Clear

Clear

Define the cross section from Landau - Lifshitz, page 323, equation 81.7

$$d\sigma = r^2 \frac{4\pi m^2 dt}{s(s-4m^2)} \left\{\frac{1}{t^2}\left[\frac{1}{2}(s^2+u^2)+4m^2(t-m^2)\right]+\frac{1}{u^2}\left[\frac{1}{2}(s^2+t^2)+4m^2(u-m^2)\right]+\frac{4}{t\,u}\left(\frac{1}{2}s-m^2\right)\left(\frac{1}{2}s-3m^2\right)\right\}$$

$$\left\{\frac{4\,dt\,m^2\pi r^2\left(\frac{4\left(-3m^2+\frac{s}{2}\right)\left(-m^2+\frac{s}{2}\right)}{t\,u}+\frac{1}{t^2}\left[4m^2(-m^2+t)+\frac{1}{2}(s^2+u^2)\right]+\frac{1}{u^2}\left[\frac{1}{2}(s^2+t^2)+4m^2(-m^2+u)\right]\right)}{s(-4m^2+s)}\right\}$$

Define a rule to get rid of half - angles

$$\text{rule} = \left\{\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2 :\rightarrow .5\,(1+\text{Cos}[\theta]),\ \left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2 :\rightarrow .5\,(1-\text{Cos}[\theta]),\ \left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^4 :\rightarrow 1/8\,(-4\,\text{Cos}[\theta]+\text{Cos}[2\theta]+3),\ \left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^4 :\rightarrow 1/8\,(4\,\text{Cos}[\theta]+\text{Cos}[2\theta]+3)\right\};$$

Define the center - of - mass terms, as declared by LL equation 81.8

$s = 4\epsilon^2;$

$t = -4p^2\left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2$ /. rule;

$u = -4p^2\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2$ /. rule;

$dt = \frac{p^2}{\pi} d\Omega;$

Make sure to get rid of the half angles

$t = -2\epsilon^2(1-\text{Cos}[\theta]);$
$u = -2\epsilon^2(1+\text{Cos}[\theta]);$

Check on the equation for the cross - section

$d\sigma$

$$\left\{\frac{1}{\epsilon^2(-4m^2+4\epsilon^2)} d\Omega\, m^2 p^2 r^2 \left(\frac{(-3m^2+2\epsilon^2)(-m^2+2\epsilon^2)}{\epsilon^4(1-\text{Cos}[\theta])(1+\text{Cos}[\theta])} + \frac{1}{4\epsilon^4(1-\text{Cos}[\theta])^2}\left[4m^2(-m^2-2\epsilon^2(1-\text{Cos}[\theta]))+\frac{1}{2}(16\epsilon^4+4\epsilon^4(1+\text{Cos}[\theta])^2)\right] + \frac{1}{4\epsilon^4(1+\text{Cos}[\theta])^2}\left[\frac{1}{2}(16\epsilon^4+4\epsilon^4(1-\text{Cos}[\theta])^2)+4m^2(-m^2-2\epsilon^2(1+\text{Cos}[\theta]))\right]\right)\right\}$$

<u>For the relativistic case, let the momentum ≈ energy</u>

$p = \epsilon;$

Check the differential cross section

$$d\sigma / d\Omega$$

$$\left\{\frac{1}{-4 m^2 + 4 \epsilon^2} m^2 r^2 \left(\frac{(-3 m^2 + 2 \epsilon^2)(-m^2 + 2 \epsilon^2)}{\epsilon^4 (1 - \text{Cos}[\theta])(1 + \text{Cos}[\theta])} + \frac{1}{4 \epsilon^4 (1 - \text{Cos}[\theta])^2}\left[4 m^2 (-m^2 - 2 \epsilon^2 (1 - \text{Cos}[\theta])) + \frac{1}{2}(16 \epsilon^4 + 4 \epsilon^4 (1 + \text{Cos}[\theta])^2)\right] + \frac{1}{4 \epsilon^4 (1 + \text{Cos}[\theta])^2}\left[\frac{1}{2}(16 \epsilon^4 + 4 \epsilon^4 (1 - \text{Cos}[\theta])^2) + 4 m^2 (-m^2 - 2 \epsilon^2 (1 + \text{Cos}[\theta]))\right]\right)\right\}$$

Try to clean it up

```
FullSimplify[dσ] / dΩ
```

$$\left\{\frac{1}{-4 m^2 + 4 \epsilon^2} m^2 r^2 \left(\frac{(3 m^4 - 8 m^2 \epsilon^2 + 4 \epsilon^4) \text{Csc}[\theta]^2}{\epsilon^4} + \frac{1}{4 \epsilon^4 (-1 + \text{Cos}[\theta])^2}\left[-4 m^4 - 8 m^2 \epsilon^2 + 10 \epsilon^4 + 4 \epsilon^2 (2 m^2 + \epsilon^2) \text{Cos}[\theta] + 2 \epsilon^4 \text{Cos}[\theta]^2\right] + \frac{1}{4 \epsilon^4 (1 + \text{Cos}[\theta])^2}\left[\frac{1}{2}(16 \epsilon^4 + 4 \epsilon^4 (-1 + \text{Cos}[\theta])^2 - 8 m^2 (m^2 + 2 \epsilon^2 + 2 \epsilon^2 \text{Cos}[\theta]))\right]\right)\right\}$$

Further define some variables (mass and Energy are in eV)

```
r = e^2 / m;
e = √(1/137);
m = .511*^6;
ϵ = 53*^6;
θ = π/2;
```

Simplify the differential cross section

```
FullSimplify[dσ] / dΩ
```

$$\left\{9.48457 \times 10^{-21} \left(1.99963 + \frac{1}{31\,561\,924\,000\,000\,000\,000\,000\,000\,000\,000\,000}\left[7.88989 \times 10^{31}\right]\right)\right\}$$

For some reason this doesn' t want to clean up, do it by hand

$$\text{DiffXSect} = 9.48457*\text{^}-21 \left(1.99963 + \frac{7.88989*\text{^}31}{31.561924*\text{^}30}\right)$$

$$4.26753 \times 10^{-20}$$

This is in eV^2, so convert to GeV^2

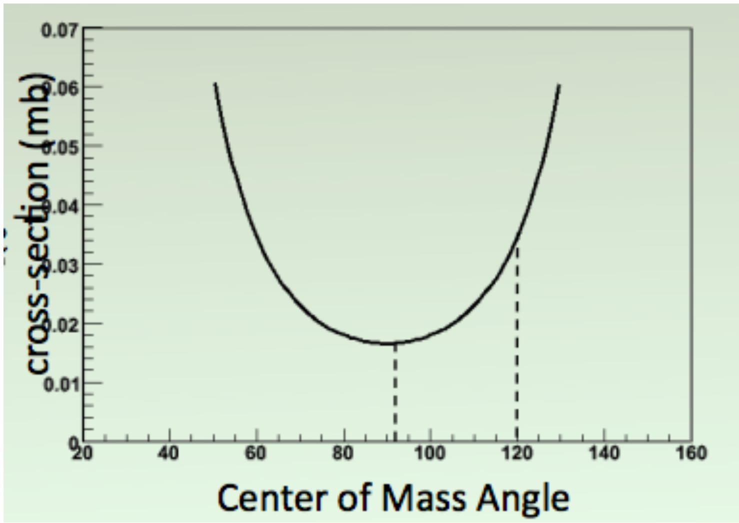
$$(4.267538*\text{^}-20) \frac{(1*\text{^}18)}{(1\ \text{GeV}^2)}$$

$$\frac{0.0426754}{\text{GeV}^2}$$

Using the conversion factor of $\frac{1}{\text{GeV}^2} \approx .3892$ mb

```
0.0426754 * .3892
0.0166093
```

Looking at Kumar' s Plot



This is in good agreement.

For the non - relativistic case, start from the beginning again.

$$d\sigma = r^2 \frac{4 \pi m^2 dt}{s (s - 4 m^2)} \left\{ \frac{1}{t^2} \left[\frac{1}{2} (s^2 + u^2) + 4 m^2 (t - m^2) \right] + \frac{1}{u^2} \left[\frac{1}{2} (s^2 + t^2) + 4 m^2 (u - m^2) \right] + \frac{4}{t u} \left(\frac{1}{2} s - m^2 \right) \left(\frac{1}{2} s - 3 m^2 \right) \right\}$$

$$\left\{ \frac{1}{s (-4 m^2 + s)} 4 \, dt \, m^2 \pi r^2 \left(\frac{4 \left(-3 m^2 + \frac{s}{2}\right) \left(-m^2 + \frac{s}{2}\right)}{t u} + \frac{1}{t^2} \left[4 m^2 (-m^2 + t) + \frac{1}{2} (s^2 + u^2) \right] + \frac{1}{u^2} \left[\frac{1}{2} (s^2 + t^2) + 4 m^2 (-m^2 + u) \right] \right) \right\}$$

$$\text{rule} = \left\{ \left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2 :\!\to .5 \, (1 + \text{Cos}[\theta]), \left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2 :\!\to .5 \, (1 - \text{Cos}[\theta]), \left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^4 :\!\to 1/8 \, (-4 \, \text{Cos}[\theta] + \text{Cos}[2 \theta] + 3), \left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^4 :\!\to 1/8 \, (4 \, \text{Cos}[\theta] + \text{Cos}[2 \theta] + 3) \right\};$$

$$s = 4 \epsilon^2;$$

$$t = -4 p^2 \left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2 \text{ /. rule};$$

$$u = -4 p^2 \left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2 \text{ /. rule};$$

$$dt = \frac{p^2}{\pi} d\Omega;$$

$$r = e^2 / m;$$

```
t = -2 ϵ^2 (1 - Cos[θ]);
u = -2 ϵ^2 (1 + Cos[θ]);
```

```
dσ
```

$$\left\{\frac{1}{\epsilon^2\left(-4 m^2+4 \epsilon^2\right)} d\Omega\, e^4 p^2\left(\frac{\left(-3 m^2+2 \epsilon^2\right)\left(-m^2+2 \epsilon^2\right)}{\epsilon^4(1-\text{Cos}[\theta])(1+\text{Cos}[\theta])}+\frac{1}{4 \epsilon^4(1-\text{Cos}[\theta])^2}\left[4 m^2\left(-m^2-2 \epsilon^2(1-\text{Cos}[\theta])\right)+\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1+\text{Cos}[\theta])^2\right)\right]+\frac{1}{4 \epsilon^4(1+\text{Cos}[\theta])^2}\left[\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1-\text{Cos}[\theta])^2\right)+4 m^2\left(-m^2-2 \epsilon^2(1+\text{Cos}[\theta])\right)\right]\right)\right\}$$

This time, since it' s non relativistic, let E ≈ m, but first we need to take care of the 1st term or else it will be 1/0,
We can say $E^2 = p^2 + m^2 \to 4E^2 - 4m^2 = 4p^2$, even when we are non-relativistic

```
rule = {-4 m^2 + 4 ϵ^2 :→ 4 p^2};
```

Check to make sure that term is clean

```
dσ /. rule
```

$$\left\{\frac{1}{4 \epsilon^2} d\Omega\, e^4\left(\frac{\left(-3 m^2+2 \epsilon^2\right)\left(-m^2+2 \epsilon^2\right)}{\epsilon^4(1-\text{Cos}[\theta])(1+\text{Cos}[\theta])}+\frac{1}{4 \epsilon^4(1-\text{Cos}[\theta])^2}\left[4 m^2\left(-m^2-2 \epsilon^2(1-\text{Cos}[\theta])\right)+\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1+\text{Cos}[\theta])^2\right)\right]+\frac{1}{4 \epsilon^4(1+\text{Cos}[\theta])^2}\left[\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1-\text{Cos}[\theta])^2\right)+4 m^2\left(-m^2-2 \epsilon^2(1+\text{Cos}[\theta])\right)\right]\right)\right\}$$

Redefine it just to be safe

$$d\sigma=\frac{1}{4 \epsilon^2} d\Omega\, e^4\left(\frac{\left(-3 m^2+2 \epsilon^2\right)\left(-m^2+2 \epsilon^2\right)}{\epsilon^4(1-\text{Cos}[\theta])(1+\text{Cos}[\theta])}+\frac{1}{4 \epsilon^4(1-\text{Cos}[\theta])^2}\left[4 m^2\left(-m^2-2 \epsilon^2(1-\text{Cos}[\theta])\right)+\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1+\text{Cos}[\theta])^2\right)\right]+\frac{1}{4 \epsilon^4(1+\text{Cos}[\theta])^2}\left[\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1-\text{Cos}[\theta])^2\right)+4 m^2\left(-m^2-2 \epsilon^2(1+\text{Cos}[\theta])\right)\right]\right)$$

$$\frac{1}{4 \epsilon^2} d\Omega\, e^4\left(\frac{\left(-3 m^2+2 \epsilon^2\right)\left(-m^2+2 \epsilon^2\right)}{\epsilon^4(1-\text{Cos}[\theta])(1+\text{Cos}[\theta])}+\frac{1}{4 \epsilon^4(1-\text{Cos}[\theta])^2}\left[4 m^2\left(-m^2-2 \epsilon^2(1-\text{Cos}[\theta])\right)+\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1+\text{Cos}[\theta])^2\right)\right]+\frac{1}{4 \epsilon^4(1+\text{Cos}[\theta])^2}\left[\frac{1}{2}\left(16 \epsilon^4+4 \epsilon^4(1-\text{Cos}[\theta])^2\right)+4 m^2\left(-m^2-2 \epsilon^2(1+\text{Cos}[\theta])\right)\right]\right)$$

Now replace ϵ with m

```
ϵ = m;
```

Check the differential cross section

$d\sigma / d\Omega$

$$\frac{1}{4\,m^2} e^4 \left(-\frac{1}{(1 - \mathrm{Cos}[\theta])\,(1 + \mathrm{Cos}[\theta])} + \frac{1}{4\,m^4\,(1 - \mathrm{Cos}[\theta])^2}\left[4\,m^2\,\left(-m^2 - 2\,m^2\,(1 - \mathrm{Cos}[\theta])\right) + \frac{1}{2}\left(16\,m^4 + 4\,m^4\,(1 + \mathrm{Cos}[\theta])^2\right)\right] + \frac{1}{4\,m^4\,(1 + \mathrm{Cos}[\theta])^2}\left[\frac{1}{2}\left(16\,m^4 + 4\,m^4\,(1 - \mathrm{Cos}[\theta])^2\right) + 4\,m^2\,\left(-m^2 - 2\,m^2\,(1 + \mathrm{Cos}[\theta])\right)\right]\right)$$

Redefine the variables for our situation

```
e = √(1/137) ;
m = .511*^6;
ϵ = 53*^6;
θ = π/2;
```

$d\sigma / d\Omega$

$$\frac{3.99926 + 2 \times \frac{1}{31\,561\,924\,000\,000\,000\,000\,000\,000\,000\,000}\left[7.88989 \times 10^{31}\right]}{210\,888\,484\,000\,000\,000\,000}$$

Do it by hand again

$$\frac{3.99926 + 2 * \frac{7.88989*\hat{}31}{31.561924*\hat{}30}}{21.0888484*\hat{}18}$$

4.26713×10^{-19}

This answer is in eV^2, convert to GeV^2

$$(4.26713*\hat{}{-19})\ \frac{(1*\hat{}18)}{1\ GeV^2}$$

$$\frac{0.426713}{GeV^2}$$

Using the conversion factor of $\frac{1}{GeV^2} \approx .3892$ mb

```
.42613 * .3892
```

0.16585

This is different from the answer that you found, looking at LL Eqn 81.9 on page 323, but breaking it into two parts

$$d\sigma = \left(\frac{e^2}{m\,v^2}\right)^2$$

$$\frac{e^4}{m^2\,v^4}$$

We can make the non - relativistic substitution they utilize v = 2 p / m;

dσ

$$\frac{e^4\, m^2}{16\, p^4}$$

If we only look at the non - trig terms, the this is of the same form of Halzen and Martin p121, where $\alpha^2 = \frac{e^4}{\hbar^2 c^2}$, $\hbar = c = 1$

The trig part according to LL can be converted to the form of Halzen and Martin, so let' s test it.

$$\frac{4\,(1+3\,(\text{Cos}[\theta])^2)}{(\text{Sin}[\theta])^4} == -\frac{4}{(1-\text{Cos}[\theta])\,(1+\text{Cos}[\theta])} + \frac{8}{3-4\,\text{Cos}[\theta]+\text{Cos}[2\,\theta]} + \frac{8}{3+4\,\text{Cos}[\theta]+\text{Cos}[2\,\theta]}$$

Get rid of those half-angles on the RHS

$$\text{rule} = \left\{1\Big/\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2 :\to \frac{2}{(1+\text{Cos}[\theta])},\ 1\Big/\left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2 :\to \frac{2}{(1-\text{Cos}[\theta])},\ 1\Big/\left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^4 :\to \frac{8}{(-4\,\text{Cos}[\theta]+\text{Cos}[2\,\theta]+3)},\ 1\Big/\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^4 :\to \frac{8}{(4\,\text{Cos}[\theta]+\text{Cos}[2\,\theta]+3)}\right\};$$

$$\text{RHS} = \text{FullSimplify}\left[\left(\frac{1}{\left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^4} + \frac{1}{\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^4} - \frac{1}{\left(\text{Sin}\left[\frac{\theta}{2}\right]\right)^2\left(\text{Cos}\left[\frac{\theta}{2}\right]\right)^2}\right) \text{/. rule}\right]$$

$$2\,(5+3\,\text{Cos}[2\,\theta])\,\text{Csc}[\theta]^4$$

$$\text{rule} = \left\{\text{Cos}[2\,\theta] :\to 2\,(\text{Cos}[\theta])^2 - 1\right\};$$

RHS = RHS /. rule

$$2\,(5+3\,(-1+2\,\text{Cos}[\theta]^2))\,\text{Csc}[\theta]^4$$

RHS

$$2\,(5+3\,(-1+2\,\text{Cos}[\theta]^2))\,\text{Csc}[\theta]^4$$

Simplifying this by hand

$$\text{RHS} = 2\,(5-3+6\,\text{Cos}[\theta]^2)\,\text{Csc}[\theta]^4$$

$$\text{RHS} = 2\,(2+6\,\text{Cos}[\theta]^2)\,\text{Csc}[\theta]^4$$

$$\text{RHS} = (4+12\,\text{Cos}[\theta]^2)\,\text{Csc}[\theta]^4$$

$$\text{RHS} = 4\,(1+3\,\text{Cos}[\theta]^2)\,\text{Csc}[\theta]^4$$

$$\text{RHS} = 4\,\frac{(1+3\,\text{Cos}[\theta]^2)}{\text{Sin}[\theta]^4}$$

LHS = RHS

The two forms, Landau - Lifshitz and Halzen - Martin is the same equation for non - relativistic Møller scattering. If we just look at the non - trig term

$$d\sigma = \frac{\alpha^2 m^2}{16 p^4}$$

$$\frac{m^2 \alpha^2}{16 p^4}$$

```
α = 1/137;
m = .511*^6;
p = 53*^6;
```

dσ

1.10199×10^{-25}

This is in eV^2, converting to GeV^2

$$(1.10199*\hat{}-25) \frac{(1*\hat{}18)}{1\ GeV^2}$$

$$\frac{1.10199 \times 10^{-7}}{GeV^2}$$

Using the conversion factor $\frac{1}{GeV}$=.3892 mb we find

(1.10199*^-7) (.3892)

4.28895×10^{-8}

We find the nanobarns you had found earlier. The difference between the two equations is that the non-relativistic equation assumes m=E, which is most defiantly not true in ultra relativistic case where p=E.