ELSEVIER Nuclear Instruments and Methods in Physics Research A 412 (1998) 174–182

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Parametric study of laser Compton-backscattering from free relativistic electrons

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Received 20 November 1996; received in revised form 23 July 1997

Abstract

The frontal collisions of a laser with free relativistic electrons result in Compton-backscattered γ-rays. The energy of these γ-rays is dependent on the laser and electron energy and ranges from keV to tens of MeV. In a sufficiently narrow backscattering angle the γ-rays are nearly monochromatic.

Over the past 30 years there have been several attempts to produce γ-ray beams by laser backscattering from relativistic electrons. One aim is to produce γ-rays in a high MeV energy range with rates useful for nuclear physics, another for medical applications in energy range from high keV to low MeV.

In order to enable conceptual studies for medical applications, a computer program IMRCBS based on the theory of laser Compton-backscattering from free relativistic electrons was written. The present document gives: (a) description of the kinematics of Compton-backscattering mechanism, and (b) parametric study of laser Compton-backscattering.

The study shows, that if variable energy of γ-beam is required, then the use of a free electron laser in combination with fixed electron energy accelerator is a better combination than the use of a fixed laser energy with electron accelerator with variable energy.

1. Kinematics of Compton-backscattering mechanism

Frontal collisions of a laser beam with an electron beam results in Compton-backscattered photons. The scattering process is shown schematically in Fig. 1.

Here T and T_s are the kinetic energies of incident and scattered electron and E_L and E_γ are energies of incident and scattered photon in the laboratory frame. T_s^{ER} is the kinetic energy of scattered electron and E_L^{ER} and E_γ^{ER} are the energies of incident and scattered photon in the electron rest frame (ER), in which the electron is initially at rest, before the collision.

After backscattering, the γ-ray emerges in the laboratory frame at a small angle θ relative to the electron beam direction. The electron emerges in the angle ϕ. In the ER frame the γ-ray emerges with a high probability at a large angle θ' and electron will move approximately in the opposite direction, i.e. at angle $\phi' \simeq \pi - \theta'$.

The theoretical basis of the Compton-backscattering mechanism was described by Federici et al. [1], and also by Sandorfi et al. [2] and Blumberg [3]. For a better understanding of our parametric study in Section 2 the most important relationships will be displayed in the next paragraphs.

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PII: S0168-9002(98)00099-0

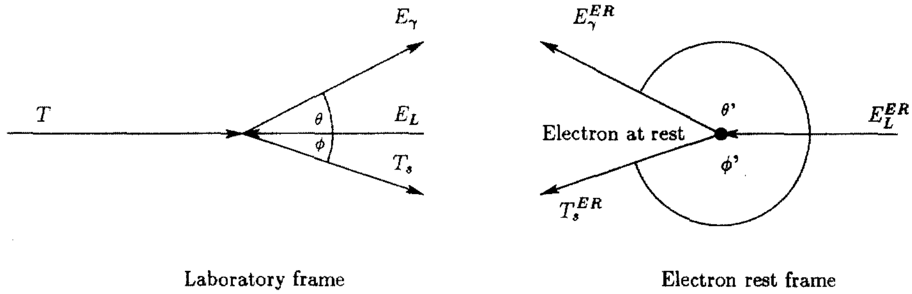

Fig. 1. The scattering process.

1.1. Energy of backscattered photon in the laboratory frame

The energy of the backscattered photon in the laboratory frame can be calculated from the equations for energy, x- and y-momentum conservation. They have the following form:

$$T + E_L = T_s + E_\gamma, \tag{1a}$$

$$P - \frac{E_L}{c} = \frac{E_\gamma}{c}\cos\theta + P_s\cos\phi, \tag{1b}$$

$$\frac{E_\gamma}{c}\sin\theta = P_s\sin\phi, \tag{1c}$$

where c is the velocity of light and P and P_s are the electron momenta before and after the scattering event:

$$P = \frac{\sqrt{T(T+2mc^2)}}{c} \quad \text{and}$$

$$P_s = \frac{\sqrt{T_s(T_s+2mc^2)}}{c}, \tag{2}$$

where m is the electron mass and mc^2 is the electron energy at rest ($mc^2 = 0.511$ MeV). Solving Eqs. (1a), (1b) and (1c) for E_γ as function of E_L and backscattering angle θ, one obtains

$$E_\gamma = \frac{(1+\beta)E_L}{1 - \beta\cos\theta + (E_L/mc^2)\sqrt{1-\beta^2}(1+\cos\theta)}, \tag{3}$$

where β is the ratio of the electron and light velocities. It can be expressed as

$$\beta = \frac{\sqrt{T(T+2mc^2)}}{T+mc^2}. \tag{4}$$

Considering electron energies above 50 MeV ($\beta \to 1$) and a small backscattering angle θ ($\theta < 10$ mrad), formula (3) can be simplified to

$$E_\gamma = \frac{4\gamma^2 E_L}{1 + (4\gamma E_L/mc^2) + \gamma^2\theta^2}, \tag{5}$$

given by Federici et al. [1], Sandorfi et al. [2] and Blumberg [3], where γ is the ratio of the total electron energy and its energy at rest:

$$\gamma = \frac{T+mc^2}{mc^2} = \frac{1}{\sqrt{1-\beta^2}}. \tag{6}$$

For a head-on collision, $\theta = 0$ in Eq. (3), the required laser energy to produce a backscattered photon of E_γ at kinetic electron energy T is given by

$$E_L = \frac{(1-\beta)E_\gamma}{1 + \beta - (2E_\gamma/mc^2)\sqrt{1-\beta^2}}. \tag{7}$$

The laser energy increases linearly in log–log scale if the energy of the incident electron is much higher then the energy of the backscattered photon. It will reach infinite value at lowest electron energy cut-off given by

$$T^{\text{cut-off}} = \frac{(E_\gamma - 0.5\,mc^2)^2}{E_\gamma}. \tag{8}$$

1.2. Differential Compton-scattering cross section

The differential Compton scattering cross section for energetic electrons in the laboratory frame can be developed starting with its form in the ER frame. In this case the electrons are at rest and the cross section was developed by Klein–Nishina [4]. It has the form

$$\frac{d\sigma}{\sin\theta'\,d\theta'} = \pi r_0^2 R^2 \left(R + \frac{1}{R} - 1 + \cos^2\theta'\right), \tag{9}$$

where r_0 is the classical electron radius and R is the ratio between the energies of scattered and incident photon, $R = E_\gamma^{ER}/E_L^{ER}$. It can be obtained from Eq. (4) considering the electron to be at rest, i.e. $\beta = 0$. In this case, the ER and laboratory frame are equivalent. The result is

$$R = \frac{E_\gamma^{ER}}{E_L^{ER}} = \frac{1}{1 + (E_L^{ER}/mc^2)(1 + \cos\theta')}. \tag{10}$$

The incident photon energy in ER frame, E_L^{ER}, can be obtained with help of Lorenz transformation of its value, E_L, in the laboratory frame. The result is

$$E_L^{ER} = (1 + \beta)\gamma E_L. \tag{11}$$

Considering a large electron energy ($T \gg mc^2$, i.e. above 50 MeV, for example, $\beta \to 1$) then Eq. (11) can be replaced with a good accuracy by $E_L^{ER} \to 2\gamma E_L$, the expression used by Federici et al. [1], Sandorfi et al. [2] and Blumberg [3].

The equality of the y-momenta of scattered photon in ER and laboratory frame leads to the requirement that

$$\frac{E_\gamma^{ER}}{c}\sin\theta' = \frac{E_\gamma}{c}\sin\theta. \tag{12}$$

Eliminating E_γ and E_γ^{ER} with help of Eqs. (3) and (10) and E_L^{ER} with help of Eq. (11) leads to the relationship between the angles θ' and θ:

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}. \tag{13}$$

Differentiating the last equation leads to

$$d\theta'\sin\theta' = d\theta\sin\theta\frac{1 - \beta^2}{(1 - \beta\cos\theta)^2}. \tag{14}$$

Multiplying now Eq. (9) by $d\theta'\sin\theta'/d\theta\sin\theta$, one obtains the Compton scattering cross section for energetic electrons in laboratory frame:

$$\frac{d\sigma}{\sin\theta\,d\theta} = \pi r_0^2 \frac{1 - \beta^2}{(1 - \beta\cos\theta)^2} R^2 \left(R + \frac{1}{R} - 1 + \cos^2\theta'\right), \tag{15}$$

where R and $\cos\theta'$ are given by Eqs. (10) and (13). The cross section for scattering a photon into a cone of angle θ_c in laboratory frame is then

$$\sigma(\theta_c) = \int_0^{\theta_c} d\theta \frac{d\sigma}{d\theta}. \tag{16}$$

The average energy of γ-rays backscattered into a cone of angle θ_c in laboratory system is then

$$E_\gamma^{av}(\theta_c) = \frac{\int_0^{\theta_c} d\theta\, E_\gamma(\theta)(d\sigma/d\theta)}{\int_0^{\theta_c} d\theta(d\sigma/d\theta)}. \tag{17}$$

1.3. Rate of backscattered photons

Sandorfi et al. [2] give the rate of γ-ray beam scattered into the cone of angle θ_c as

$$I_\gamma = 2.604\frac{I\rho_L L_L \sigma(\theta_c)}{E_L A}\ \text{(photons/s)}, \tag{18}$$

where I is the electron beam current in Amps, ρ_L is the peak laser bunch power in watts, $\sigma(\theta_c)$ is the cross section given by Eq. (16) in milibarns, L_L is the length of the incident laser bunch in centimeters, E_L is the incident laser energy in eV, and A is the effective overlap area of the photon and electron bunch at the interaction point in cm^2 given by

$$A = 2\pi\sqrt{\sigma_L^2 + \sigma_{ex}^2}\sqrt{\sigma_L^2 + \sigma_{ey}^2}, \tag{19}$$

where σ_L^2 is the RMS radius of laser beam, σ_{ex}^2 is the RMS width of electron beam and σ_{ey}^2 is RMS height of electron beam.

2. Calculations

Based on the equations described above, the computer program IMRCBS was written. A series of test cases was selected which could be calculated using the program IMRCBS and program obtained from Blumberg [3]. A good agreement was found.

For calculations in this work, selected were the energies of the backscattered γ-rays of 10, 100 keV, 1 and 10 MeV in combination with laser energies of 0.1, 1 and 10 eV. These three energies correspond, since $\lambda_1 = hc/E_L$ with $hc = 1.2398$ eV μm, to $\lambda = 0.12398$, 1.2398 and 12.398 μm. Table 1 displays the related electron energies calculated with help of Eq. (3).

The left-hand picture of Fig. 2 displays the laser energy as a function of the electron energy required to obtain the backscattered 10, 100 keV, 1 and 10 MeV γ-ray beam. At electron energies much higher then the energy of the backscattered photon it increases linearly in log–log scale. Only if the electron energy will be near to its low-energy cut-off given by Eq. (8), the required laser energy will raise to infinity.

The relationship (13) between the backscattering angles in the ER and laboratory frames is the basis of the laser Compton-backscattering from relativistic electrons. The right-hand picture of Fig. 2 displays backscattering angle in the laboratory frame as a function of the backscattering angle in the ER frame for electron energies $T = 1$, 10, 100 and 1000 MeV. With increasing electron energy the angular range in ER frame will be compressed into a small region about the backscattering direction in the laboratory frame. Only at $\theta' = \pi$ also $\theta = \pi$. This compression leads to the increasing probability of the backscattering into the cone at angle θ (see left-hand picture of Fig. 3) and the compression of a large portion of the Compton scattering cross section into a narrow region about $\theta = 0$.

The right-hand picture of Fig. 3 displays the Compton cross section for scattering γ-rays into a cone of angle 1 mrad in laboratory system for γ-ray energy 0.01, 0.1, 1 and 10 MeV at $\theta = 0$ as function of the electron energy. With exception at low electron energies, it is only dependent on the electron energy. This indicates, that the rate of γ-ray beam scattered into the cone of 1 mrad at a required energy of the γ-ray will grow with increasing electron and therefore decreasing laser energy. In the electron energy range above 1 GeV,

Table 1
Electron energies required to obtain 10, 100 keV, 1 and 10 MeV γ-rays due to collisions with 0.1, 1 and 10 eV laser beam

E_γ	T (MeV)		
	$E_L = 0.1$ eV	$E_L = 1$ eV	$E_L = 10$ eV
10 keV	80.3	25.0	7.6
100 keV	255.0	80.3	25.1
1 MeV	807.9	255.5	80.8
10 MeV	2559.5	812.4	259.9

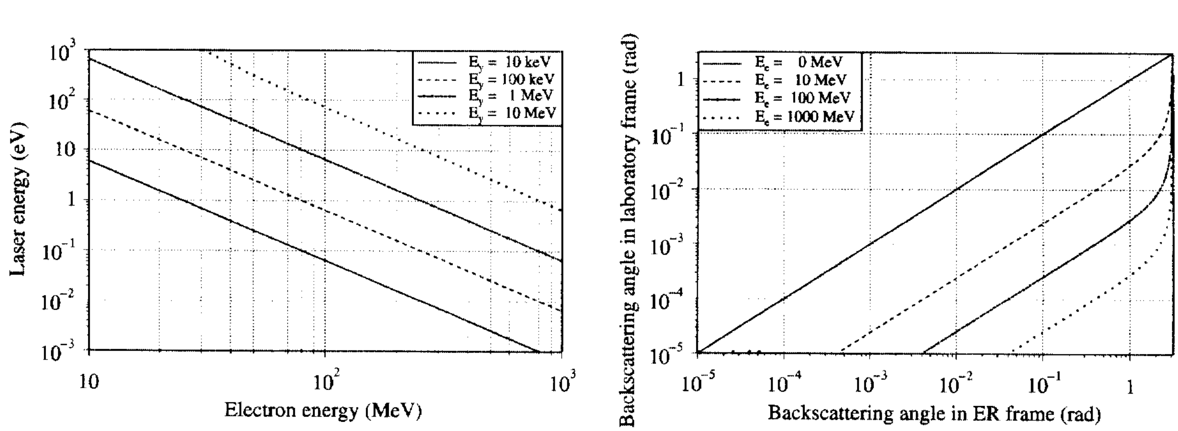

Fig. 2. Laser energy as function of the electron energy required to obtain 10, 100 keV, 1 and 10 MeV γ-ray energy at $\theta = 0$; Backscattering angle in laboratory frame as a function of the backscattering angle in ER frame for electron energies of 1, 10, 100 and 1000 MeV.

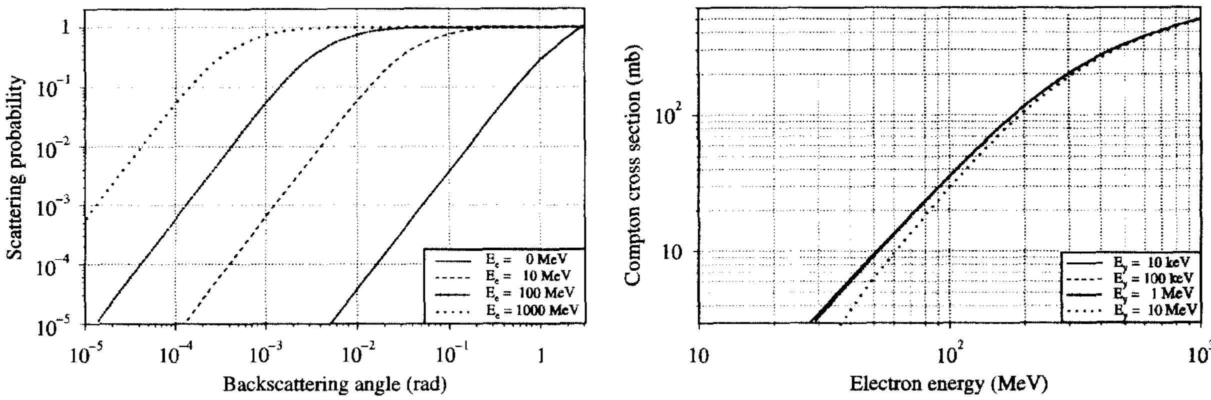

Fig. 3. Scattering probability into the cone of the backscattering angle in laboratory frame at electron energies of 1, 10, 100 and 1000 MeV; The Compton cross section for scattering the γ-rays into a cone of angle 1 mrad in laboratory frame at γ-ray energies of 10, 100 keV, 1 and 10 MeV and at $\theta = 0$, as a function of the electron energy.

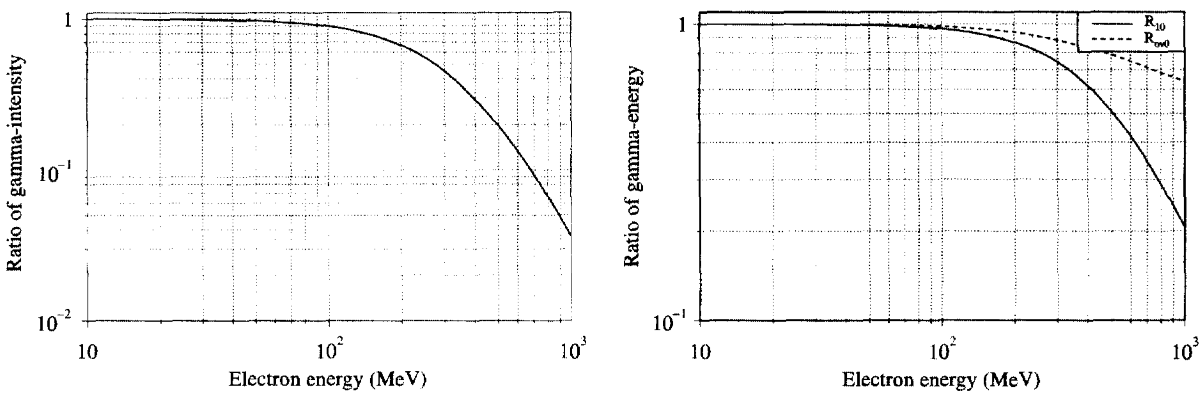

Fig. 4. Ratio of γ-intensity at angle $\theta = 1$ mrad and $\theta = 0$ as function of the electron energy; Ratio of γ-energy at angle $\theta = 1$ mrad and $\theta = 0$ as function of the electron energy.

relativistic saturation of the Compton cross section could be observed.

The left-hand picture of Fig. 4 displays the ratio of γ-rate at angle $\theta = 1$ mrad and $\theta = 0$ as a function of the electron energy. In the energy ranges of interest it is a function only of the electron energy. It decreases at energies above 100 MeV strongly with increasing electron energy.

The right-hand picture of Fig. 4 displays the ratio of γ-energy at angle $\theta = 1$ mrad and $\theta = 0$ as a function of the electron energy. Also the energy-ratio is only a function of the electron energy. Below the electron energy of about 150 MeV it is greater than 0.9 but above this energy it strongly decrease. The decreases is much smaller than that of the gamma-rate ratio. This indicates that the ratio of the average energy and the energy at $\theta = 0$ (R_{av0}) will be less energy dependent. Right-hand picture of Fig. 4 shows that it decreases with increasing electron energy only slowly and reaches at 1 GeV value of 0.64. At very high energies this value tends to 0.5.

The rate of γ-ray beam is linearly dependent on Compton cross section. As discussed above the Compton cross section grows with increasing electron energy (see right-hand picture of Fig. 3).

Therefore also the rate of γ-ray beam of a given energy grows with increasing electron and decreasing laser energy.

Figs. 5–7 display γ-ray energy (E_γ) and its the relative rate $I_\gamma(\theta)/I_\gamma(0)$ as functions of the backscattering angle θ for three laser energies $E_L = 0.1$, 1 and 10 eV.

The energies of the backscattered photons were calculated using Eq. (3) and the relative rate is given by $I_\gamma(\theta)/I_\gamma(0) = \sigma(\theta)/\sigma(0)$, where $\sigma(\theta)$ was obtained using Eq. (15).

It is clear that E_γ as well as the relative rate are decreasing functions of θ. They are first almost constant at a small values of θ but drop rapidly for θ larger than a critical value θ_c. The value of θ_c increases with decreasing electron energy (or with increasing laser energy). This suggests the use of highest possible laser energy in combination with lowest possible electron energy for an γ-ray energy of interest. In such a case, the γ-ray source will be almost monochromatic and will have a constant rate in a largest cone. At low electron energy $T = 0.5\,(E_L)^2/mc^2$ the γ-energy will be constant in the whole angular range. Below this energy E_γ will be an increasing function of θ.

In practice, the γ-source can be constructed using, (a) a fixed laser energy combined with electron accelerator with variable energy, or (b) a free electron laser combined with electron accelerator with fixed energy.

First, lasers with fixed energy combined with electron accelerator with variable electron energy is assumed. Table 2 contains, for 12 combinations of laser and electron energies required to produce γ-ray beams at $\theta = 0$ of energy 10, 100 keV, 1 and 10 MeV, (a) rate of the γ-ray beam scattered into the cone of θ_c, (b) relative energy spread between γ-rays backscattered $\theta = 0$ and $\theta = \theta_c$ and (c) the energy spread between γ-rays backscattered $\theta = 0$ and average energy of γ-ray backscattered into the cone. Selected were two angles $\theta_c = 0.1$ and 0.2 mrad.

The rate of γ-ray beam is linearly dependent on Compton cross section. As discussed above the Compton cross section grows with increasing electron energy (see right-hand picture of Fig. 3). Therefore, also the rate of γ-ray beam grows with increasing electron and therefore decreasing laser energy.

Important is the size of the energy spread. The right-hand picture of Fig. 4 shows that it is only a function of the electron energy and increases with increasing electron energy. If we accept the maximum of the energy spread $S_e = 2.5\%$ then at a laser energy of 1 eV the X-rays in the energy range up to 100 keV can be scattered into the cone of 1 mrad with cross sections of 59.83 and 571.7 mb. The required electron energies are 25 and 80.3 MeV, respectively. If the γ-rays of energies between 1 and 10 MeV have to be produced with the above energy spread, then the angle of the cone has to be reduced to 0.1 mrad with cross sections of 59.59 and 565.2 mb. The required electron energies are 255.5 and 812.4 MeV, respectively.

Table 2 indicates that the cross sections and both, the angular energy and rate deviations of photon beam, S_e and S_i, are in the relativistic electron energy range almost independent from the laser energy. They are increasing with increasing kinetic electron energy T. This implies that the quality photon beam can be obtained keeping the electron energy constant and small and to vary the laser energy to obtain the photon beam in the energy range between 34 keV and 3 MeV. Important for selection of the electron energy is also the size of the cross section for scattering a photon into a cone of angle θ_c. It is a linear measure of the γ-beam rate so that it should be so large as possible. Again it is almost independent from the laser energy at a constant electron energy. Unfortunately, it decreases with decreasing electron energy so that a reasonable compromise must be found.

Table 3 gives the laser energies E_L, the cross sections $\sigma(\theta_c)$ and the angular deviations of the photon energy S_e and rate S_i as function of the electron energy and photon energies. In this case, a free electron laser with variable laser energy in combination with electron accelerator with fixed electron energy is assumed.

The size of the cross section is reasonable and angular deviation of the energy and rate is small in cone of $\theta_c = 0.1$ mrad. It is still acceptable even in cone of $\theta_c = 0.2$ mrad. At $T = 600$ MeV the cross sections are high but the angular deviation of the rate in the cone of $\theta_c = 0.2$ mrad is near to the limit of acceptance.

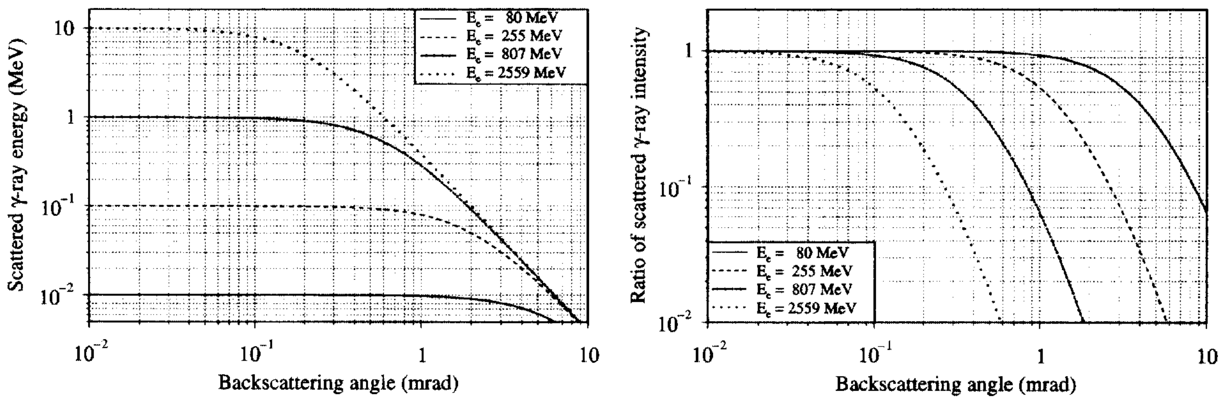

Fig. 5. γ-ray energy as function of the backscattering angle (left-hand picture) and relative gamma intensity $I_\gamma(\theta)/I_\gamma(0)$ as function of the backscattering angle; Both at laser energy of 0.1 eV and electron–beam energies of 80, 255, 807 and 2559 MeV.

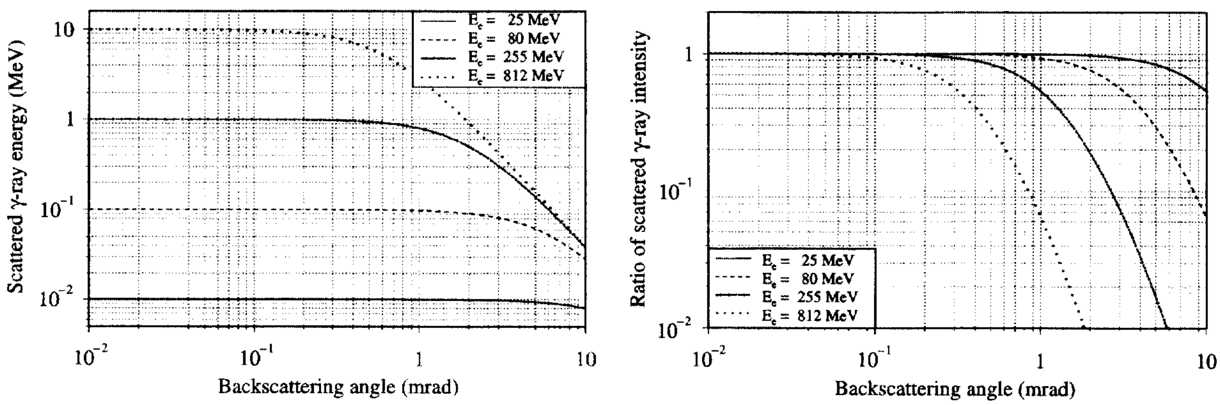

Fig. 6. γ-ray energy as function of the backscattering angle (left-hand picture) and relative gamma intensity $I_\gamma(\theta)/I_\gamma(0)$ as function of the backscattering angle; Both at laser energy of 1 eV and electron–beam energies of 25, 80, 255 and 812 MeV.

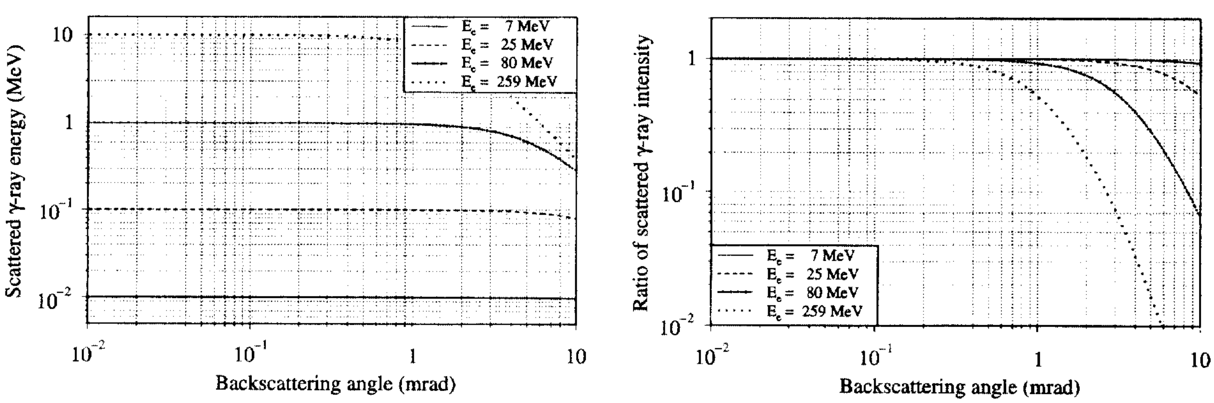

Fig. 7. γ-ray energy as function of the backscattering angle (left-hand picture) and relative gamma intensity $I_\gamma(\theta)/I_\gamma(0)$ as function of the backscattering angle; Both at laser energy of 10 eV and electron-beam energies of 7, 25, 80 and 259 MeV.

Table 2
Energies and rates of γ-ray beam scattered into the cone of θ_c as function of the laser and electron energy; $S_e = 1 - E_\gamma/E_\gamma^{(\theta=0)}$ and $S_i = 1 - I_\gamma/I_\gamma^{(\theta=0)}$.

θ_c (mrad)	E_L (eV)	T (MeV)	$E_\gamma^{(\theta=0)}$ (MeV)	$\sigma(\theta_c)$ (mb)	S_e (%)	S_i (%)
0.1	0.1	80.3	0.01	0.249	0.0	0.0
		255.0	0.1	2.484	0.2	0.7
		808.0	1.0	24.02	2.4	7.0
		2559.5	10.0	181.5	20.0	44.9
	1.0	25.0	0.01	0.025	0.0	0.0
		80.3	0.1	0.249	0.0	0.1
		255.5	1.0	2.476	0.2	0.7
		812.4	10.0	23.75	2.4	7.1
	10.0	7.6	0.01	0.002	0.0	0.0
		25.1	0.1	0.025	0.0	0.0
		80.8	1.0	0.246	0.0	0.1
		259.9	10.0	2.392	0.2	0.8
0.2	0.1	80.3	0.01	0.996	0.1	0.3
		255.0	0.1	9.827	1.0	2.9
		807.9	1.0	86.73	9.1	23.9
		2559.5	10.0	414.0	50.0	80.0
	1.0	25.0	0.01	0.100	0.0	0.0
		80.3	0.1	0.995	0.1	0.3
		255.5	1.0	9.793	1.0	2.9
		812.4	10.0	85.74	9.1	24.1
	10.0	7.6	0.01	0.010	0.0	0.0
		25.1	0.1	0.099	0.0	0.0
		80.8	1.0	0.984	0.1	0.3
		259.9	10.0	9.459	1.0	3.0

Table 3
Laser energies, cross sections and angular deviations of the photon energy and rate as function of photon and electron energy

E_γ (MeV)	E_L (eV)	T (MeV)	$\sigma(\theta_c)$ (mb) $\theta_c = 0.1$ mrad	S_e (%)	S_i (%)	$\sigma(\theta_c)$ (mb) $\theta_c = 0.2$ mrad	S_e (%)	S_i (%)
0.010	0.0041	400.0	6.074	0.61	1.82	23.65	2.40	6.97
0.100	0.0407		6.053	0.61	1.82	23.36	2.40	6.97
1.0	0.4080		6.025	0.61	1.81	23.25	2.40	6.96
10.0	4.174		5.777	0.60	1.79	22.50	2.34	6.86
0.010	0.0018	600.0	13.50	1.36	4.01	50.92	5.23	14.7
0.100	0.0181		13.41	1.36	4.01	49.58	5.23	14.6
1.0	0.1813		13.37	1.36	4.01	49.44	5.23	14.6
10.0	1.8409		13.06	1.34	3.97	49.29	5.15	14.5

3. Conclusion

If variable energy of γ-beam should be obtained, then the combination of a free electron laser with fixed energy electron accelerator offers almost constant quality of the γ-beam in the whole energy range. Therefore, this combination offers a better solution than the use of a laser with a fixed energy and electron accelerator with variable energy.

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