamma(2) = (2-1)! = 2$ finally

$$< v >= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^2 \frac{\Gamma(2)}{2} \tag{6}$$

that is exactly the equation 2 above. End of prove.

1.2 Energy Fluctuation (Grad)

Show that the energy fluctuation is

$$\frac{1}{4} m^2 \left\langle \left(v^2 - < v^2 >\right)^2 \right\rangle = \frac{3}{2}(kT)^2 \tag{7}$$

Solution:

$$\frac{m^2}{4} \left\langle \left(v^2 - < v^2 >\right)^2 \right\rangle = \frac{m^2}{4}\left(< v^4 > -2 < v^2 >< v^2 > + < v^2 >^2\right) = \frac{m^2}{4}\left(< v^4 > - < v^2 >^2\right) \tag{8}$$

Using table integral (`http://en.wikipedia.org/wiki/Lists_of_integrals`):

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2^{n+1}} \sqrt{\frac{\pi}{a^{2n+1}}} \tag{9}$$

we can calculate:

$$< v^2 >= \int_0^\infty v^2 N(v) dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-\frac{mv^2}{2kT}} dv \tag{10}$$

as

$$< v^2 >= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{3!!}{2^3} \sqrt{\frac{\pi}{\left(\frac{m}{2kT}\right)^5}} = \frac{3}{2}\left(\frac{m}{2kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{5/2} = \frac{3}{2}\left(\frac{2kT}{m}\right) \tag{11}$$

Again using the table integral 9 we can calculate:

$$< v^4 >= \int_0^\infty v^4 N(v) dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^6 e^{-\frac{mv^2}{2kT}} dv \tag{12}$$

as

$$< v^4 >= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{5!!}{2^4} \sqrt{\frac{\pi}{\left(\frac{m}{2kT}\right)^7}} = \frac{15}{4}\left(\frac{m}{2kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{7/2} = \frac{15}{4}\left(\frac{2kT}{m}\right)^2 \tag{13}$$

Using equation 8 above

$$\frac{m^2}{4}\left\langle \left(v^2 - < v^2 >\right)^2 \right\rangle = \frac{m^2}{4}\left(< v^4 > - < v^2 >^2\right) = \frac{m^2}{4}\left(\frac{15}{4}\left(\frac{2kT}{m}\right)^2 - \left(\frac{3}{2}\left(\frac{2kT}{m}\right)\right)^2\right) = \frac{3}{2}\frac{kT}{m} \tag{14}$$

that is exactly the equation 7 above. End of prove.

2 MC calculation of π

Calculate π using the Monte Carlo method described in the Notes: https://wiki.iac.isu.edu/index.php/Simulations_of_Particle_Interactions_with_Matter#Example_2_Calculation_of_.CF.80

Below is the script of my program Roman_Pi.C:

```
//This program to calculat the Pi number
//using Monte-Carlo method
//written for g++ compiler

//Created May 31, 2011 by Roman S.
//Modified Sep 12, 2011

using namespace std;
#include <iostream>
#include <iomanip>
#include <fstream>
#include <ctime>
#include <cstdlib>
#include <cmath>

main()
{

  fprintf(stderr,"\n******Welcome to C++ program to calculate Pi using Monte Carlo method******\n");
  fprintf(stderr,"******Created May 31, 2011 by Roman S.\n");
  fprintf(stderr,"******Modified Sep 12, 2011\n\n");

 double x,y;
 int i;
 double hit_max;
 double hit_circle;

 hit_max = 10;
 hit_circle = 0;
 i = 0;

 srand((unsigned)time(0));

 while(i<hit_max){
     x = double(rand())/double(RAND_MAX);
     y = double(rand())/double(RAND_MAX);
     if (y<=sqrt(1-pow(x,2)))
       hit_circle++;

     fprintf(stdout,"%g\t%g\n",x,y);
     i++;
 }

 fprintf(stderr,"\nhit_circle=%g\thit=%g\tPi=%g\n\n",hit_circle,hit_max,4*hit_circle/hit_max);

}
```

To compile the script I have used the following command:

```
 g++ Roman_Pi.C -o Roman_Pi
```

Below is the screenshot of the results of running the program. The `hit_max = 10` here to be able to fit the results into screen. I will re-run the program later with the `hit_max = 10,000` to more accurate calculate the Pi.

```
roman@roman-laptop:~/Classes_Manuals/Phys599_NucSim/Homework/hw1/MonteCarlo_Pi$ ./Roman_Pi

******Welcome to C++ program to calculate Pi using Monte Carlo method******
******Created May 31, 2011 by Roman S.
******Modified Sep 12, 2011

0.864968        0.102693
0.136565        0.143102
0.140669        0.284003
0.448752        0.208196
0.343608        0.861973
0.793586        0.467476
0.323104        0.120766
0.953589        0.759857
0.00992611      0.970646
0.728003        0.703635

hit_circle=8    hit=10  Pi=3.2
```

3 Histograms using ROOT

To create a file with the 2 columns of random numbers I have used the command:

```
./Roman_Pi > data.txt
```

The program here was recompiled with `hit_max = 10,000` and below the screenshot of results:

```
roman@roman-laptop:~/Classes_Manuals/Phys599_NucSim/Homework/hw1/MonteCarlo_Pi$ ./Roman_Pi > data.txt

******Welcome to C++ program to calculate Pi using Monte Carlo method******
******Created May 31, 2011 by Roman S.
******Modified Sep 12, 2011


hit_circle=7830 hit=10000       Pi=3.132
```

Now I did the following from the command line:

- root (to start root). The following inside the root.
- `root [0] .L ascii2root.C`
- `root [1] asci2nt(''data.txt'')`
- `root [2] new TBrowser();`
- inside the opened window found the `rns.root` file and navigated to the leaf "rnd1" and "rnd2" and tried the following command to manipulate with histogram:
- `rns->Draw(''rnd1*4'');` see figure 1
- `rns->Draw(''rnd1:rnd1'',''rnd1>0.5'');` see figure 2
- `rns->Draw(''rnd1:rnd2'');` see figure 3
- `rns->Draw(''asin(rnd1)'');` see figure 4
- `rns->Draw(''asin(rnd1-rnd2)'');` see figure 5. It does look like a Normal/Gaussian Distribution.

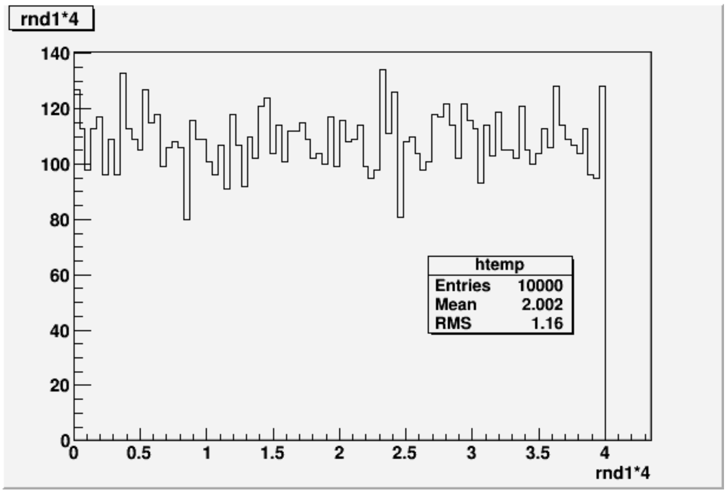

Figure 1: Histogram of rnd1 multiplied by 4

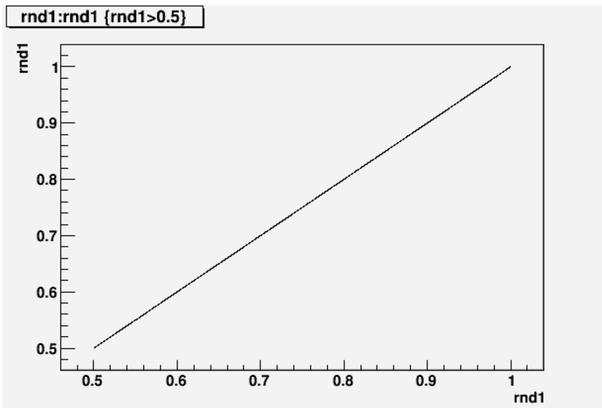

Figure 2: Convolution of rnd1 and rnd1 for rnd1 ¿ 0.5

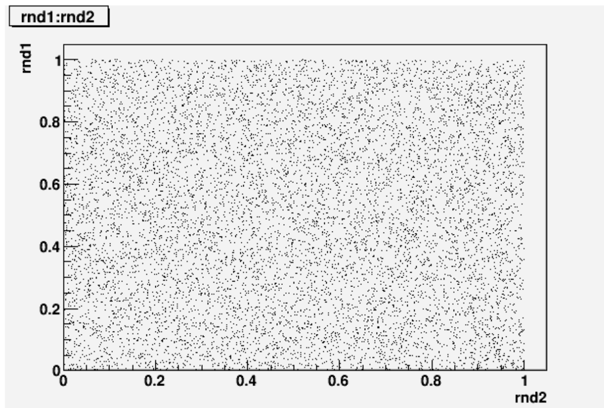

Figure 3: Convolution of rnd1 and rnd2

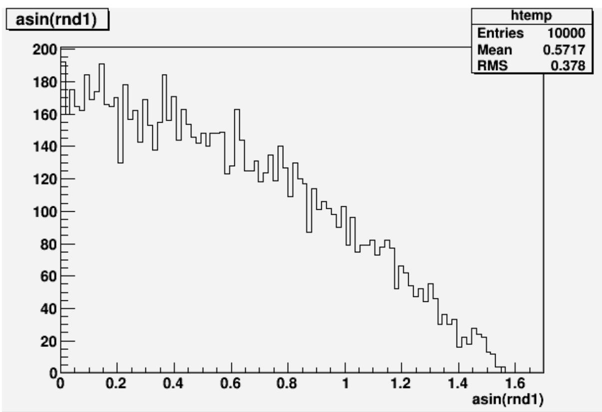

Figure 4: Histogram of asin(rnd1)

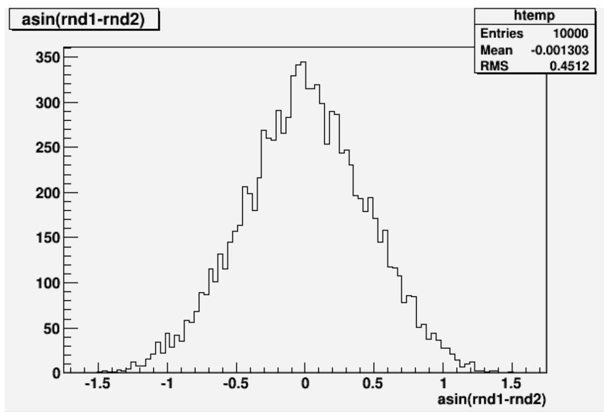

Figure 5: Histogram of asin(rnd1-rnd2)