Operational Amplifiers

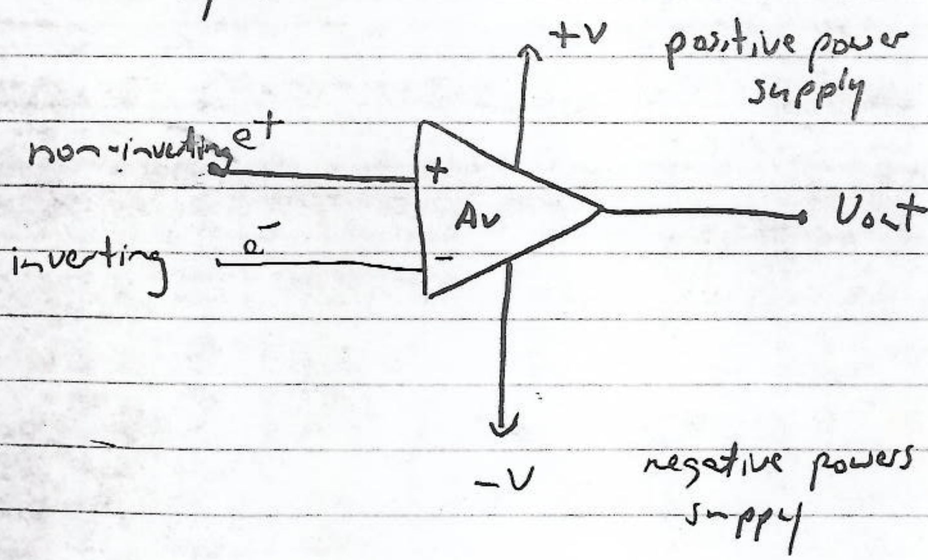
Some points:

integrated circuits

- many, many transistors
- black boxes, sort of
  - — can't take high power
  - — No big capacitors
- high input Z
- low output Z
- 2 inputs
- 1 output, high gain

Use feedback

Circuit symbol



Key thing

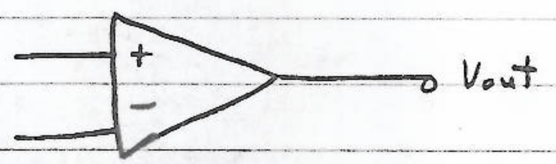
$$\boxed{V_{out} = A_0 \left(e^+ - e^- \right)}$$

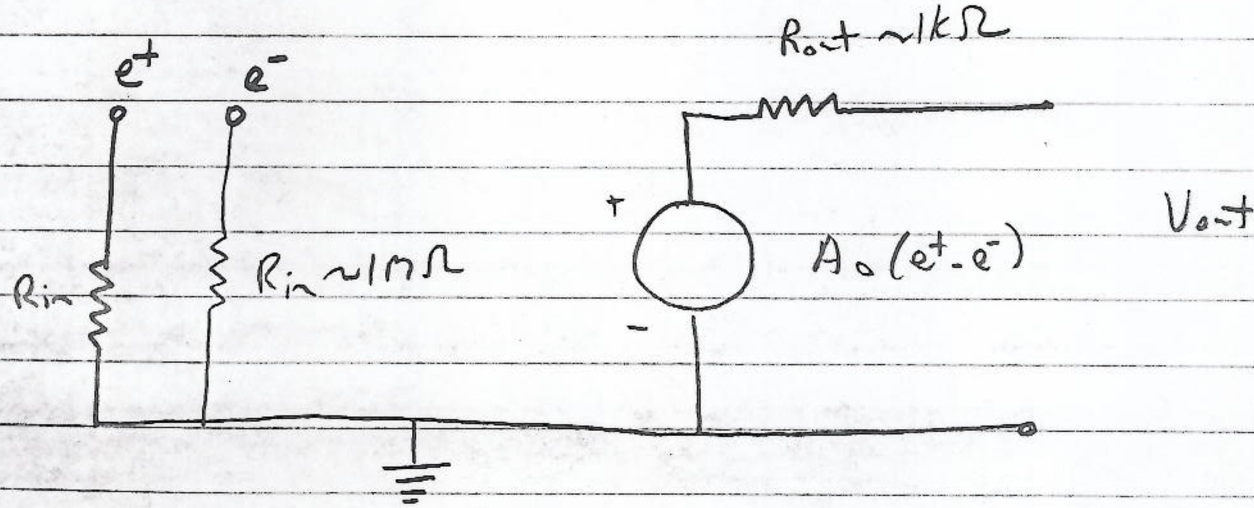
A_0: open loop voltage gain

for ideal op amp, $A_o = \infty$ reality $\sim 10^5$

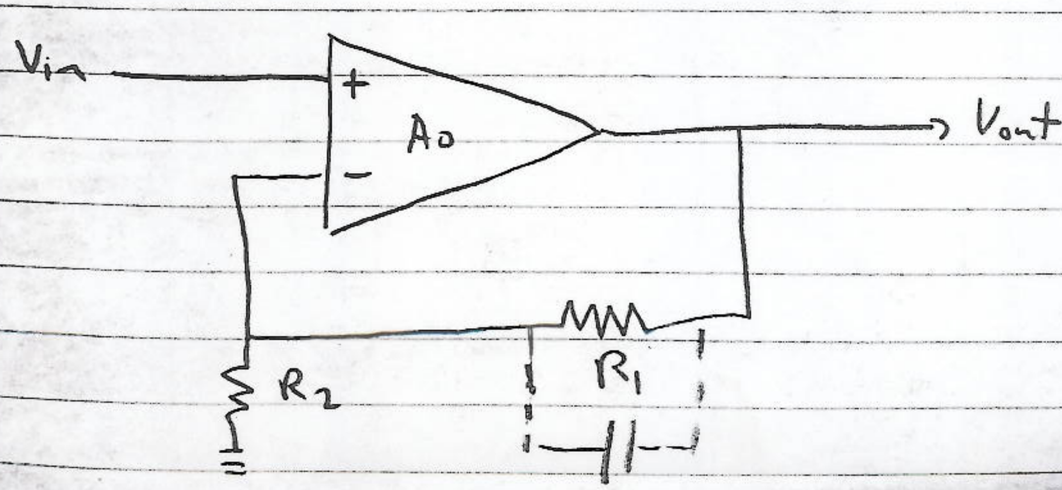
\+ input, signal at output in phase
− input inverted

Equivalent circuit:





Example 1: Linear Amplification



R_1, R_2 form negative feedback network.

Derive "transfer characteristic" ie $\frac{V_{out}}{V_{in}}$

1) $e^+ = V_{in}$

2) $e^- = V_{out} \frac{R_2}{R_1 + R_2}$

3) $V_{out} = A_o (e^+ - e^-) = A_o \left(V_{in} - V_{out} \frac{R_2}{R_1 + R_2} \right)$

$$V_{out} \left(1 + A_o \frac{R_2}{R_1 + R_2} \right) = A_o V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_o}{1 + A_o R_2 / (R_1 + R_2)}$$

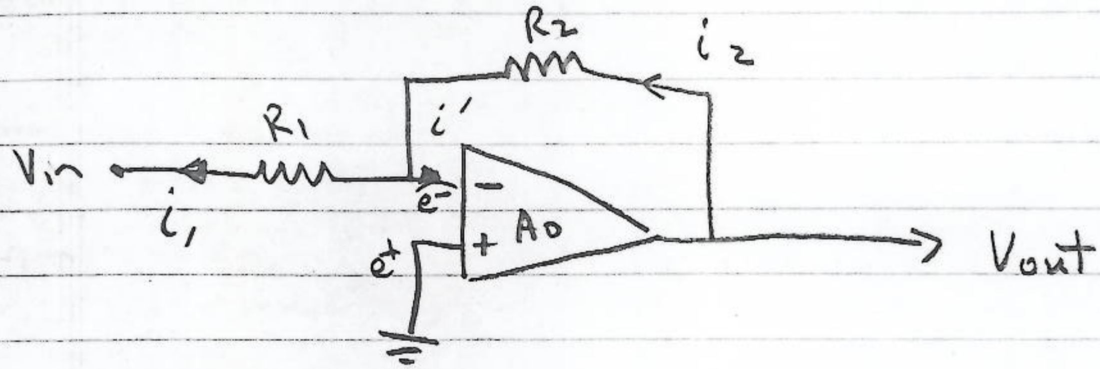
Take limit $A_o \to \infty$ (ideal opamp)

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

(1) Indep of parameters of op amp

(2) Assumed $Z_{in} \to \infty$ (op amp draws no current)

Example 2: Inverting Linear Amplifier



Feed back through R2

$$i_2 = i_1 + i'$$

$$\frac{V_{out} - e^-}{R_2} = \frac{e^- - V_{in}}{R_1} + i'$$

Always

$$V_{out} = A_0\left(e^+ - e^-\right)$$

$e^+ = 0$

$$V_{out} = -A_0 e^-$$

$i' \to 0$, eliminate e^-

$$V_{out} - e^- = \frac{R_2}{R_1}\left(e^- - V_{in}\right)$$

$$e^-\left(\frac{R_2}{R_1} + 1\right) = \frac{R_2}{R_1} V_{in} + V_{out}$$

$$V_{out} = -A_o \frac{V_{in}\frac{R_2}{R_1} + V_{out}}{R_2/R_1 + 1}$$

$$\left(\frac{R_2}{R_1}+1\right) V_{out} = -A_o\left(V_{in}\frac{R_2}{R_1} + V_{out}\right)$$

$$V_{out}\left(1 + A_o + \frac{R_2}{R_1}\right) = -A_o\frac{R_2}{R_1} V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{A_o R_2}{(A_o+1)R_1 + R_2}$$

For ideal (or even pretty good) op amp

$$(A_o+1)R_1 \gg R_2 \qquad A_o + 1 \simeq A_o$$

$$A_v = -\frac{R_2}{R_1}$$

inverting

Two key rules for op amps:

1) Current into inputs is zero. High input Z.
Op Amp current rule OACR

2) Voltage difference between inputs $e^+ - e^-$ is zero. Op Amp voltage rule OAVR or,

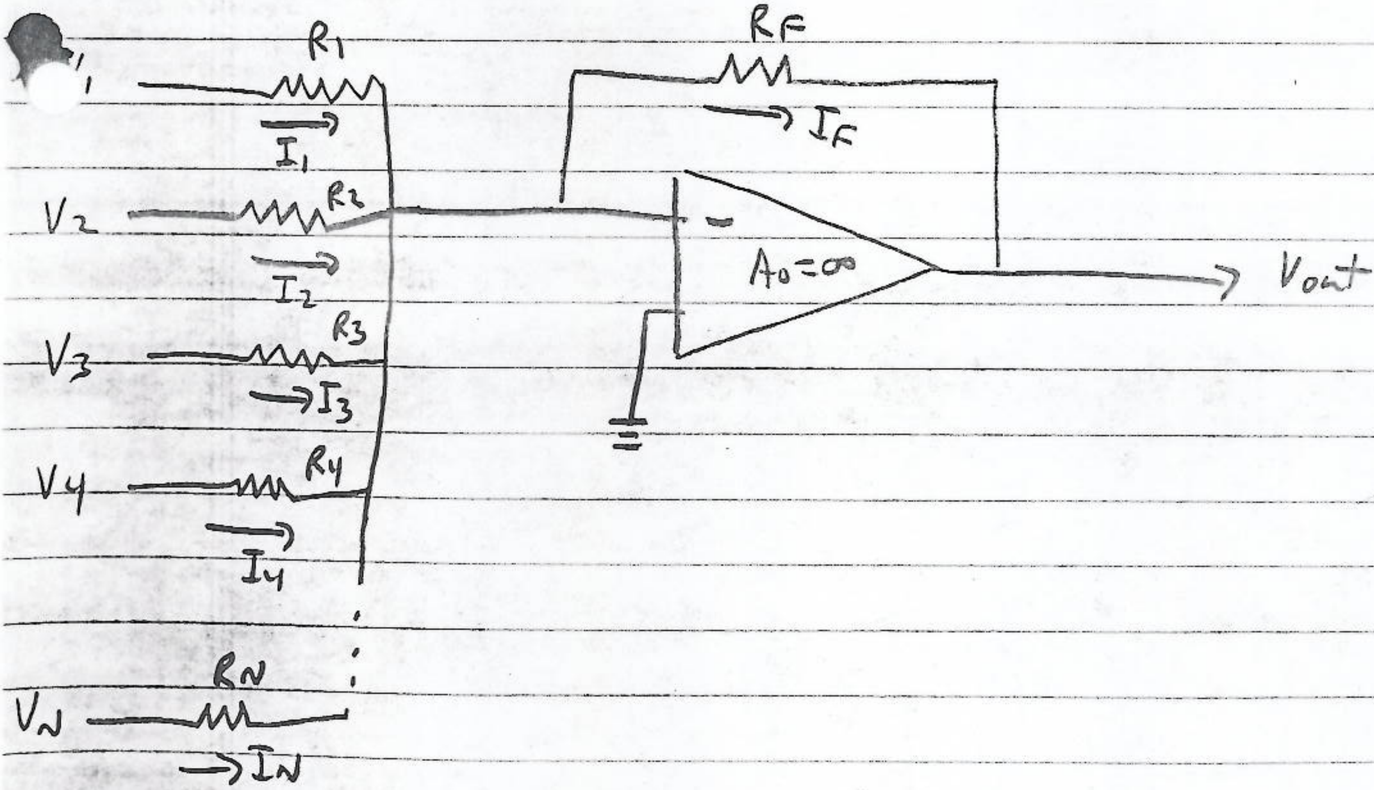
PVN - Principle of virtual null

Arguement

$$V_{out} = A_0(e^+ - e^-)$$

Power supplies to op amp $\sim \pm 15V$
V_{out} can't be ∞
If $A_0 \to \infty$ $\quad e^+ - e^- \to 0$

Example 3: Linear Summation with Gain



By P.V.N. $e^- = e^+ = 0$

So, each input voltage V_i contributes

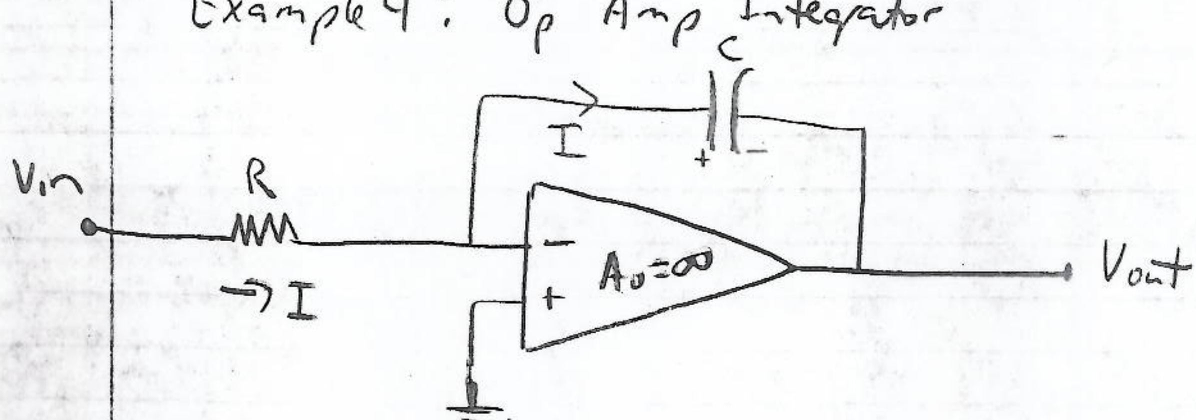
$$I_i = \frac{V_i}{R_i}$$

which goes through R_F

$$V_{out} = -R_F \sum_{i=1}^{N} I_i = -R_F \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots \frac{V_N}{R_N} \right)$$

Adds voltages, multiplied by const $-\frac{R_f}{R_1}$

Example 4: Op Amp Integrator



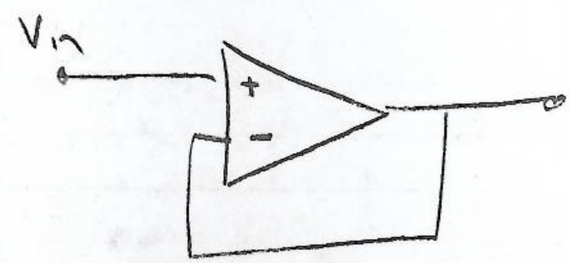
$e^- = e^+ = 0 \qquad I = V_{in}/R \qquad C = Q/V$

$$V_{out} = -V_c = -\frac{Q}{C} = -\frac{1}{C} \int I\,dt$$

If $V_c = 0$ @ $t = 0$

$$\boxed{V_{out}(t) = -\frac{1}{RC}\int_{t=0}^{t} V_{in}\,dt}$$

Example 5: Voltage Follower



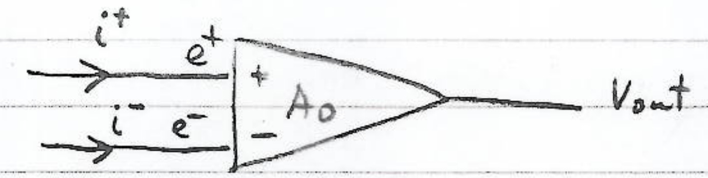
$V_{out} = V_{in}$

$$\underset{\substack{\uparrow \\ \text{PVN}}}{V_{in} = e^+ = e^-} = V_{out}$$

gain = 1, but input impedance large

Better than emitter follower.

Op Amps: key Points



(1) $i^+ = i^- = 0$ (ideal)

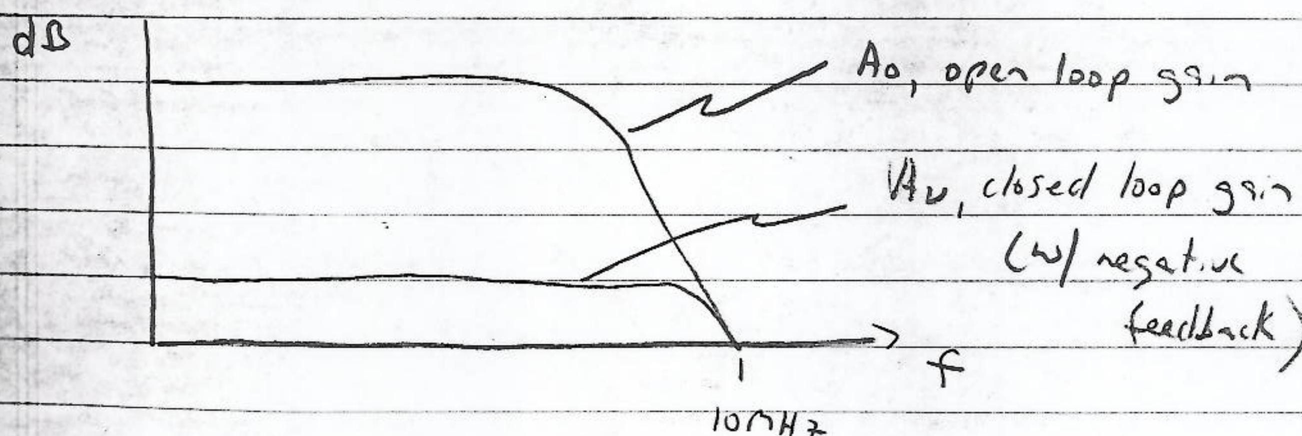
(2) $e^+ = e^-$. (ideal)

(3) $V_{out} = A_0 (e^+ - e^-)$ (ideal)

$A_0 \to \infty$

Real Op Amps

Gain & Bandwidth



$f \uparrow$ $A_0 \downarrow$ approximations break down

Slew rate

$$\frac{dV}{dt} = \frac{dQ}{dt}\frac{1}{C} = \frac{I}{C}$$

some capacitance, either inside IC or load

For sine wave

$$\frac{dV}{dt} = \frac{d(V_0 \sin\omega t)}{dt} = \omega V_0 \cos\omega t$$

$$\left(\frac{dV}{dt}\right)_{max} = \omega V_0$$

frequency amplitude

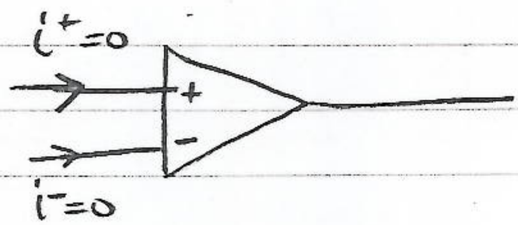
Highest ωV_0 op amp can handle — slew rate

S.R. ~ 0.5 V/μs – 500 V/μs

$f = \frac{S.R.}{2\pi V_0}$ If f higher → distortion

Input Bias Current

Ideal

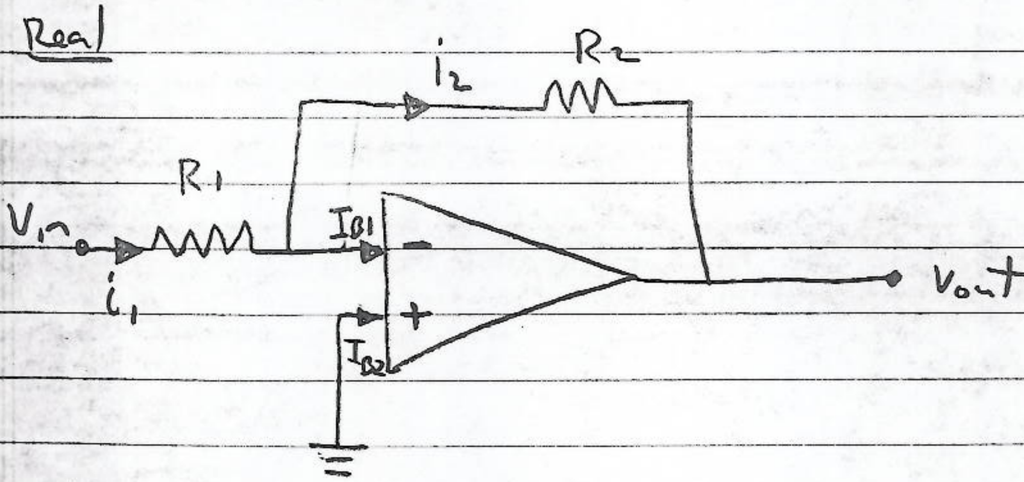


Real

Input	i
BPJT	10 – 100 nA
JFET	1 – 500 pA
MOSFET	1 – 10 pA

needed to bias input transistor

Real



$$i_1 = I_{B1} + i_2$$

$$\frac{V_{in} - e^-}{R_1} = I_{B1} + \frac{e^- - V_{out}}{R_2}$$

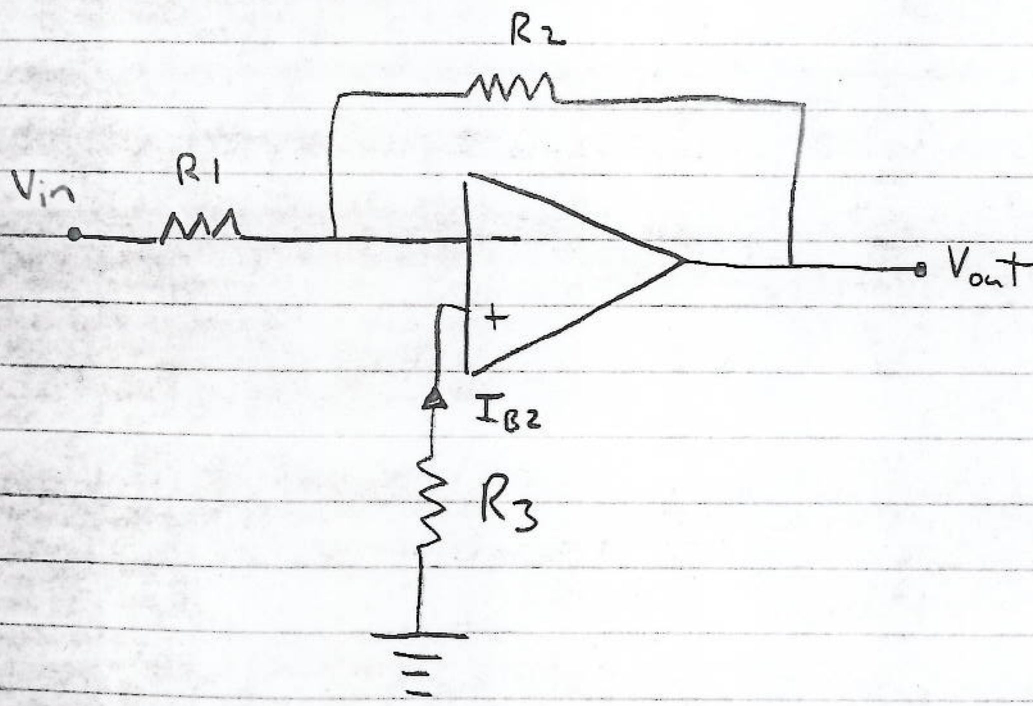
$$e^+ = e^- = 0$$

$$\frac{V_{in}}{R_1} = I_{B1} - \frac{V_{out}}{R_2}$$

$$V_{out} = -\frac{R_2}{R_1} V_{in} + \underbrace{I_{B1} R_2}_{\text{New}}$$

$$V_{out} = -\frac{R_2}{R_1}\left[V_{in} - I_{B1} R_1\right]$$

Either live with it or, trick:



$$e^+ = -I_{B2} R_3$$

Go back to

$$\frac{V_{in} - e^-}{R_1} = I_{B1} + \frac{e^- - V_{out}}{R_2}$$

e^+, e^- not longer zero

when $V_{in} = 0$ want $V_{out} = 0$

$$\frac{-e^-}{R_1} = I_{B1} + \frac{e^-}{R_2}$$

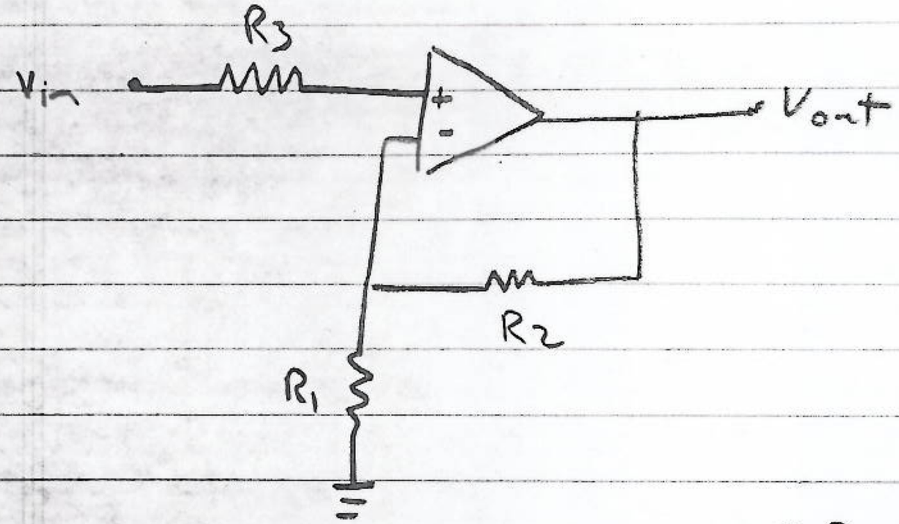
$$e^-\left(1 + \frac{R_2}{R_1}\right) + I_{B1} R_2 = 0$$

$$I_{B1} \approx I_{B2}$$

$$I_{B2} R_3 \left(1 + \frac{R_2}{R_1}\right) = I_{B2} R_2$$

$$R_3 = \frac{R_2}{1 + R_2/R_1} = \frac{R_1 R_2}{R_1 + R_2}$$

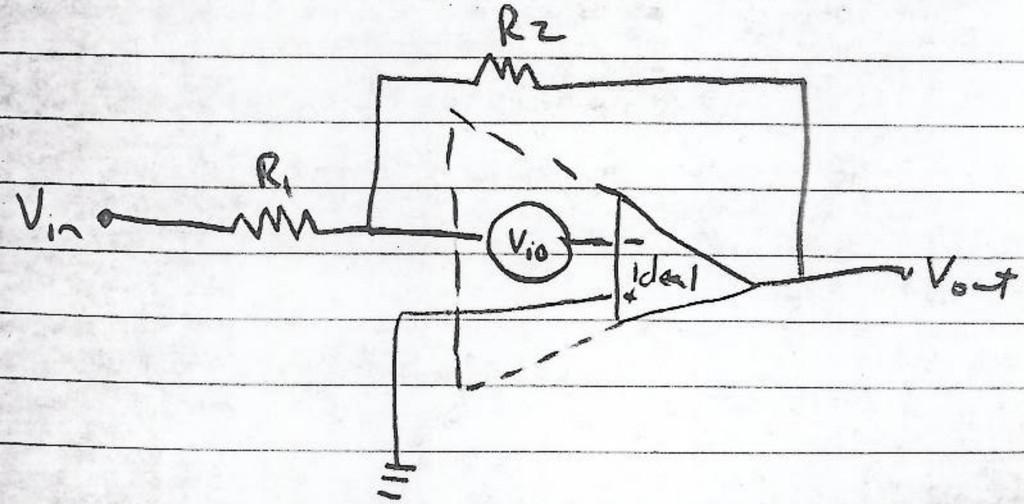
For noninverting



Similar arguement $R_3 = \frac{R_1 R_2}{R_1 + R_2}$

Input Offset Voltage

Two inputs → different B-E characteristics



ICBST

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{io}$$

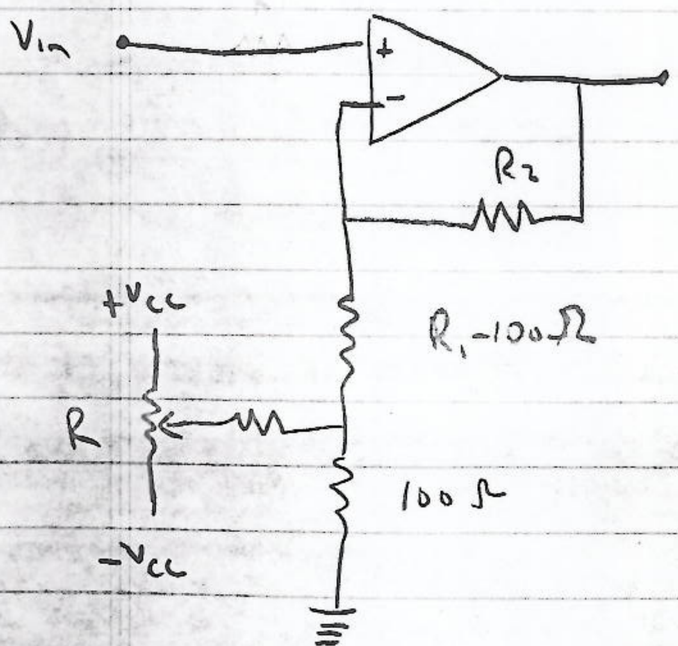
~1mV

w/ no input

big

The fix

non inverting offset null circuit



adjust R so

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

Total noninverting

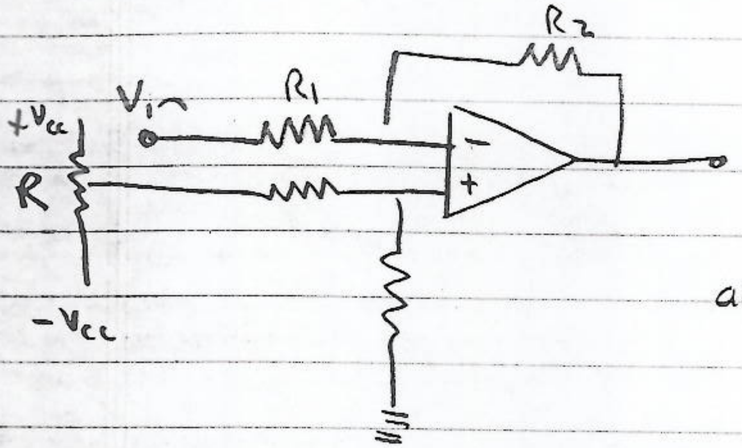
$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in} \pm \left(1 + \frac{R_2}{R_1}\right) V_{io} + I_B R_2$$

good guy

input offset, bad guy, eliminated w/ null circuit

input bias current, bad guy, fix w/ "R_3"

The Fix: inverting offset null circuit



adjust R so

$$V_{out} = -\left(\frac{R_2}{R_1}\right) V_{in}$$

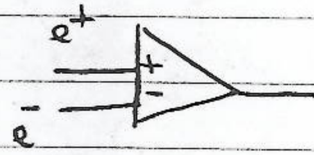
Total inverting

$$V_{out} = -\left(\frac{R_2}{R_1}\right) V_{in} \pm \left(1+\frac{R_2}{R_1}\right) V_{io} + I_B R_2$$

usual stuff — input offset — input bias current

Common Mode Rejection Ratio (CMRR)

Ideally



$e^+ = e^-$

$$V_{out} = A_o\,(e^+ - e^-)$$

A_o ← differential gain

For real op amp

$e^+ = e^- \rightarrow$ small V_{out}

$\rightarrow A_{cm}$ common mode gain

$A_{cm} = 0$ ideal

Common mode rejection ratio

$$CMRR = \frac{A_{diff}}{A_{cm}}$$

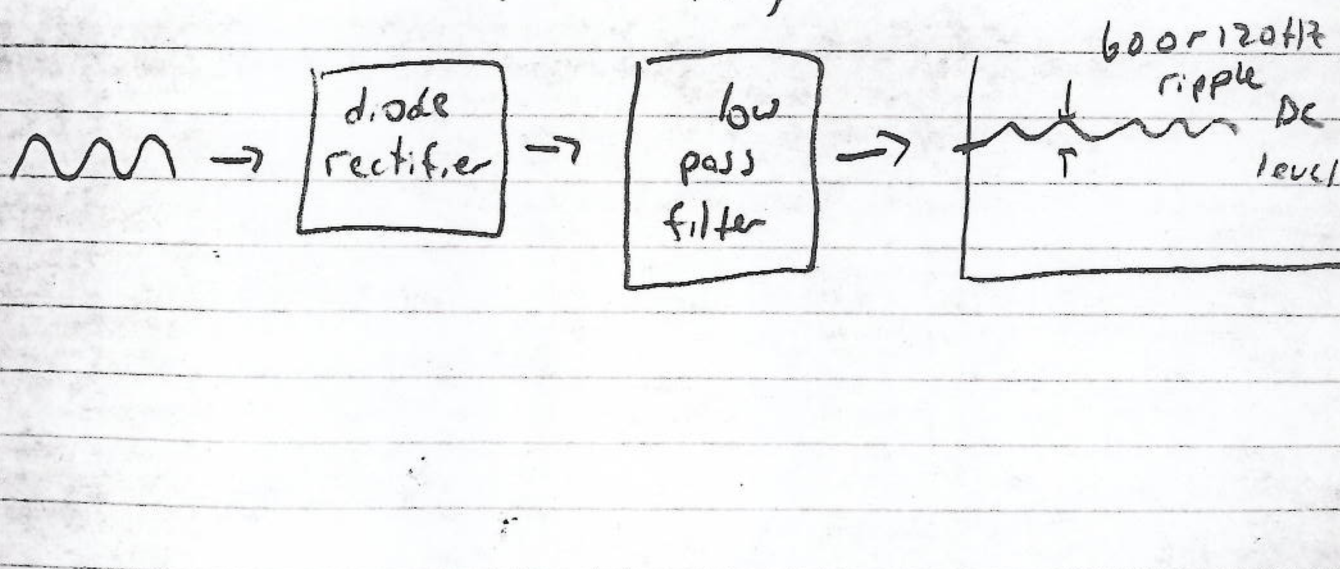
A_{diff} ← differential

A_{cm} ← common mode

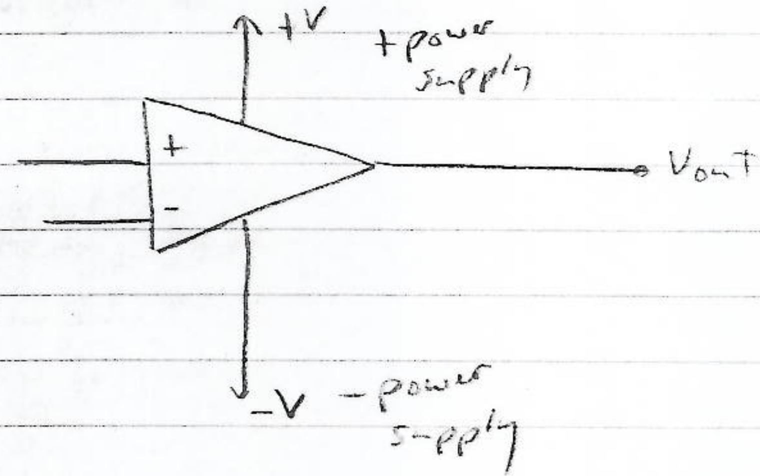
ideal: $= \frac{\infty}{0}$

Power Supply Rejection Ratio

Remember our D.C. power supply



Op Amp



Ripple in power supply affects output
→ effective change in V_{io}

$$PSRR = \frac{\Delta V_{io}}{\Delta V_{cc}}$$

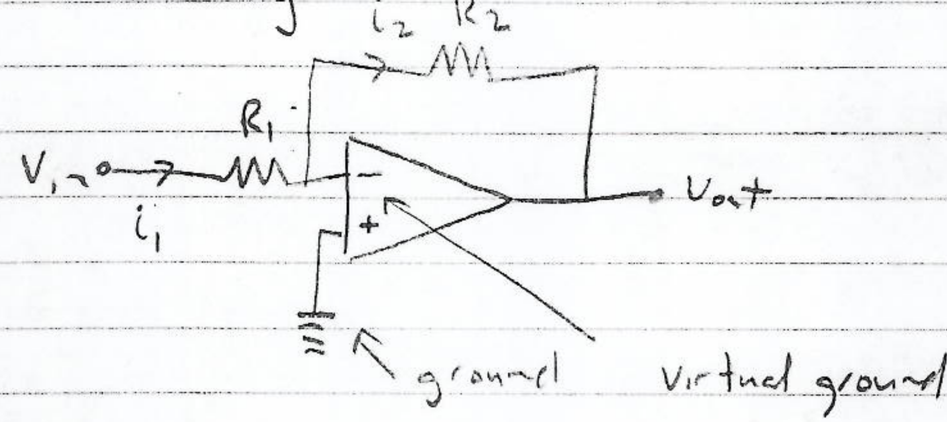
ideal = 0 or $-\infty$ dB

Applications

Rule 1: No currents into inputs

Rule 2: Magically $e^+ = e^-$
PVN

Inverting

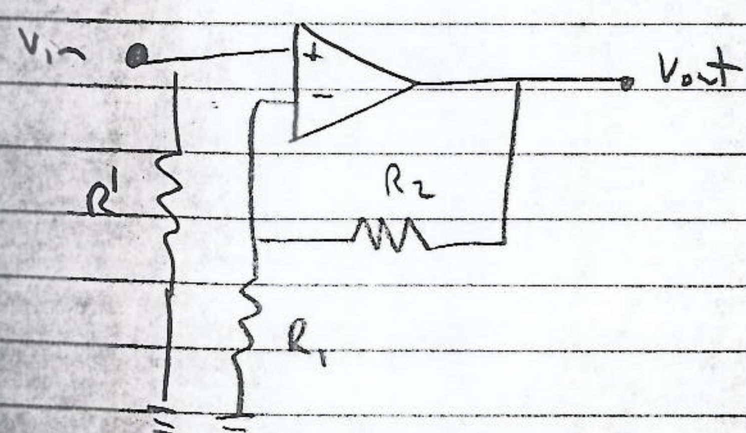


Input R is R_1

$$i_1 = i_2$$

$$\frac{V_{in}}{R_1} = \frac{-V_{out}}{R_2} \qquad \frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1}$$

Non Inverting



What is B?

$$\frac{R_1}{R_1+R_2} \qquad \frac{V_{out}}{V_{in}} = \frac{1}{B} = 1 + \frac{R_2}{R_1}$$

$$e^+ = e^-$$

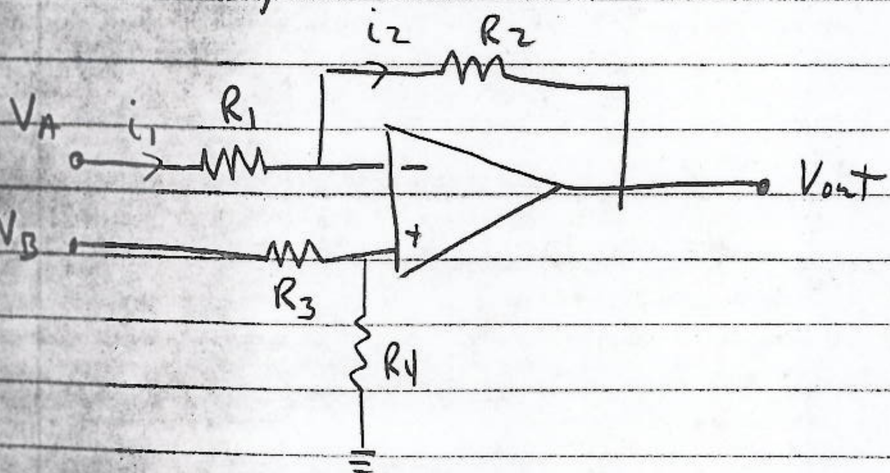
$$V_{in} = \frac{R_1}{R_1+R_2} V_{out}$$

PVN

The Differential Amplifier

$$V_{out} = A_2 (V_B - V_A)$$

Good if
- Two wires have same pickup - twisted pairs
- High DC levels



$i_1 = i_2$

$$\frac{V_A - e^-}{R_1} = \frac{e^- - V_{out}}{R_2}$$

$$e^+ = \frac{R_4}{R_3 + R_4} V_B$$

Eliminate $e^+ = e^-$

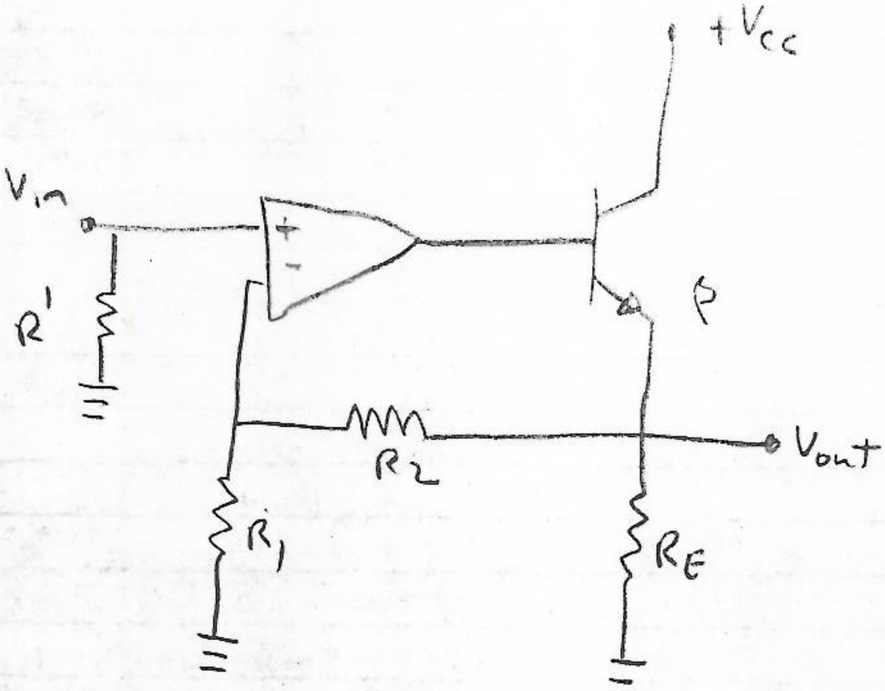
$$V_{out} = \frac{R_2}{R_1} \frac{1 + \frac{R_1}{R_2}}{1 + R_3/R_4} (V_B - V_A)$$

If $R_1/R_2 = R_3/R_4$

$$V_{out} = \frac{R_2}{R_1} (V_B - V_A)$$

Op Amp Power Booster

- Op Amp current limit ~20mA
- Want to boost power



Only works for +Vin

For − swings, replace

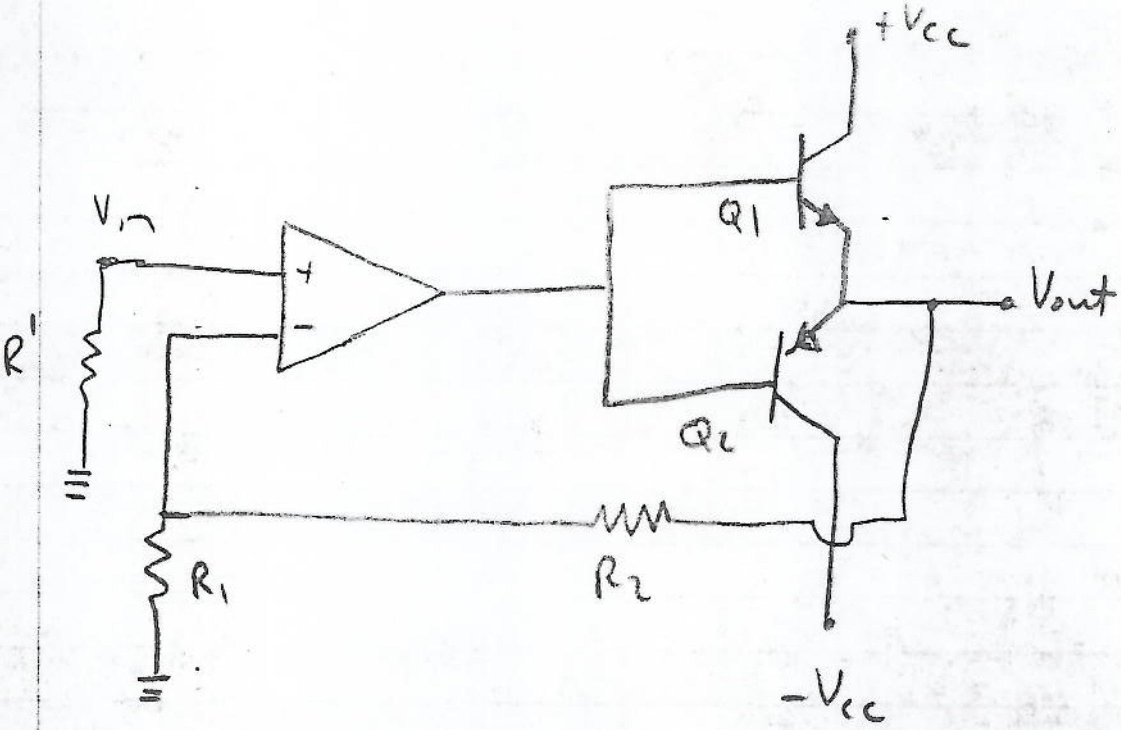
with

or Push-Pull

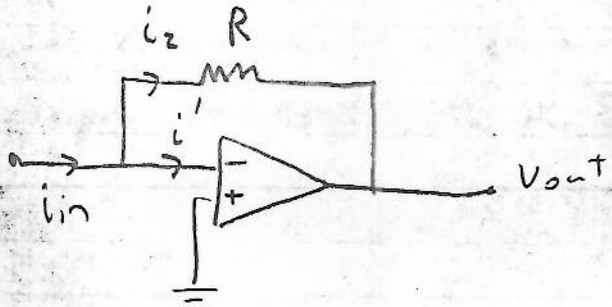
185

*

skipped



Current-to-Voltage Converter



$$i_2 = \frac{e^- - V_{out}}{R} = -\frac{V_{out}}{R}$$

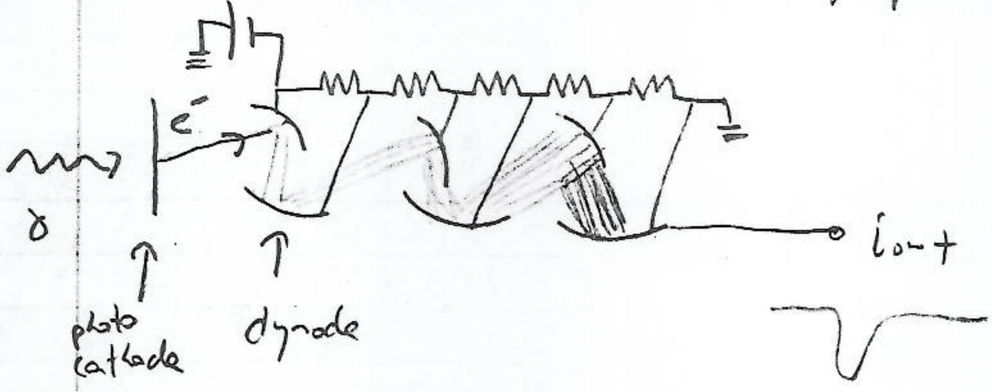
$$i_2 = i_{in}$$

$$V_{out} = -i_{in} R$$

i_{in} converted to voltage

For example Photomultiplier tube PMT

How to measure a single photon

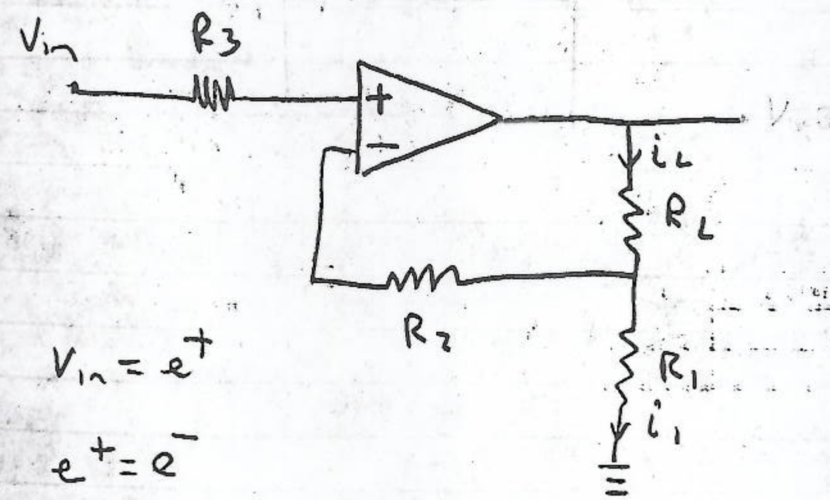


↑

e⁻ ejected via photoelectric effect

⟶ send i_{out} to i-to-v converter

V to I converter



$V_{in} = e^+$

$e^+ = e^-$

$V_{in} = e^+ = e^-$

$V_{R_2} = 0$

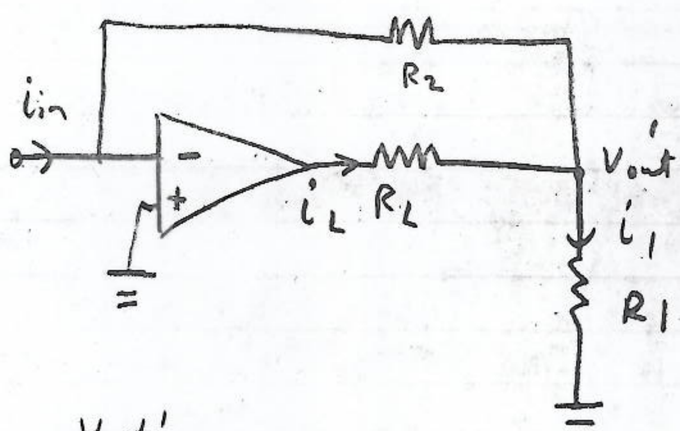
$e^- = i_1 R_1$

$i_1 = i_L$

$V_{in} = i_L R_1 \qquad i_L = \frac{V_{in}}{R_1}$

- No current drawn from source
- Floating load
- Indep of R_L

Current-to-Current Converter



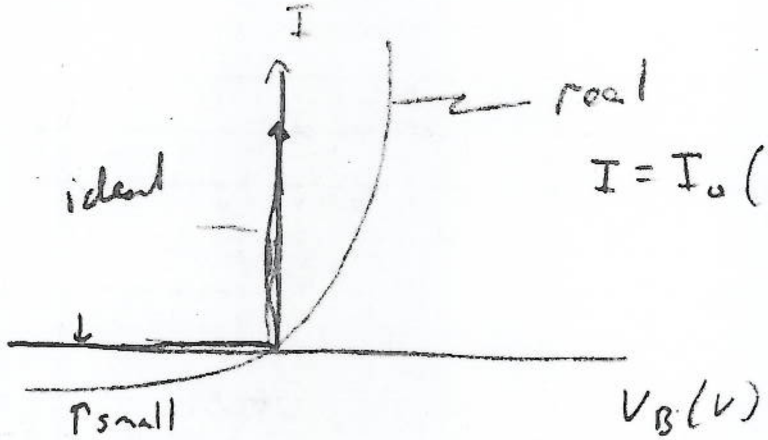
$i_{in} = -\frac{V_{out}'}{R_2}$

$i_{in} + i_L = i_1 = \frac{V_{out}'}{R_1}$

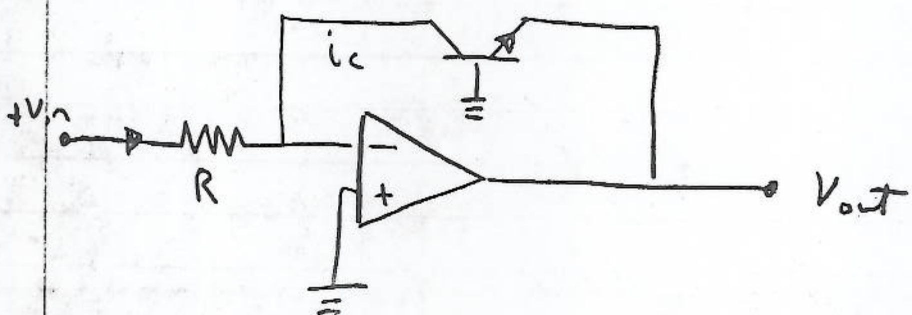
$i_{in} + i_L = \frac{-i_{in} R_2}{R_1} \longrightarrow \boxed{i_L = -i_{in}\left(1 + \frac{R_2}{R_1}\right)}$ both ends R_L above ground

The Logarithmic Amplifier : $V_{out} \propto \log V_{in}$

Recall our p-n junction diode



$$I = I_0\left(e^{eV_B/kT} - 1\right)$$



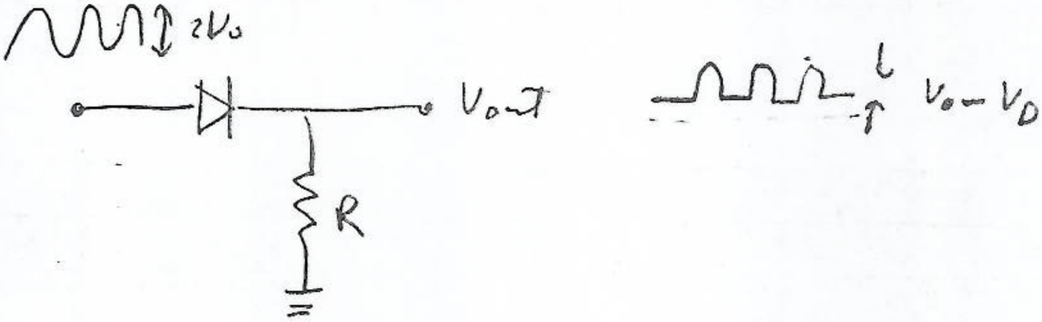
$$i_c = \frac{V_{in}}{R} \approx I_0 \, e^{eV_{BE}/kT}$$

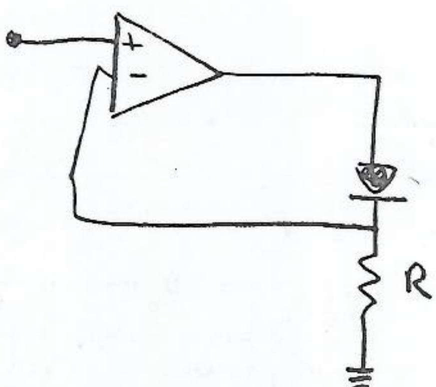
$$V_{BE} = -V_{out}$$

$$\frac{V_{in}}{R} = I_0 \, e^{-eV_{out}/kT}$$

$$V_{out} = -\frac{kT}{e} \ln\left(\frac{i_{in}}{I_0}\right)$$

Making a diode closer to ideal





e^+ goes + by a little $\quad V_{out} = A_0(e^+ - e^-)$

V_{out} goes + by a lot

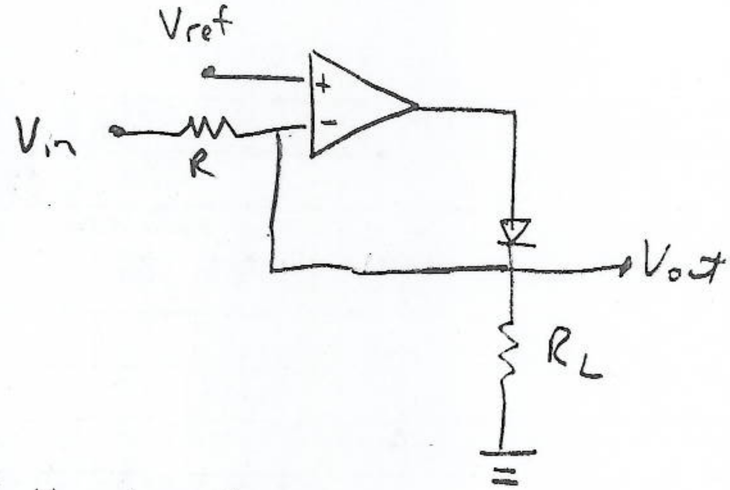
→ diode turns on

e^+ goes − by a little

V_{out} goes −

→ diode off

Diode Clamp



If $V_{in} < V_{ref}$

$$V_{out} = A_o(e^+ - e^-)$$

Op amp → +, diode on

$V_{out} = V_{ref}$ clamped

If $V_{in} > V_{ref}$

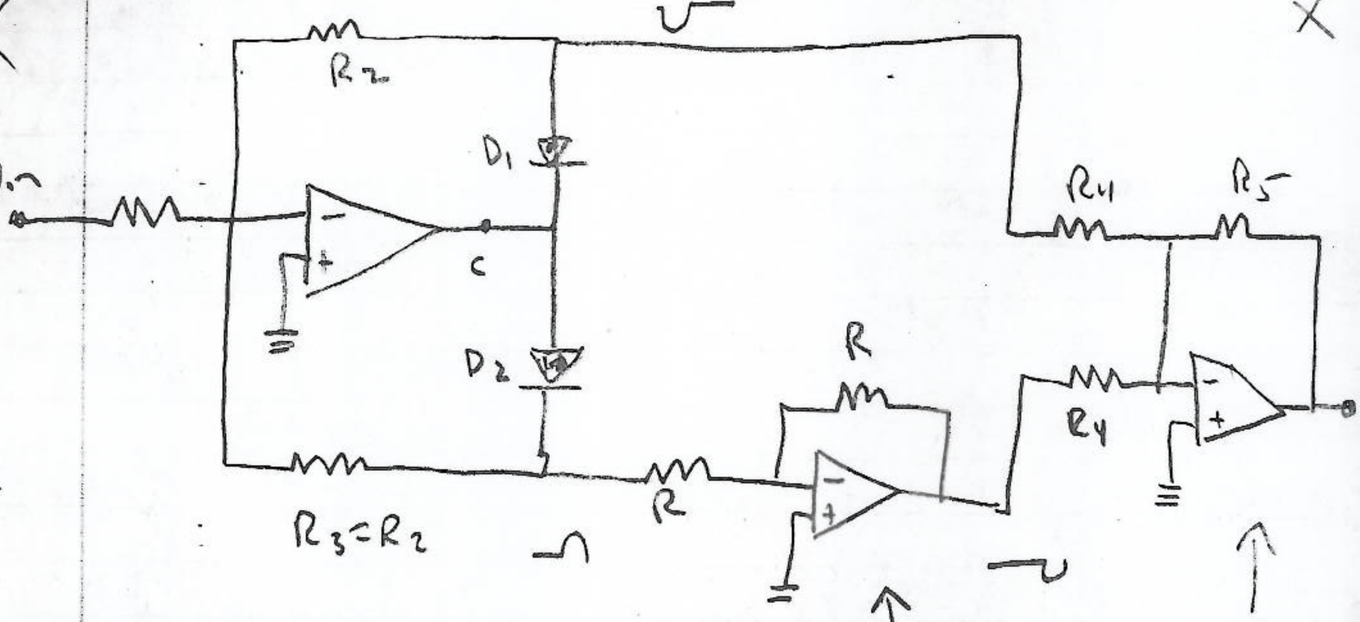
op amp → −, diode off

$$V_{out} = \frac{R_L V_{in}}{R + R_L}$$

$R \ll R_L$ $\quad V_{out} = V_{in}$

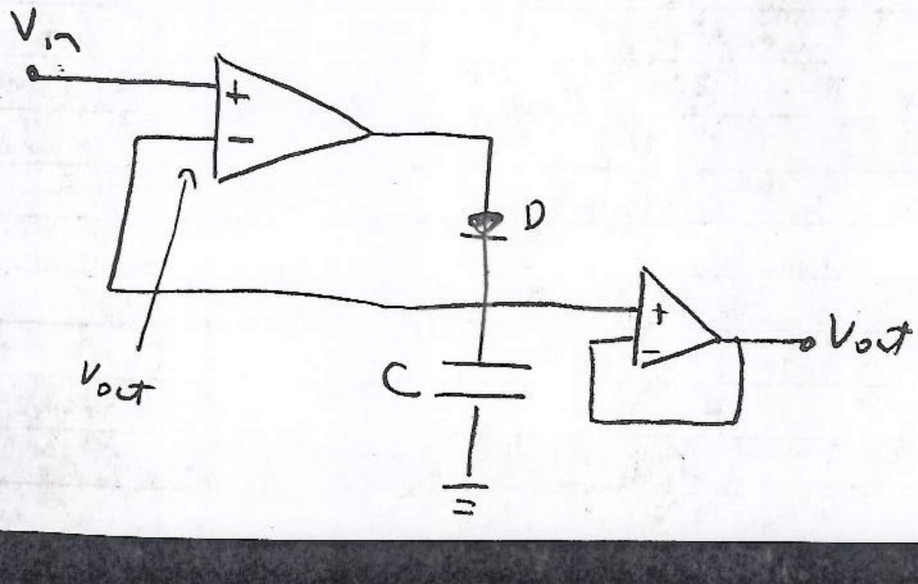
Full wave rectifier

skip



V_{in}	V_c	D_1	D_2
+	− 0.6V	on	off
−	+	off	on

Peak detector



If $V_{in} > V_{out}$
D turns on

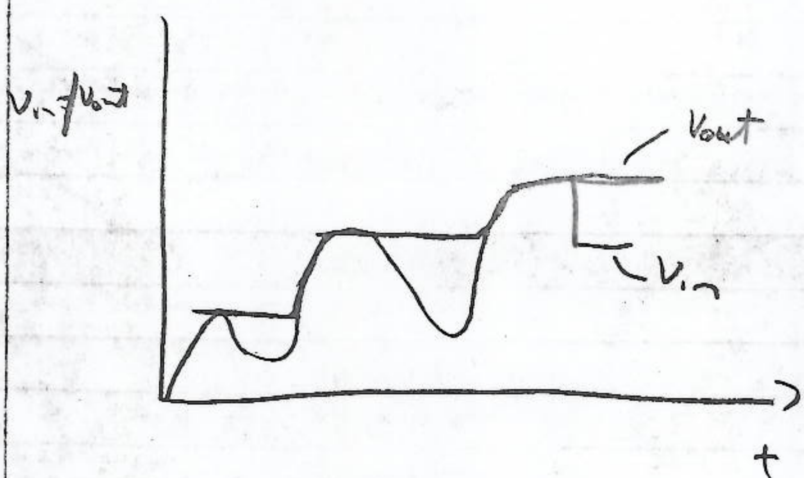
C charged to new voltage

If V_{in} gets even bigger

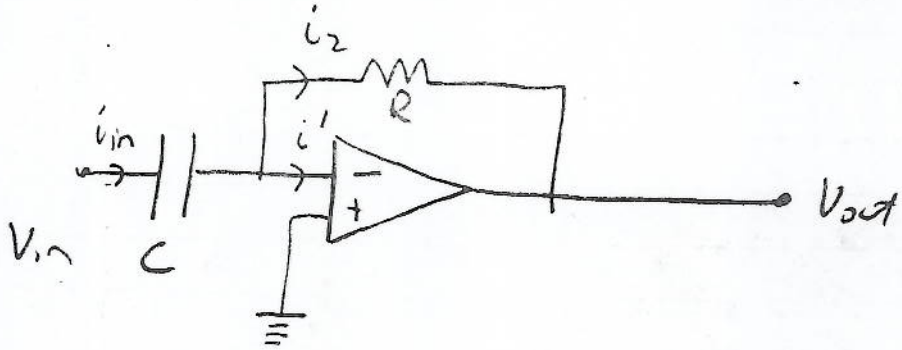
C charged to new bigger voltage

If $V_{in} < V_{out}$

D off
C sits there still holding biggest voltage



Op Amp Differentiator



$$i_{in} = i_2 + i'$$

$$\frac{dQ}{dt} = i_{in} = \frac{-V_{out}}{R}$$

$$Q = CV_{in}$$

$$\frac{d(CV_{in})}{dt} = -\frac{V_{out}}{R}$$

$$V_{out} = -RC\frac{dV_{in}}{dt}$$

No restrictions on RC

Op Amp Comparators

Recall

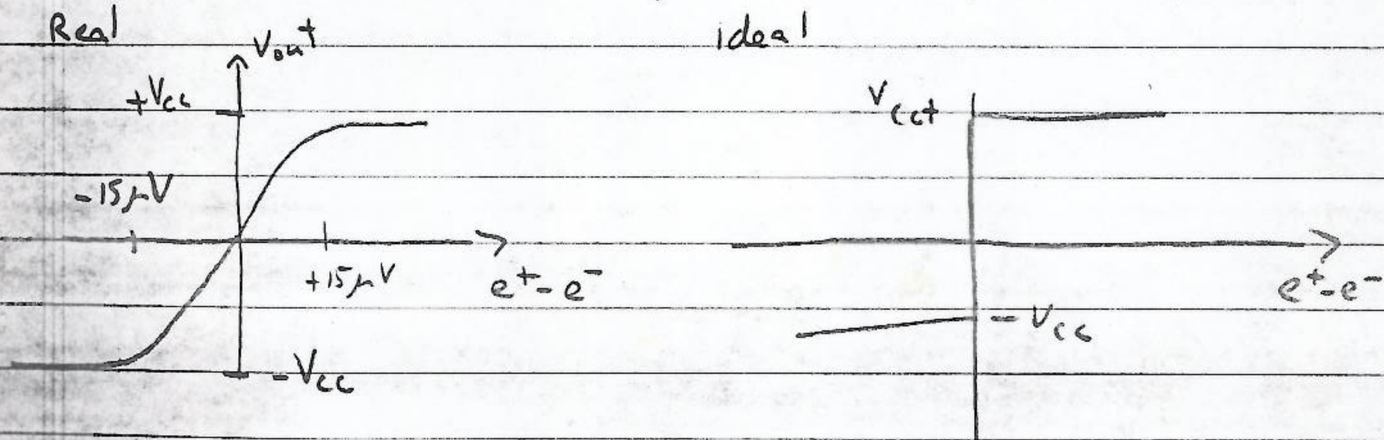
$$V_{out} = A_o (e^+ - e^-)$$

e^+, e^-: voltages at terminals

$A_o \sim 10^5 - 10^6$

$V_{out\,max} \sim 13V$

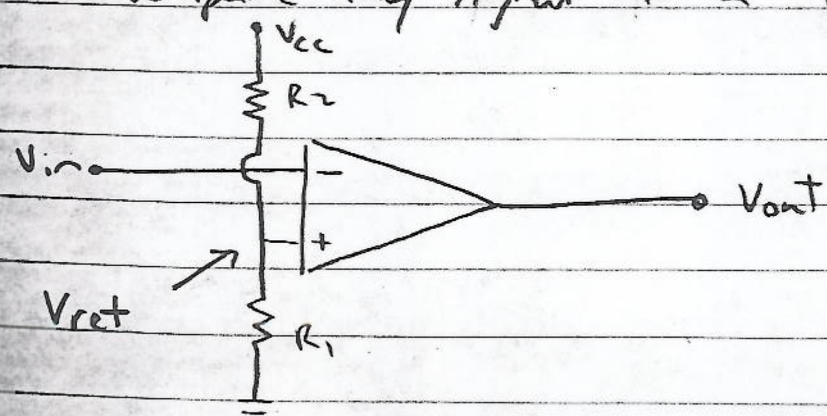
If $e^+ - e^- > \frac{15V}{10^6} \approx 15\mu V$, op amp output pinned



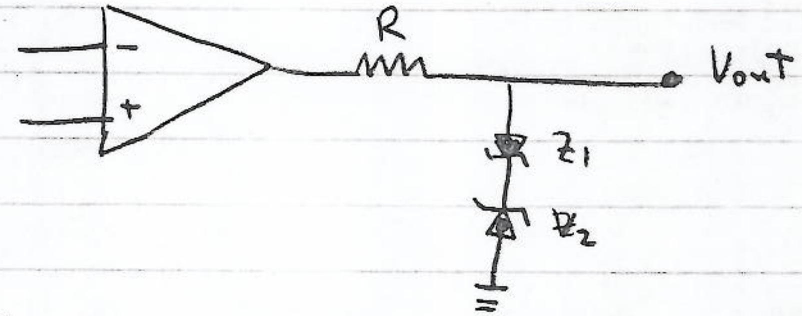
Comparator → tells which input is bigger

slew rates

Can compare any signal to a <u>reference</u> <u>voltage</u>



Can clip output

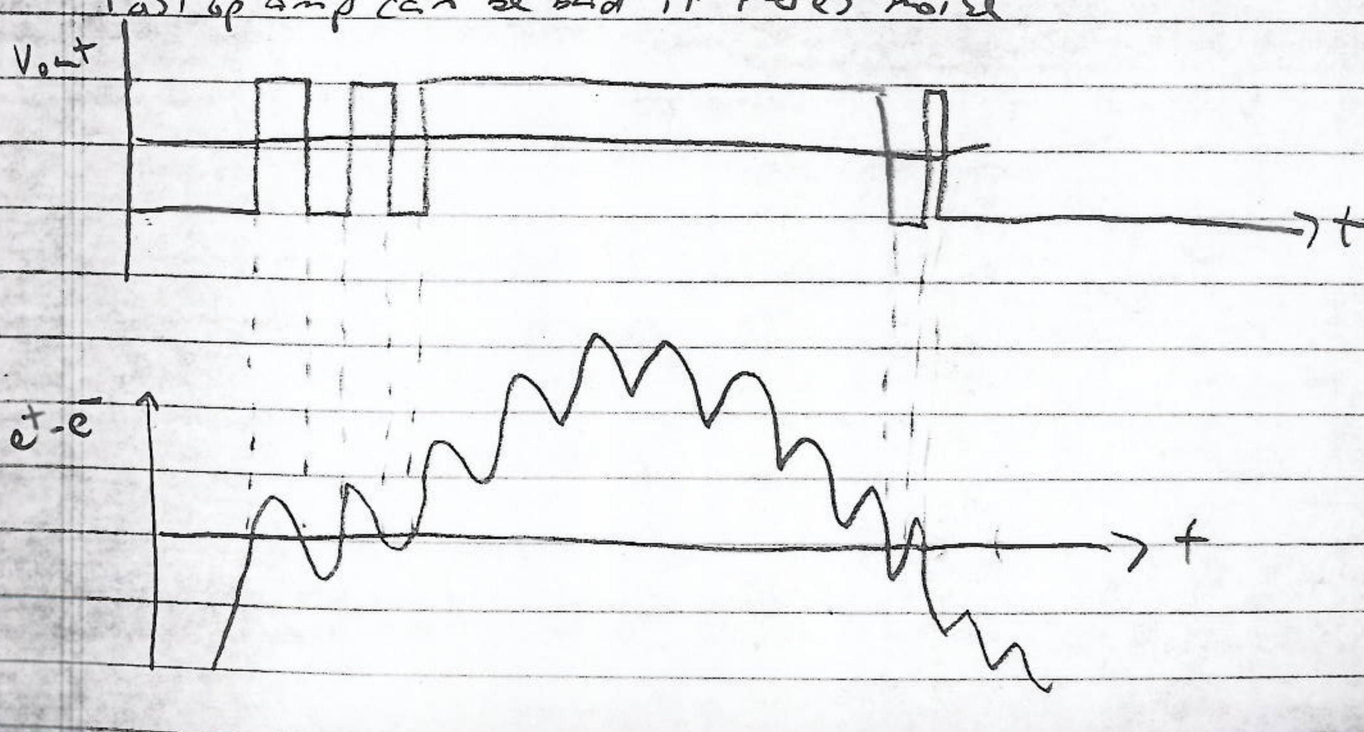


$$-(V_D + V_{Z1}) < V_{out} < V_D + V_{Z2}$$

Slew rates:

ordinary op amp   1-5 V/μsec
specially fast   500-1000 V/μsec

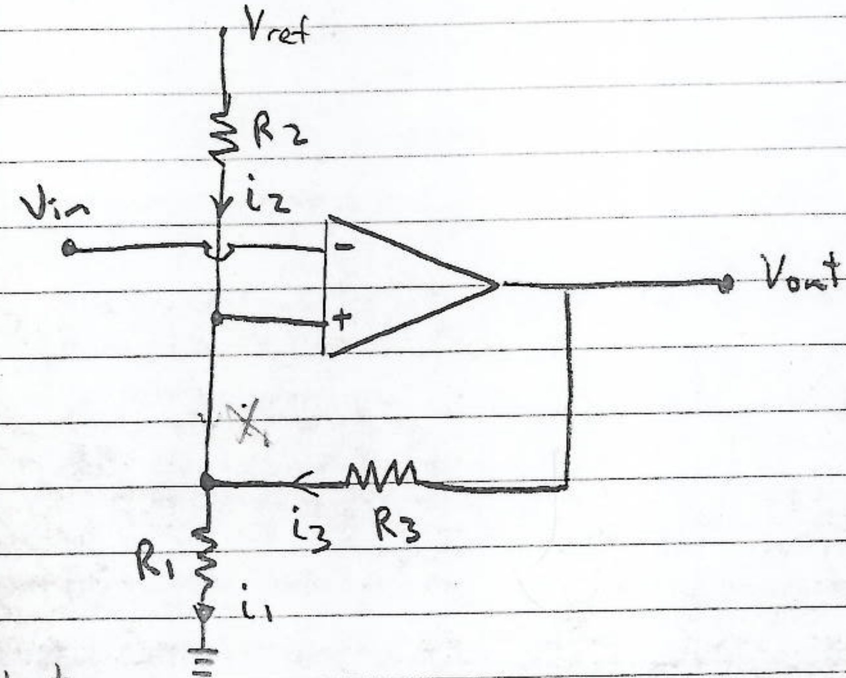
Fast op amp can be bad if there's noise



Trick to solve this:

The Schmitt Trigger

exhibits hysteresis



$V_{thresh} = e^+$, depends on V_{out}

whether or not comparator flips depends upon previous history.

$$i_2 + i_3 = i_1$$

$$\frac{V_{ref} - e^+}{R_2} + \frac{V_{out} - e^+}{R_3} = \frac{e^+}{R_1}$$

$$\frac{V_{ref}}{R_2} + \frac{V_{out}}{R_3} = e^+\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{e^+}{R_{123}}$$

$$e^+ = \frac{R_{123}}{R_2} V_{ref} + \frac{R_{123}}{R_3} V_{out}$$

- Threshold depends on V_{out}

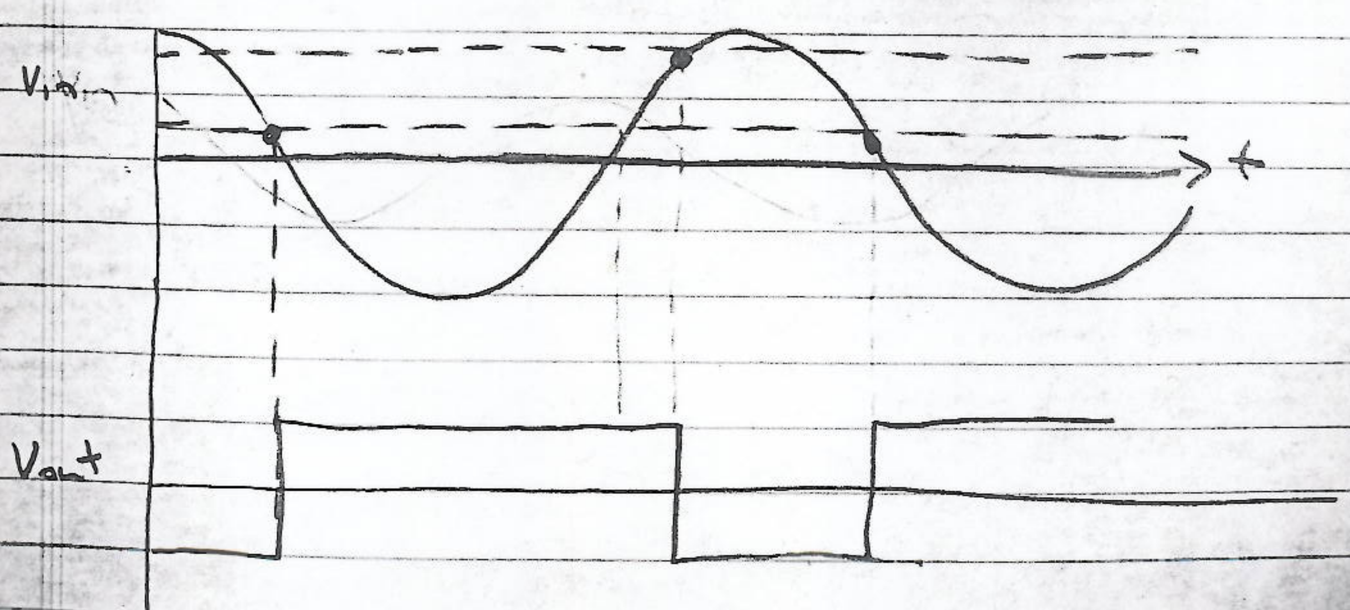
V_{out} high

$$e^+ = \frac{R_{123}}{R_2} V_{ref} + \frac{R_{123}}{R_3} V_{cc}$$

V_{out} low

$$e^+ = \frac{R_{123}}{R_2} V_{ref} - \frac{R_{123}}{R_3} V_{cc}$$

$$\Delta \text{ threshold} = \frac{2R_{123}}{R_3} V_{cc}$$



Two kinds

(1) Peak sensing – maximum voltage digitized
(2) charge sensitive – total integrated current digitized

Best technique: successive approximations
→ compare incoming pulse to a bunch of reference voltages

Example:

Suppose ADC accepts 0–10V input
Let's give it an 8V signal

(1) Compare input to 5V
$\boxed{\text{bigger}} \rightarrow$ set 1st bit to 1

(2) Compare to $5V + \frac{1}{2}5V = 7.5V$
$\boxed{\text{bigger}} \rightarrow$ set 2nd bit to 1

(3) Compare to $5V + \frac{1}{2}5V + \frac{1}{4}5V = 8.75V$
$\boxed{\text{smaller}} \rightarrow$ set 3rd bit to 0

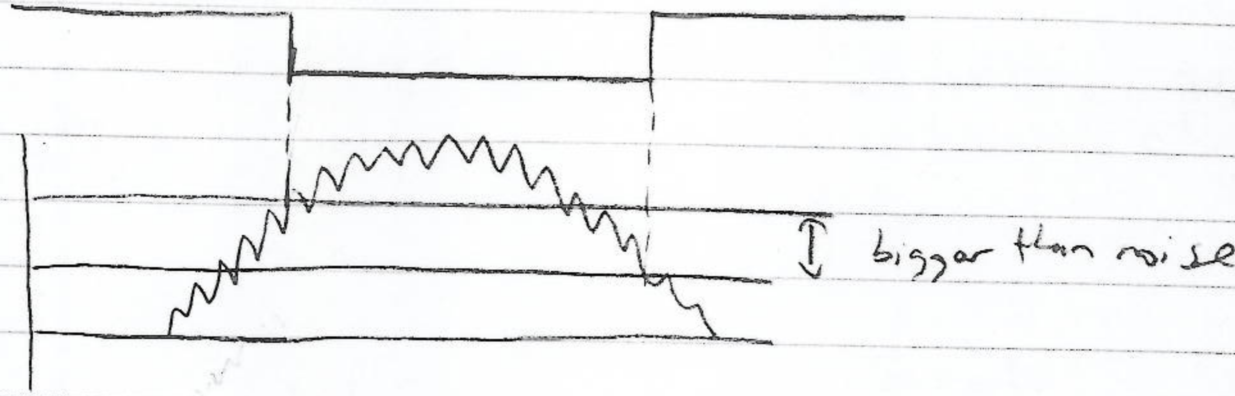
(4) Compare to $5V + \frac{1}{2}5V + \frac{1}{4}5V - \frac{1}{8}5V$

→ etc

Do until required accuracy (# of bits) is obtained.

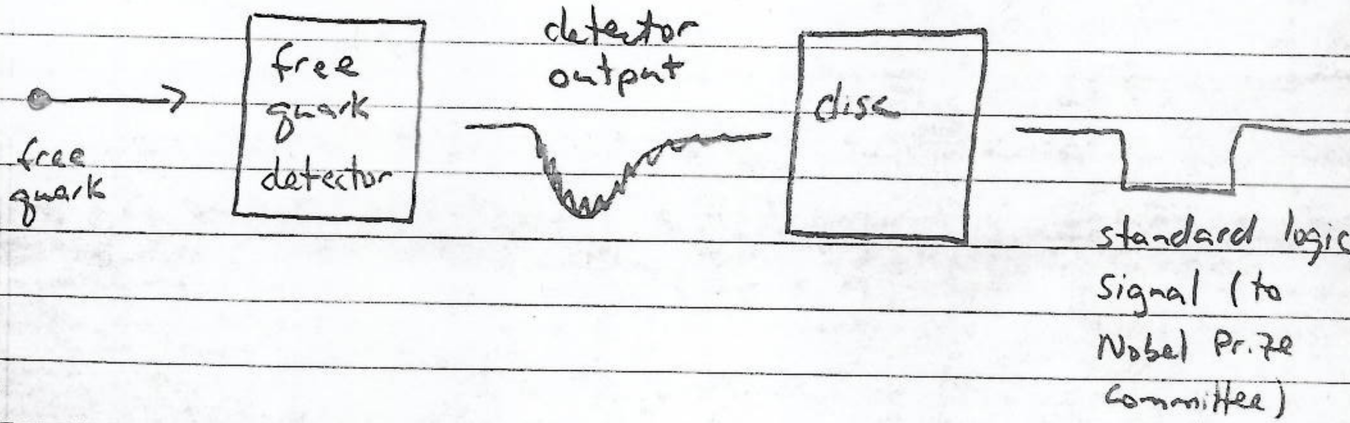
So what?

noise



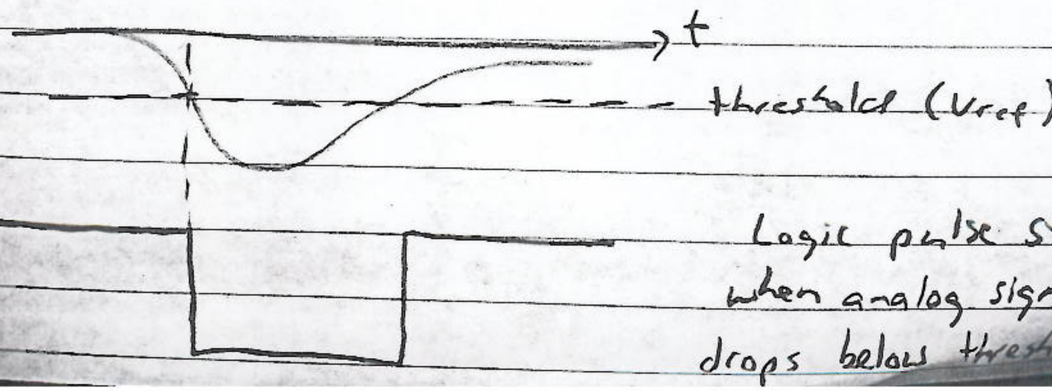
Comparator Examples

<u>Discriminators</u>



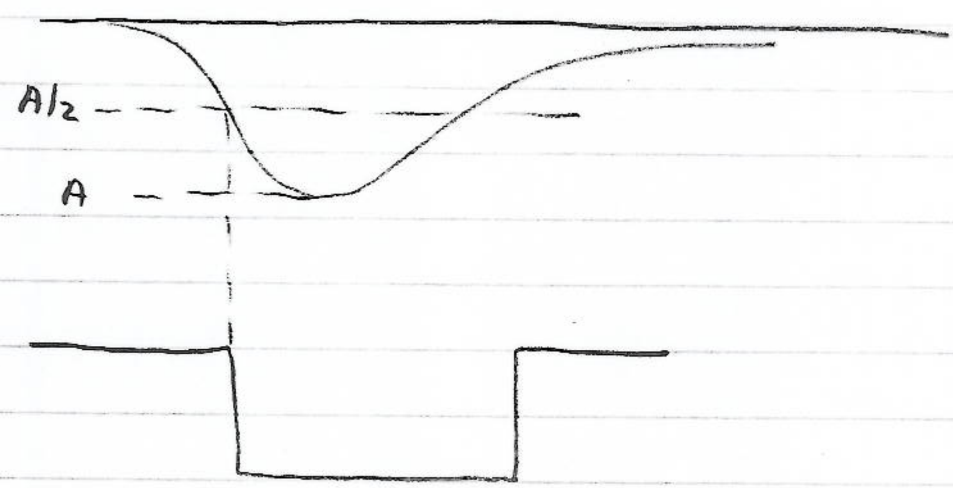
Two kinds of discrimination

(1) leading edge



Logic pulse starts when analog signal drops below threshold

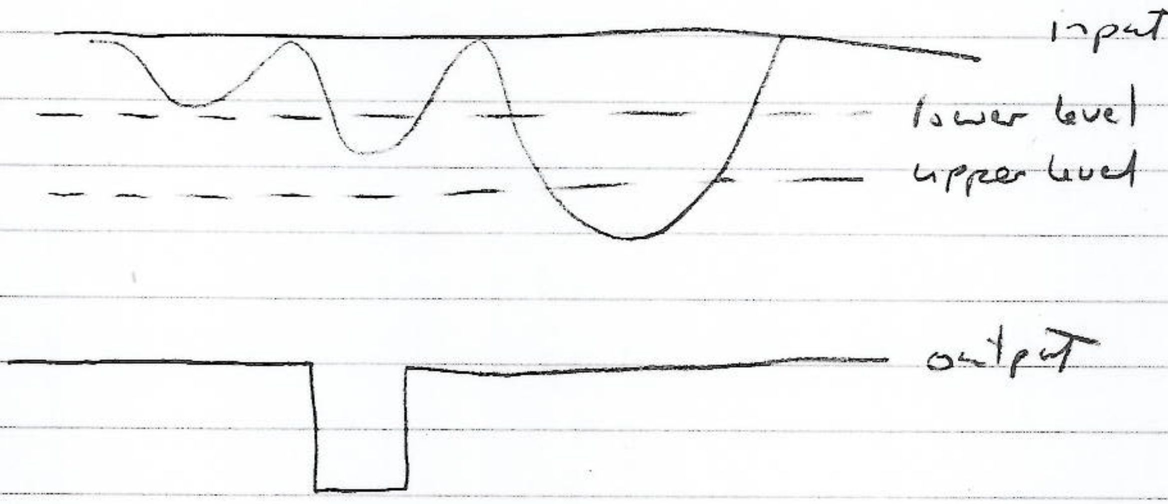
Constant Fraction Discriminator



Logic pulse starts when analog pulse drops below some preset <u>fraction</u> of its amplitude

Single - Channel Analyzer (Differential Discriminator)
- sorts incoming analog signals according to amplitudes

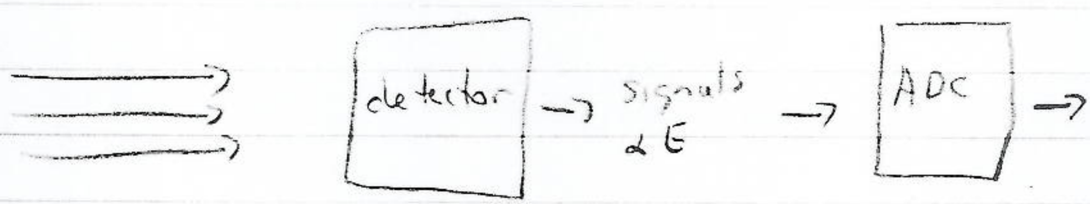
Discriminator - one threshold
SCA - two thresholds
→ window

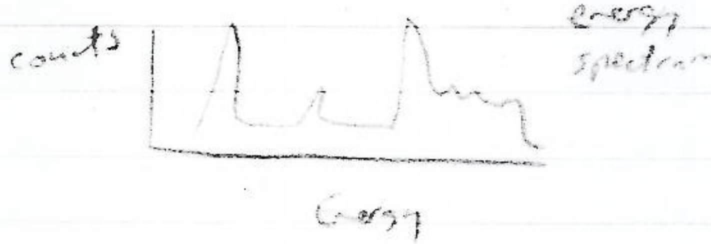


Analog-to-Digital Converter (ADC or A/D)

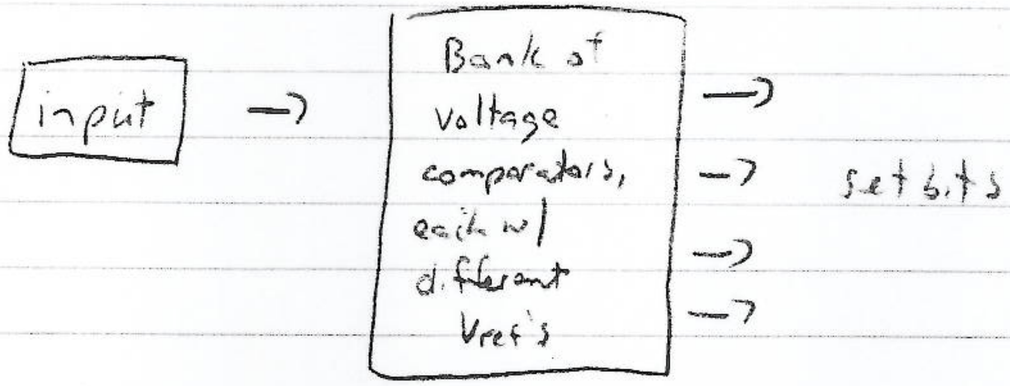
Can sort signals by size w/ many SCA's w/ different windows

particles of different energies





Flash ADC

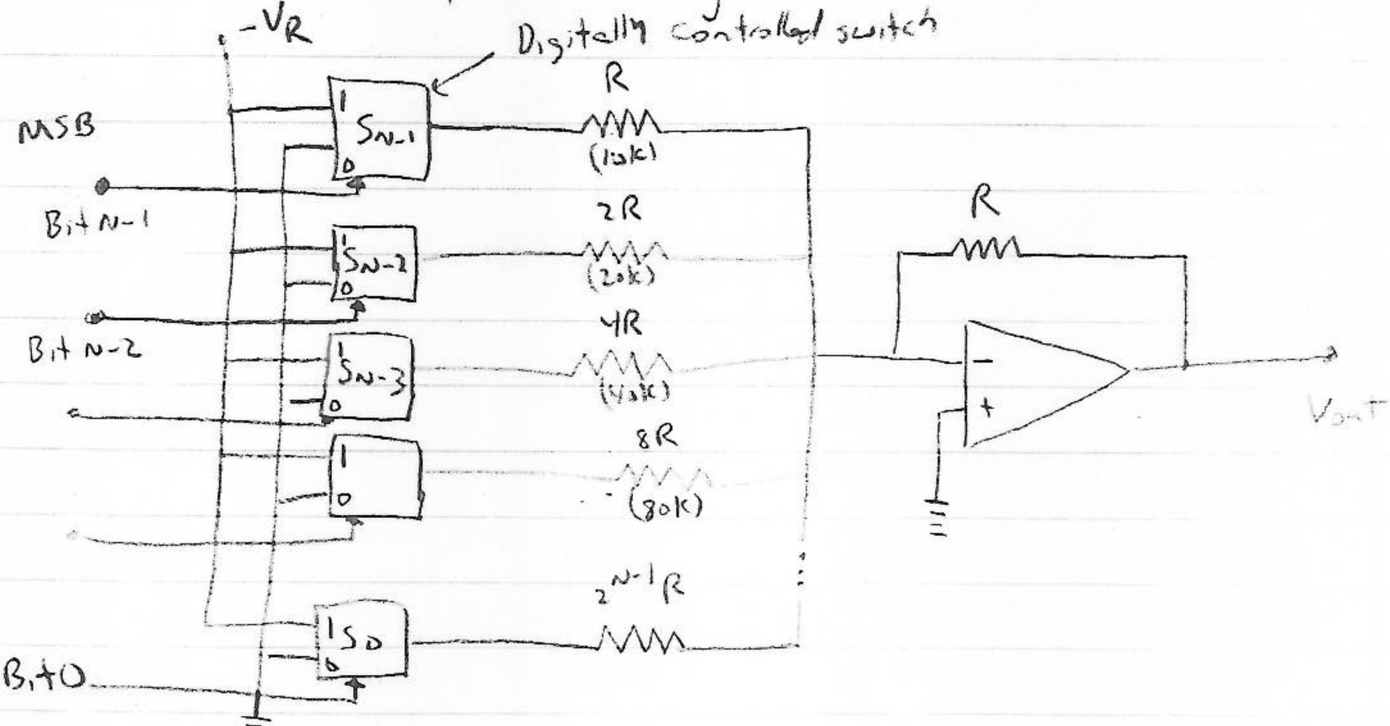


very fast, conversion time ~1 μsec

Digital-to-Analog Converters (DAC or D/A)

Drive servo-mechanism, etc

Recall our totally cool summing circuit



(1) Binary word passed through network of parallel branches, one branch for each bit

(2) Electronic switch

V_r if bit is 1

0 if bit is 0

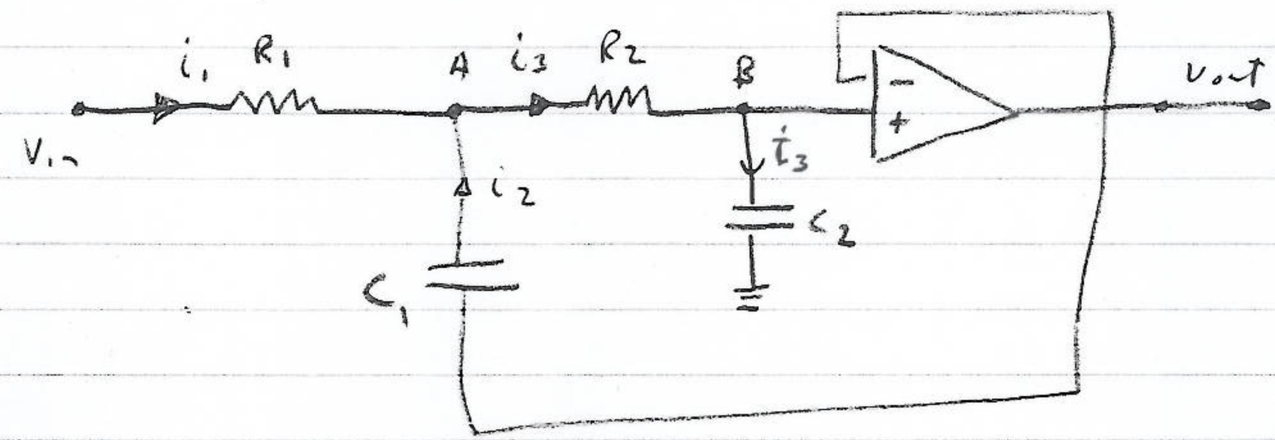
(3) Each branch has resistance weighted by inverse of significance of bit

R, 2R, 4R, etc

(4) Currents from each branch summed

→ analog output

Sallen Key Active Filter (low pass)



$$i_1 + i_2 = i_3$$

$$i_1 = \frac{V_{in} - V_A}{R_1}$$

$$i_2 = \frac{V_{out} - V_A}{\frac{1}{j\omega C_1}} = j\omega C_1 (V_{out} - V_A)$$

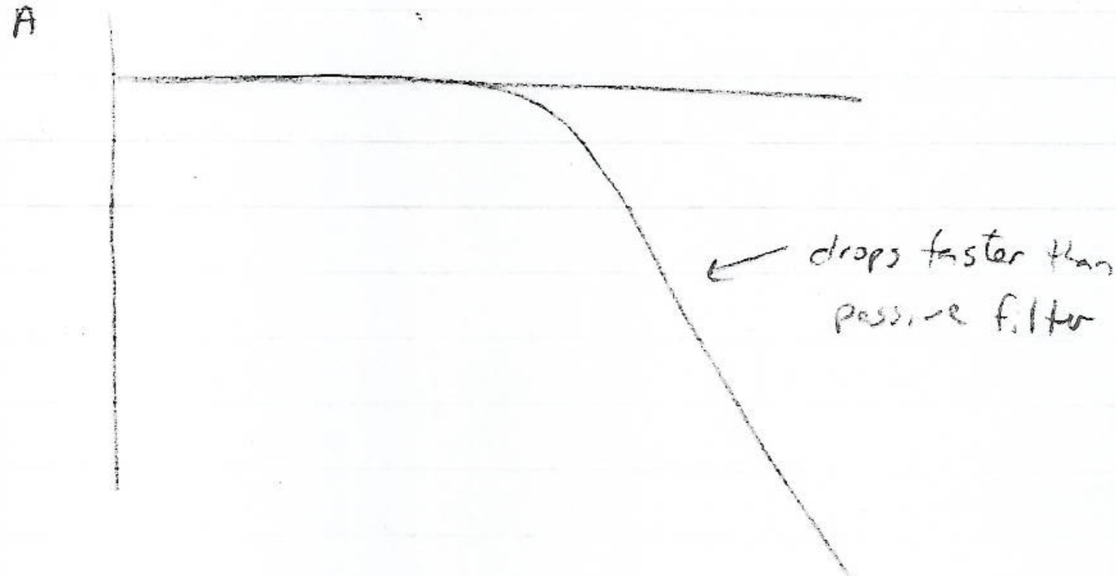
$$i_3 = \frac{V_A}{R_2 + \frac{1}{j\omega C_2}}$$

$$V_B = V_{out}$$

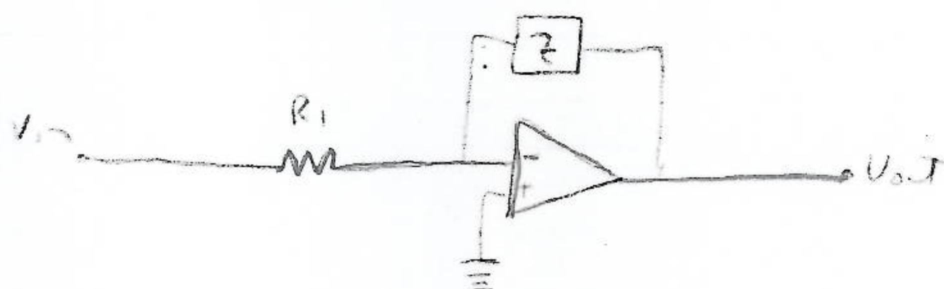
$$i_3 = \frac{V_B}{\frac{1}{j\omega C_2}} = V_{out}\, j\omega C_2$$

5 eqn, 6 unknowns $\quad i_1, i_2, i_3, V_{in}, V_{out}, V_A$

$$A = \frac{V_{out}}{V_{in}} = \frac{1}{1 - \omega^2 R_1 R_2 C_1 C_2 + j\omega (R_1 + R_2) C_2}$$

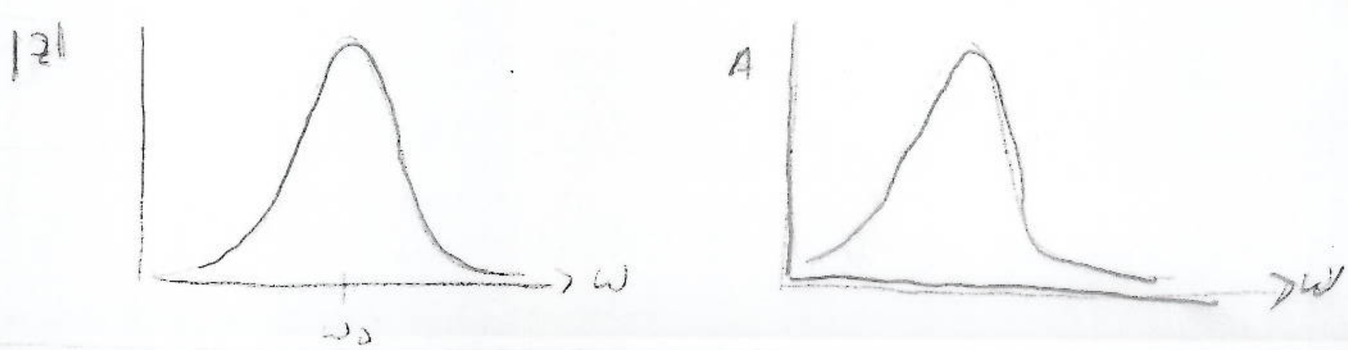


Active Bandpass Filters



V_{in}, R_1 specify current through Z

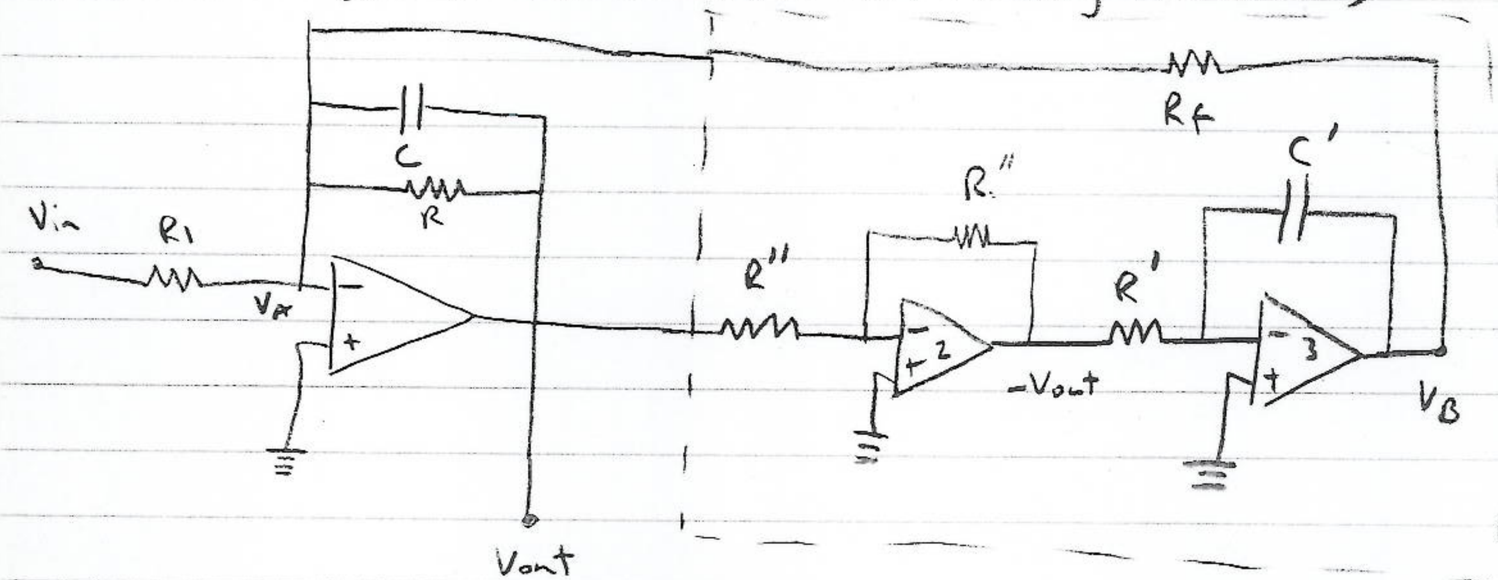
$V_{out} = IZ$



X

Active inductor

(How to make an inductor without using an inductor)



$$V_B = -\frac{1}{R'C'}\int(-V_{out})\,dt$$

$$\left\{ I = \frac{-V_{out}}{R'} \; ; \; V_B = \frac{Q}{C'} \qquad V_B = \frac{1}{R'C'}\int -V_o\,dt \right.$$

$$\frac{dV_B}{dt} = \frac{1}{R'C'}V_{out}$$

$$i_f = \frac{V_B - V_A}{R_f} = \frac{V_B}{R_f}$$

$$R_f\frac{di_f}{dt} = \frac{1}{R'C'}V_{out}$$

$$V_{out} = R_f R'C'\frac{di_f}{dt}$$

like $\quad V_{out} = L_{eff}\dfrac{di_f}{dt}$

$L_{eff} = R_f R' C'$

So resonant f is

$$\omega_0 = \frac{1}{\sqrt{L_{eff} C}} = \frac{1}{\sqrt{R_f R' C' C}}$$