Development of MATLAB based Emittance Measurement Tools

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Accelerator Physics Class Term Project

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Project Description

1. Design a beamline to measure emittance with the quadrupole scan method with ELEGANT code. The beamline is same as that of the ELEGANT sample files for the HRRL beamline01.

2. Turn off all QMs at T2 (Q1@T2, Q2@T2, and Q3@T2).

3. Scan Q1@T1 and use a screen located at TCOL2 position to measure beam sizes. All other initial beam parameters are same as those in the ELEGANT sample file. Except there are two cases for the rms energy spreads (1% or 4.23%). Generate scanned beam images on the screen at TCOL2 by scanning Q1@T1 with ELEGANT code

Emittance: $\varepsilon_n = 16\ \mu m$

Quadrupole Strength: $k1 = -6.5 \sim +6.5\ 1/m^2$

Scanning Quadrupole Length: $L = 0.15$ m

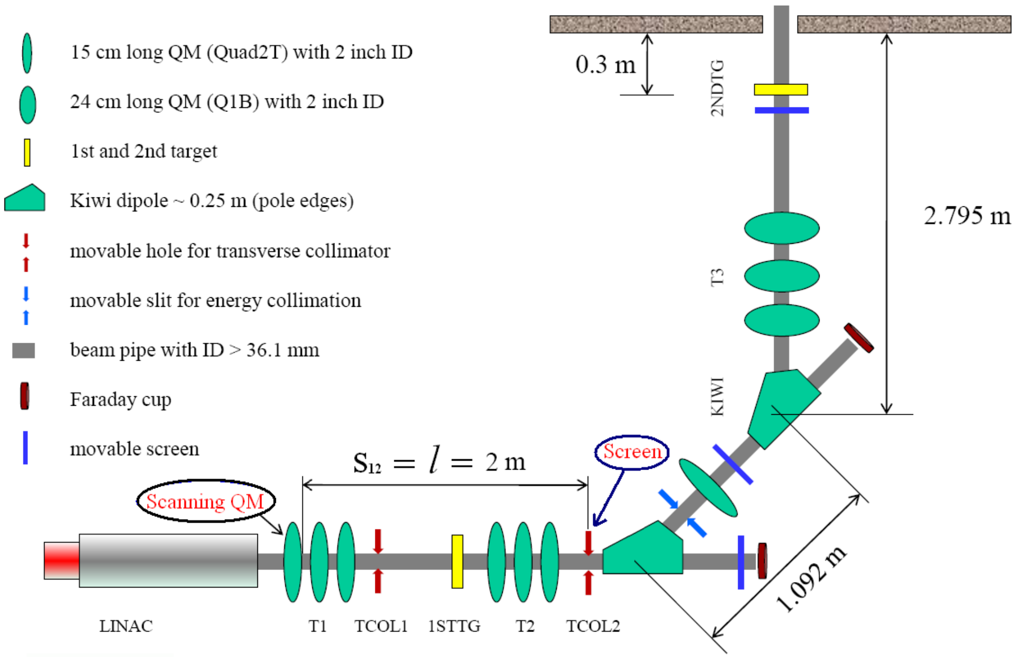
Distance from QM to the view screen: $l = 2$

Beam Energy : 10 MeV

Beam Distribution Chop-off : 6σ

Number of Particles Per Bunch: 150,000

Energy Spread: 1% and 4.23%

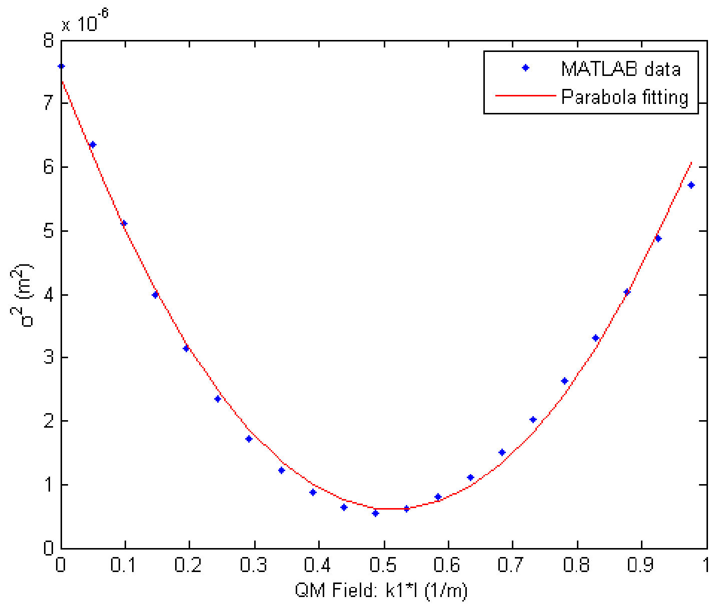


Project Description

4. Estimate the rms beam size on the screen by using MATLAB codes and Gaussian fitting.

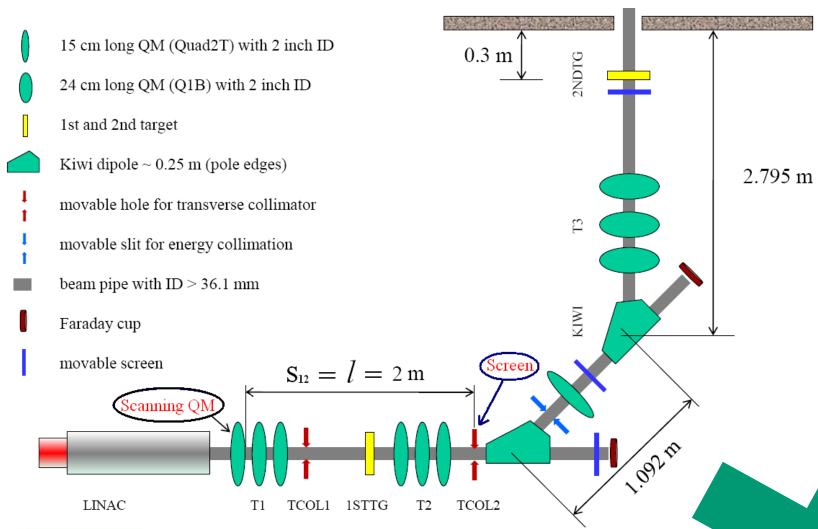
5. Estimate the normalized rms emittance by using a parabola fitting (square of rms beam size vs. quadrupole strength). **This parabola fitting should also be done by programming a MATLAB code.**

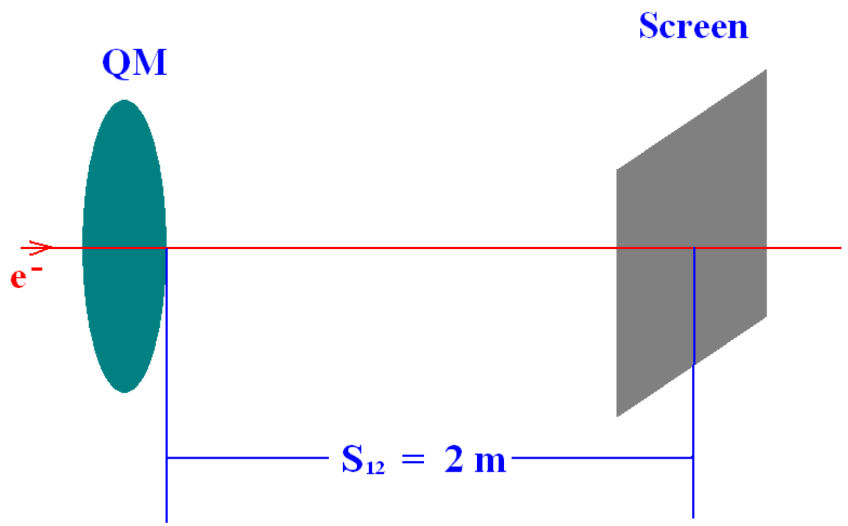
6. Compare the estimated normalized rms emittance with pre-assumed beam emittance in the ELEGANT code (= 16 μm). **If there is difference, what is source of the difference?**



Basic Setup

One quadrupole magnet and one OTR screen





Theory of Quadrupole Magnet Scanning

Transfer matrix of a quadrupole magnet under thin lens approximation:

$$\mathrm{Q} = \begin{pmatrix} 1 & 0 \\ -kl & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Here $kl > 0$ for x-plane, and $kl < 0$ for y-plane.

Transfer matrix of a drift space between quadrupole and screen:

$$\mathrm{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Transfer matrix of the scanned region is:

$$\mathrm{M} = \mathrm{SQ} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -kl & 1 \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} S_{11} - klS_{12} & S_{12} \\ S_{21} - klS_{22} & S_{22} \end{pmatrix}$$

Theory of Quadrupole Magnet Scanning

$$\mathbf{M}=\mathbf{SQ}=\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ -kl & 1 \end{pmatrix}\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}=\begin{pmatrix} S_{11}-klS_{12} & S_{12} \\ S_{21}-klS_{22} & S_{22} \end{pmatrix}$$

M is related with the beam matrix σ as:

$$\sigma_{\mathrm{screen}}=\mathbf{M}\sigma_{\mathrm{quad}}\mathbf{M}^{\mathrm{T}}=\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} \sigma_{quad11} & \sigma_{quad12} \\ \sigma_{quad21} & \sigma_{quad22} \end{pmatrix}\begin{pmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{pmatrix}$$

Since:

$$\sigma_x=\sqrt{\varepsilon_x\beta},\sigma_{x'}=\sqrt{\varepsilon_x\gamma},\sigma_{xx'}=-\varepsilon_x\alpha$$

$$\sigma=\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}=\begin{pmatrix} \sigma_x^2 & \sigma_{xx'} \\ \sigma_{xx'} & \sigma_{x'}^2 \end{pmatrix}$$

σ matrix can be written:

$$\sigma_{quad}=\begin{pmatrix} \sigma_{quad,x} & \sigma_{quad,xx'} \\ \sigma_{quad,xx'} & \sigma_{quad,x'} \end{pmatrix}=\varepsilon_{rms,x}\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

Theory of Quadrupole Magnet Scanning

$$\sigma_{quad} = \begin{pmatrix} \sigma_{quad,x} & \sigma_{quad,xx'} \\ \sigma_{quad,xx'} & \sigma_{quad,x'} \end{pmatrix} = \varepsilon_{rms,x} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\sigma_{screen} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \varepsilon_{rms,x} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} \begin{pmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{pmatrix}$$

Drop off subscript "rms" on emittace.

$$\sigma_{screen11} = \sigma^2_{screen,x} = \varepsilon_x \left(m_{11}^2 \beta - 2 m_{12} m_{11} \alpha + m_{12}^2 \gamma \right)$$

Using σ matrix relations:

$$\varepsilon\beta = \sigma_{11}, \varepsilon\alpha = \sigma_{12}, \varepsilon\gamma = \varepsilon \frac{1+\alpha^2}{\beta} = \frac{\varepsilon^2}{\varepsilon} \frac{1+\alpha^2}{\beta} = \frac{\varepsilon^2 + \sigma_{12}^2}{\sigma_{11}}$$

$$\sigma^2_{screen,x} = m_{11}^2 \sigma_{11} + 2 m_{12} m_{11} \sigma_{12} + m_{12}^2 \frac{\varepsilon_x^2 + \sigma_{12}^2}{\sigma_{11}}$$

Theory of Quadrupole Magnet Scanning

$$\sigma^2_{screen,x} = m_{11}^2\sigma_{11} + 2m_{12}m_{11}\sigma_{12} + m_{12}^2\frac{\varepsilon_x^2 + \sigma_{12}^2}{\sigma_{11}}$$

Remember: $m_{11} = S_{11} - klS_{12}$ $\quad m_{12} = S_{12}$

$$\sigma^2_{screen,x} = (S_{11} - klS_{12})^2\sigma_{11} + 2(S_{11} - klS_{12})S_{12}\sigma_{12} + S_{12}^2\frac{\varepsilon_x^2}{\sigma_{11}} + S_{12}^2\frac{\sigma_{12}^2}{\sigma_{11}}$$

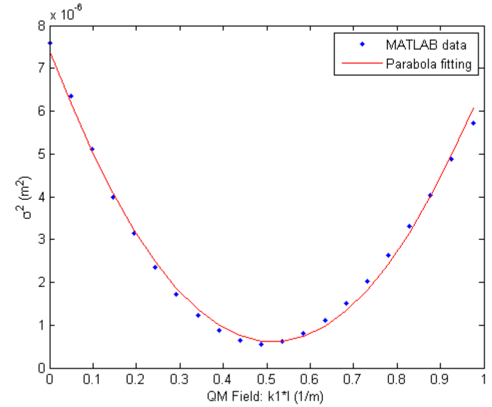
$$\sigma^2_{screen,x} = \sigma_{11}\left((S_{11} - klS_{12})^2 + 2(S_{11} - klS_{12})S_{12}\frac{\sigma_{12}}{\sigma_{11}} + S_{12}^2\frac{\sigma_{12}^2}{\sigma_{11}^2}\right) + S_{12}^2\frac{\varepsilon_x^2}{\sigma_{11}}$$

$$\sigma^2_{screen,x} = \sigma_{11}\left((S_{11} - klS_{12}) + S_{12}\frac{\sigma_{12}}{\sigma_{11}}\right)^2 + S_{12}^2\frac{\varepsilon_x^2}{\sigma_{11}}$$

$$\sigma^2_{screen,x} = \sigma_{11}S_{12}^2\left(kl - \left(\frac{S_{11}}{S_{12}} + \frac{\sigma_{12}}{\sigma_{11}}\right)\right)^2 + S_{12}^2\frac{\varepsilon_x^2}{\sigma_{11}}$$

Theory of Quadrupole Magnet Scanning

$$\sigma^2_{screen,x} = \sigma_{11} S_{12}^2 \left(kl - \left(\frac{S_{11}}{S_{12}} + \frac{\sigma_{12}}{\sigma_{11}} \right) \right)^2 + S_{12}^2 \frac{\varepsilon_x^2}{\sigma_{11}}$$



Introducing constants A, B and C:

$$A = \sigma_{11} S_{12}^2, \quad B = \left(\frac{S_{11}}{S_{12}} + \frac{\sigma_{12}}{\sigma_{11}} \right), \quad C = S_{12}^2 \frac{\varepsilon_x^2}{\sigma_{11}}.$$

$$\sigma_{screen,11} = \sigma^2_{screen,x} = A(kl - B)^2 + C = A(kl)^2 - 2AB(kl) + (C + AB^2)$$

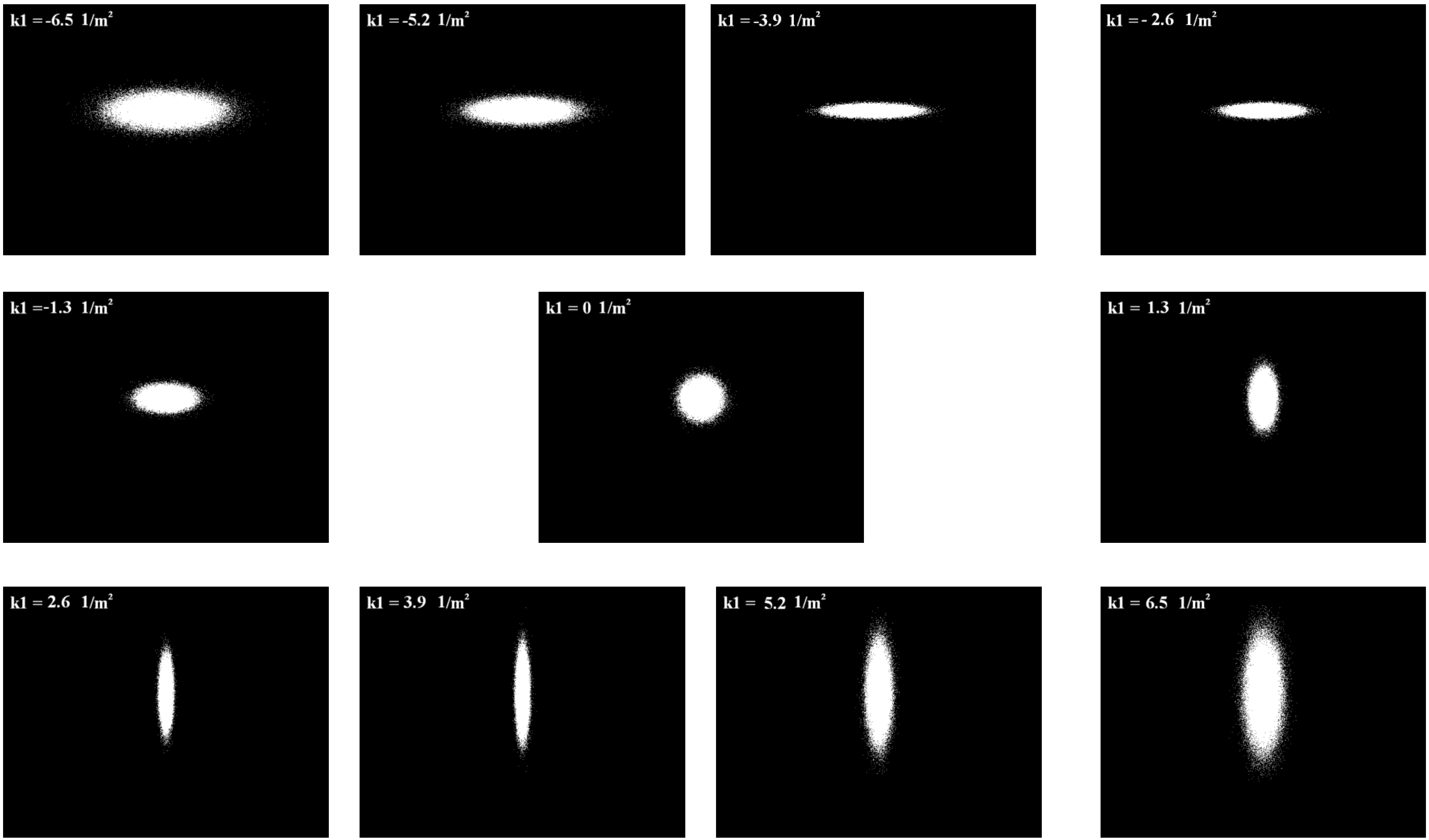
$$\varepsilon_x^2 = \frac{C\sigma_{11}}{S_{12}^2} = \frac{C\sigma_{11} S_{12}^2}{S_{12}^4} = \frac{AC}{S_{12}^4}$$

$$\varepsilon = \frac{\sqrt{AC}}{S_{12}^2}$$

By varying quadrupole magnetic field strength k1, we can change beam size on the OTR screen ($\sigma^2_{screen,x}$). We make projection to the *x*, *y* axes, then fit them with Gaussian fittings to extract rms values, then plot σ^2 vs k1L and fit parabola to find *A*, *B*, and *C*. Then, we can get emittance.

QM Scanning with ELEGANT code

Beam images on the screen during QM scan to measure emittance



Gaussian Fitting

Beam images from ELEGANT simulation.

These figures are saved in PNG format to be imported in MATLAB code.

image in sddsplot

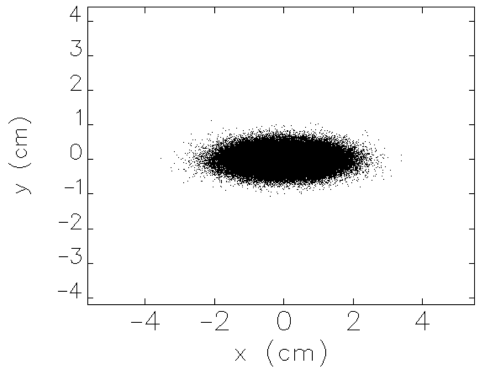
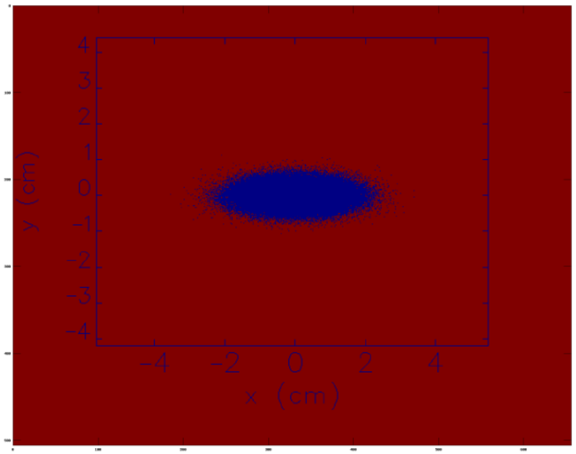
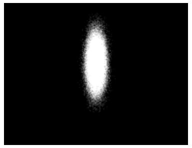


image in MATLAB (656 × 506 pixels)





ELEGANT sddsplot

↓

saved to PNG format

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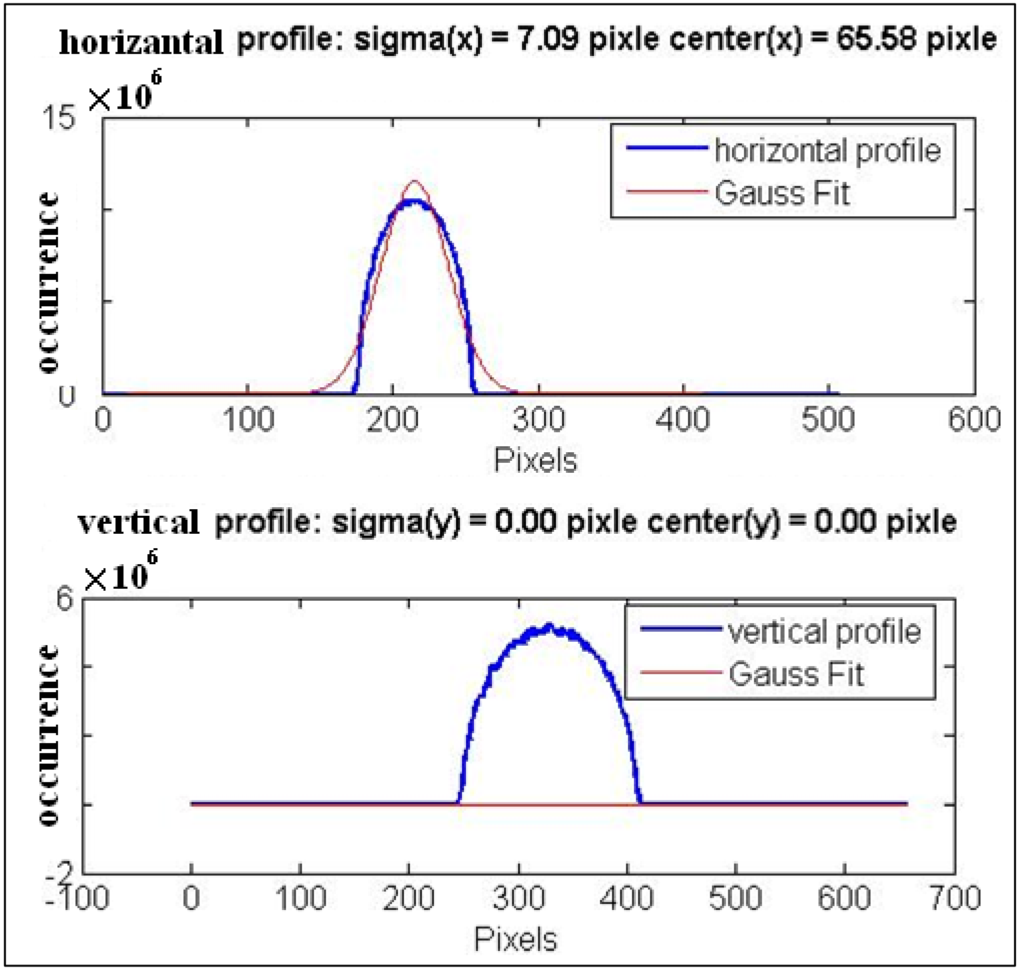
import PNG file in MATLAB for Gaussian fitting & parabola plotting and fitting

Gaussian Fitting – Problem

Fitting failure due to sharp edge and large binning



Solutions:

1. **Modify ELEGANT input file: using less number of particles (150000) to make beam halo on purpose and 6 simga-chopping instead of 3 sigma-chopping in distribution.**

2. **improving MATLAB script with a finer binning.**

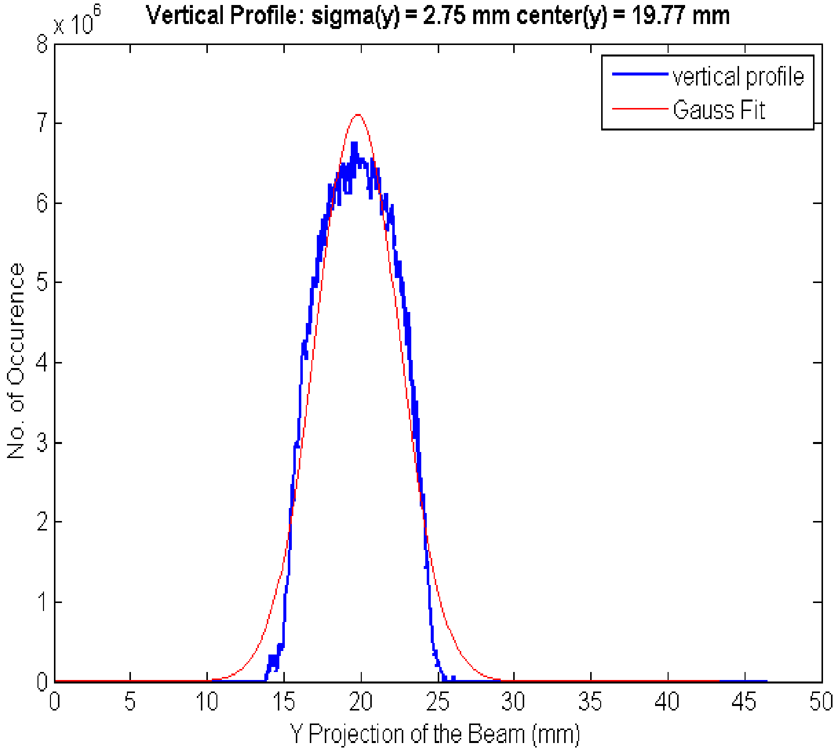
Gaussian Fitting – Improved

Fitting failure due to sharp edge and large binning

Solutions:

1. **Modify ELEGANT input file: using less number of particles (150000) to make beam halo on purpose and 6 simga-chopping instead of 3 sigma-chopping in distribution.**

2. **improving MATLAB script with a finer binning.**

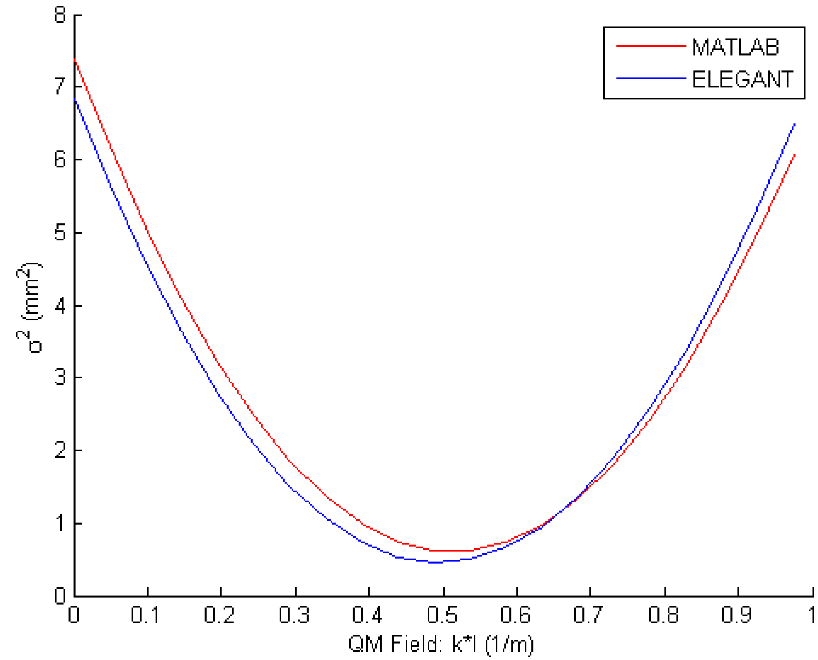


Gaussian Fitting

RMS values extracted from Gaussian fittings

0.1 % Energy spread, 6 sigma chopping, no fringe field					
Figure	K1 ($1/m^2$)	$\sigma_{Ma,x}$ (mm)	$\sigma_{EL,x}$ (mm)	$\sigma_{Ma,y}$ (mm)	$\sigma_{EL,y}$ (mm)
1	0.000	2.755	2.63	2.745	2.63
2	0.325	2.520	2.38	3.011	2.88
3	0.650	2.258	2.14	3.283	3.13
4	0.975	1.998	1.9	3.562	3.38
5	1.300	1.771	1.67	3.836	3.63
6	1.625	1.534	1.44	4.139	3.89
7	1.950	1.317	1.22	4.456	4.14
8	2.275	1.110	1.02	4.779	4.4
9	2.600	0.932	0.85	5.107	4.65
10	2.925	0.807	0.72	5.443	4.91
11	3.250	0.747	0.67	5.715	5.17
12	3.575	0.785	0.71	5.952	5.43
13	3.900	0.893	0.83	6.160	5.69
14	4.225	1.052	0.99	6.310	5.95
15	4.550	1.229	1.18	6.482	6.22
16	4.875	1.422	1.39	6.644	6.48
17	5.200	1.620	1.61	6.734	6.74
18	5.525	1.819	1.84	6.897	7.01
19	5.850	2.010	2.07	7.036	7.27
20	6.175	2.209	2.3	7.163	7.54
21	6.500	2.392	2.54	7.302	7.8

MATLAB ELEGANT MATLAB ELEGANT



Great Agreement!

Parabola Fitting

Performing parabola fitting with MATLAB data to extract *A*, *B* and *C*.

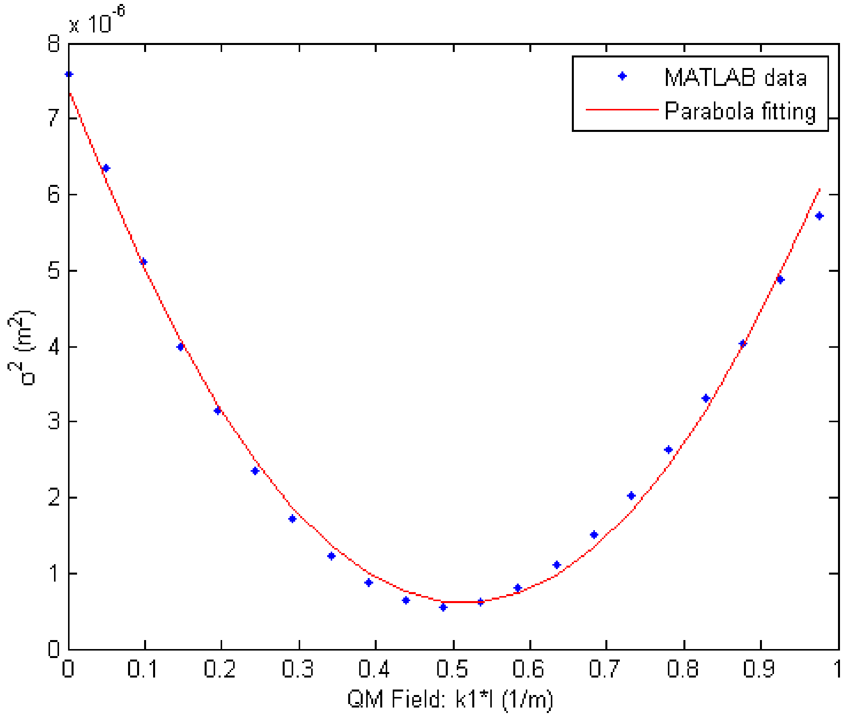
$$\sigma_{screen,11} = \sigma^2_{screen,x} = A(kl - B)^2 + C = A(kl)^2 - 2AB(kl) + \left(C + AB^2\right)$$

A = 2.5768e-005
B = 0.5140
C = 6.0309e-007

$$\varepsilon_x = \frac{\sqrt{AC}}{S_{12}^2} \approx 0.986\,\mu\text{m} \rightarrow \varepsilon_{nx} = 19.3\,\mu\text{m}$$

emittance in ELEGANT = 16 μm

estimated error between
MATLAB & ELEGANT ~ 20.5%



With *kl* < 0 region, we can estimate the vertical emittance.

Estimated Emittance and Errors

We got results for different energy spread (0.1%, 1.0%, and 4.23%).
We can see that bigger energy spread result in bigger emittance growth due to chromatic effects.

Errors of estimated emittances are about 20% due to various sources!
Good estimation!

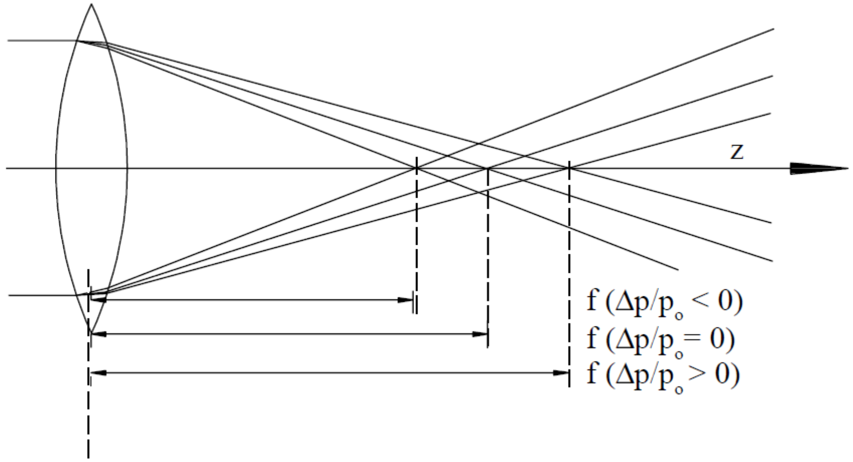
Energy Spread (%)	ε_x (μm)	ε_x error (%)	ε_y (μm)	ε_y error (%)
0.1	19.286	20.54	19.312	20.7
1	19.326	20.79	19.327	20.79
4.23	19.904	24.4	19.897	24.36

Emittance Growth - Chromatic Effects

From our class note: **focal property of a lattice with QM depend on the normalized strength k1, which, in turns, depend on the momentum of the charge particle (chromatic effect) → emittance growth**

Emittance Growth due to Chromatic Effects
Becomes stronger when:

1. **larger β-function**
2. **larger energy spread σ_δ**
3. **stronger QM strength k1**
4. **longer QM length L_{QM}**



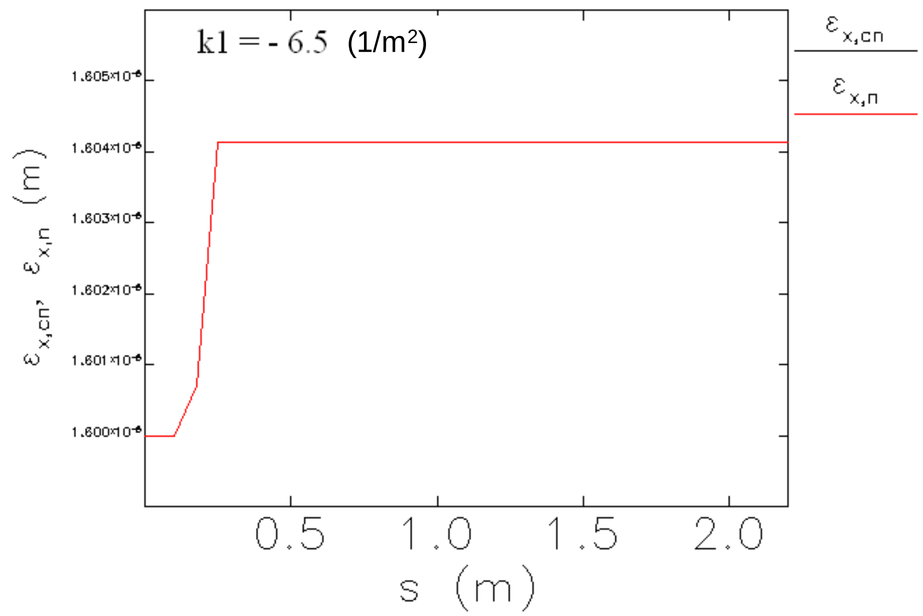
If $\beta\sigma_\delta k1 L_{QM}$ << 1, chromatic effect can be ignored.

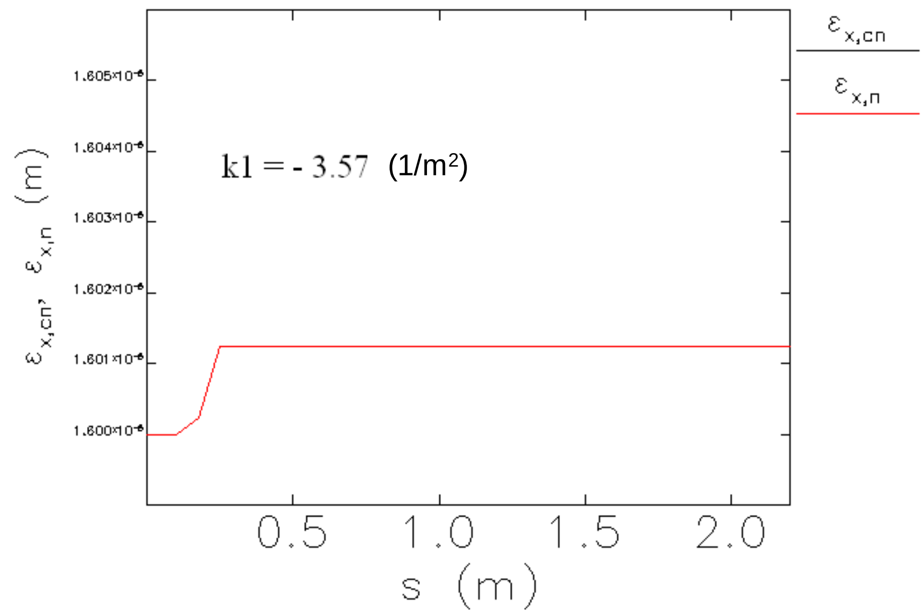
During QM scanning, k1 becomes bigger → emittance can be increased.

Emittance Growth – Chromatic Effects

During QM scanning, bigger k1 result in bigger emittance growth due to chromatic effects.

Here, σ_δ = 0.1%

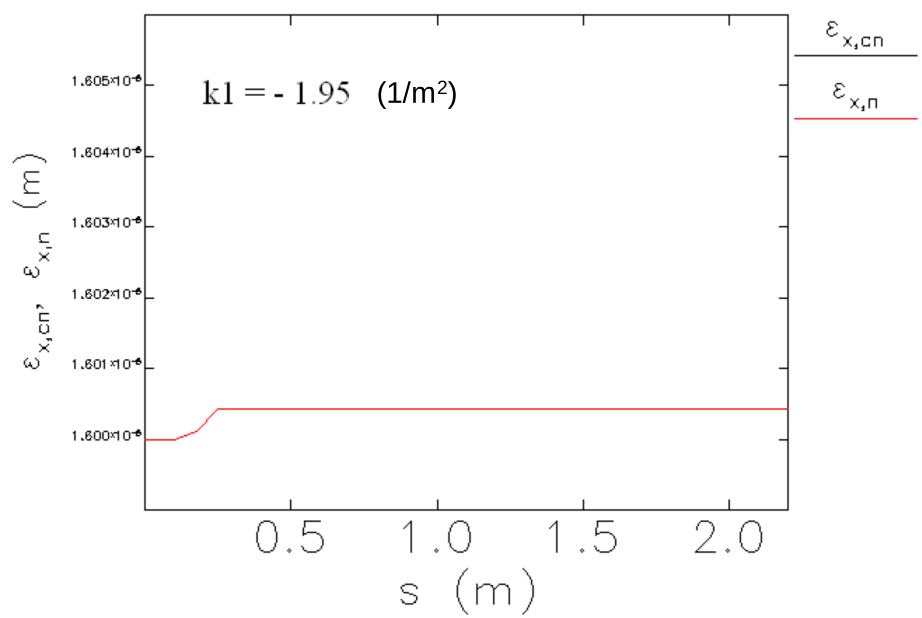


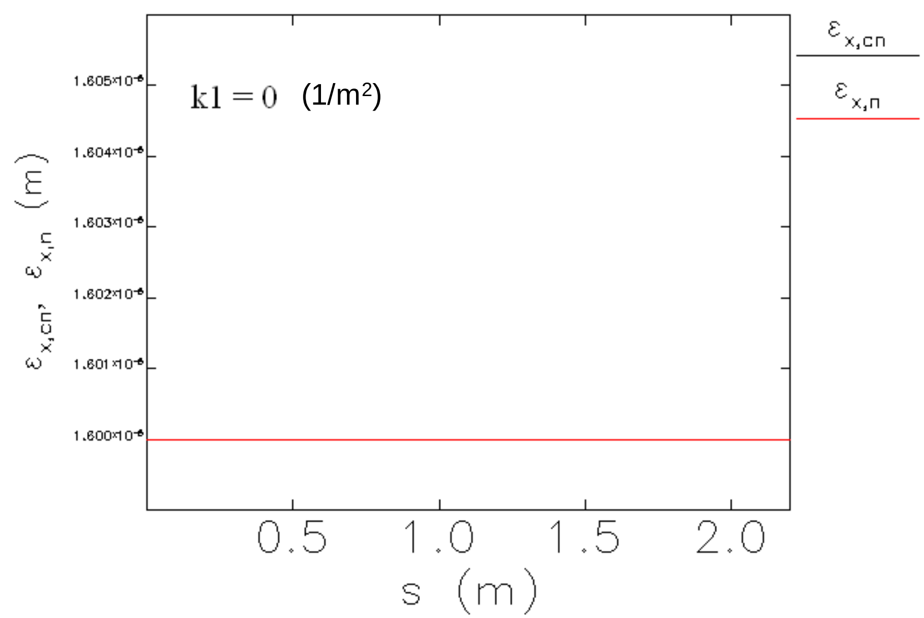


Emittance Growth – Chromatic Effects

During QM scanning, bigger k1 result in bigger emittance growth due to chromatic effects.

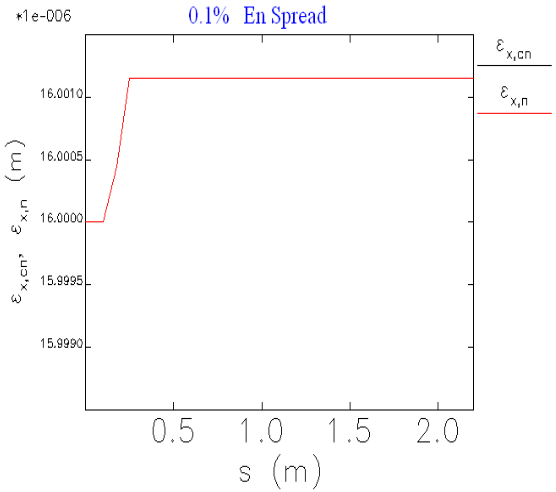
Here, $\sigma_\delta = 0.1\%$

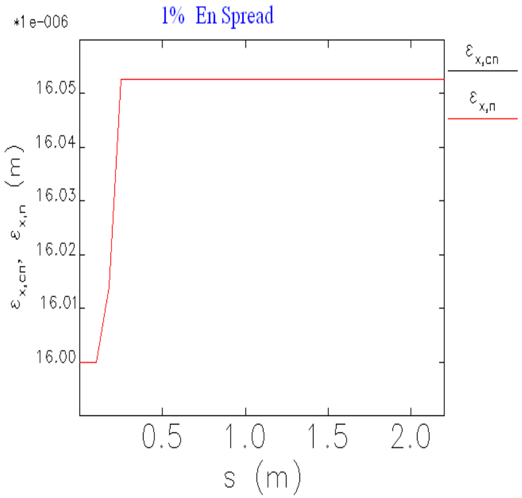


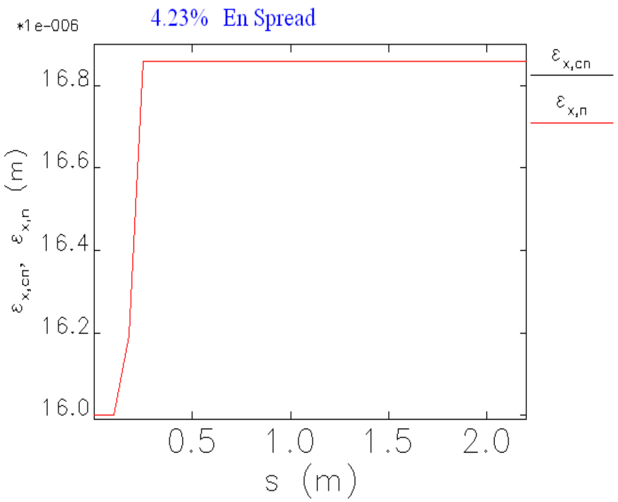


Emittance Growth – Chromatic Effects

During QM scan, bigger σ_δ result in bigger emittance growth due to chromatic effects







Emittance in ELEGANT vs. MATLAB

From ELEGANT simulation, we found that the emittance growth due to chromatic effects is within 1 µm range even for the biggest energy spread.

But MATLAB tool estimated abou 3~4 µm bigger emittance (~ 20% error).

Then, what are other sources of errors beside of chromatic effects?

?

Other Source – Thin Lens Approximation

Transfer matrix for QF :

$$\mathbf{M}_F = \begin{pmatrix} \cos\sqrt{k}l & \frac{1}{\sqrt{k}}\sin\sqrt{k}l \\ -\sqrt{k}\sin\sqrt{k}l & \cos\sqrt{k}l \end{pmatrix}$$

Thin lens approximation condition:

$$\sqrt{k}l << 1$$

Then, we have:

$$\mathbf{M}_F = \begin{pmatrix} 1 & 0 \\ -kl & 1 \end{pmatrix}$$

However, in our case:

$$\sqrt{k}l = \sqrt{6.5} \times 0.15 \approx 0.38 < 1$$

Thin Lens Approximation does not hold perfectly.

Conclusions

1. Emittance growth due to chromatic effects gives some error in emittance estimation.

2. Thin lens approximation does not hold well for our case because of big k1×l.

3. Fringe field effect: we ignored it intestinally in our simulation to make life easier. But in real experiment, fringe field effect is one of the main source of emittance growth. We did consider it at the beginning and we found that it give a big impact on the emittance estimation.

4. Fitting Method: ELEGANT uses particle coordinates to determine rms beamsizes. But MATLAB does fitting to estimate rms beamsizes. Therefore, there are some differences between two values.

5. We have to consider and minimize these effects in real emittance measurements.

Improvements

Do the fit for the region of the parabola with smaller k1, so that Thin Lens approximation does hold properly.

Do not use the thin lens approximation, derive the full equation without approximation.

Consider fringe field effects and estimate better effective QM length by measuring QM fields precisely.

Acknowledgements

Thanks to Dr. Kim. Without his help, it is impossible to finish this project. He really cared to the details of my project. He helped me to go though ELEGANT and MATLAB codes. He helped to correct my presentation. He went through each slide and did the corrections. Not only he taught us physics, but also art of presentation. You don’t really get this much care in most of the places.

I also thank Mayir. He helped me to listed to Dr. Kim’s class through internet while I was in China. Without it, I would have lost a lot of classes.

Also thanks to my classmates, Bindu, Bibek, Roman, Jason for their presence and patience.

Thank you !!!

감사합니다 !!!

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Спасибо !!!

მადლობა !!!

رەخمەت!!!

and

Questions are welcomed!!!