of insufficient data. For extreme relativistic energies, the most accurate estimates (3%) for $d\sigma_{k,\theta_0\phi}$ are given by Formula 2CS.

III. ELECTRON-ELECTRON BREMSSTRAHLUNG

The bremsstrahlung cross-section formulas for electron-nuclear interactions in Sec. IIC vary as Z^2. For targets with high atomic numbers, the additional influence of electron-electron bremsstrahlung can be included approximately by replacing Z^2 by $Z(Z+1)$. However for very low Z elements such as hydrogen or beryllium, the electron-electron bremsstrahlung contributions must be included more accurately. Cross-section calculations for this process are complicated because of the exchange character of the interaction in which there is a large energy and momentum transfer to the recoil electron, in contrast to the electron-nuclear bremsstrahlung process in which the nucleus is assumed to be infinitely heavy. No complete calculations are available for predicting the detailed features of electron-electron bremsstrahlung.[39] A summary of pertinent results that have been obtained is given in the following.

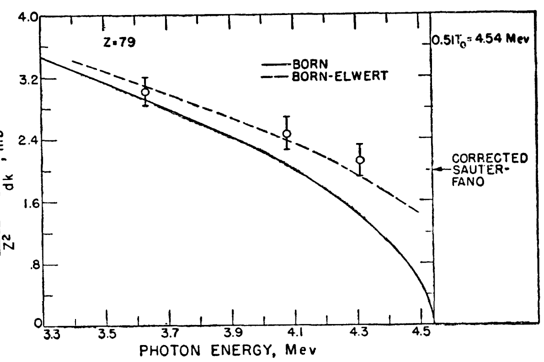

FIG. 21. Dependence of the bremsstrahlung cross section integrated over photon angle on the photon energy for 4.54-Mev electrons. The Born-approximation cross sections shown by the solid curve are calculated from Formula 3BN, and the Born-Elwert cross sections shown by the dashed curve are obtained from the product of Formula 3BN and the Elwert factor, Formula (II-4). The experimental values[36] are shown by the open circles for gold. The corrected Sauter-Fano values at the high-frequency limit are estimated in reference 21.

A. Maximum Photon Energy

In the electron-electron bremsstrahlung process, the maximum photon energy that is available in the laboratory system at the laboratory angle θ_0 is[35]

$$k_{max}=F/(1-\sqrt{F}\cos\theta_0), \qquad \text{(III-1)}$$

where F is equal to $(E_0-1)/(E_0+1)$. Table VI gives some values of k_{max} at zero and 90 degrees obtained from Formula (III-1) for various incident electron kinetic energies. From the very sparse experimental information[35,36] available on electron-electron bremsstrahlung, some results[35] have shown reasonably good agreement with the values of k_{max} predicted by Formula (III-1).

[39] For a general review of the available theories on electron-electron bremsstrahlung, see J. Joseph and F. Rohrlich, Revs. Modern Phys. 30, 354 (1958).

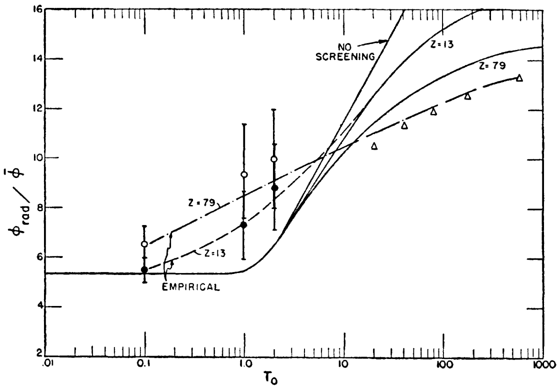

FIG. 22. Dependence of the total radiation cross section, $\phi_{rad}[=(1/E_0)\int_0^{T_0}kd\sigma_k]$, on the initial electron kinetic energy, T_0. The solid lines are obtained from Formula 4BN for no screening, and from the numerical integration of Formulas 3BS(c) and 3BS(d) with screening corrections for Z equal to 13 and 79. The experimental points[33,35] are shown by the open and closed circles for a gold and aluminum target, respectively. The values shown by the triangles are estimated by numerical integration of Formula 3CS for $Z=79$. On the basis of the experimental data at the low energies and the theoretical values (triangles) predicted by the exact theory at the extreme relativistic energies, the dashed curves have been drawn as an estimate of the most accurate ϕ_{rad} values for Z equal to 13 and 79.

B. Cross-Section Formulas for Free Electrons

(1) Nonrelativistic Energies

In contrast to the electron-nucleus and electron-positron systems, the electron-electron system has no

TABLE V. Corrected cross-section formulas for $d\sigma_k$.

Kinetic energy range for incident electron, Mev	Corrected cross-section formula[a]	Restrictions	Estimated accuracy[b]
0.01–0.10	$d\sigma_k=f_E d\sigma_k^{3BN(a)}$	$k>0.01T_0$	±5%
0.10–2.0	$d\sigma_k=Af_E d\sigma_k^{3BN}$	$k>0.01T_0$	±20%
2.0–15	$d\sigma_k=Ad\sigma_k^{3BN}$	$\gamma>15$	b
	$=Ad\sigma_k^{3BS(d)}$	$2<\gamma<15$	±5%
	$=Ad\sigma_k^{3BS(c)}$	$\gamma<2$	±5%
15–50	$d\sigma_k=d\sigma_k^{3BN}$	$\gamma>15$	b
	$=Ad\sigma_k^{3BS(d)}$	$2<\gamma<15$	±3%
	$=Ad\sigma_k^{3BS(c)}$	$\gamma<2$	±3%
50–500	$d\sigma_k=d\sigma_k^{3BN}$	$\gamma>15$	b
	$=d\sigma_k^{3CS(a)}$	$2<\gamma<15$	±3%
	$=d\sigma_k^{3CS(b)}$	$\gamma<2$	±3%

where f_E is defined in Formula (II-6), A is the correction factor given in Fig. 23, γ is equal to the quantity $100k(E_0EZ^{\frac{1}{3}})^{-1}$.

[a] The superscripts for $d\sigma_k$ give the formula numbers defined in Sec. IIC.
[b] No estimated accuracy is given at photon energies near the high-frequency limit of the spectrum. If better accuracy is desired in this region, the cross section at the high-frequency limit can be obtained from the dashed curves in Fig. 12, and the spectrum shape may be adjusted by fitting this end to the curves given by the formulas in column 2 above.

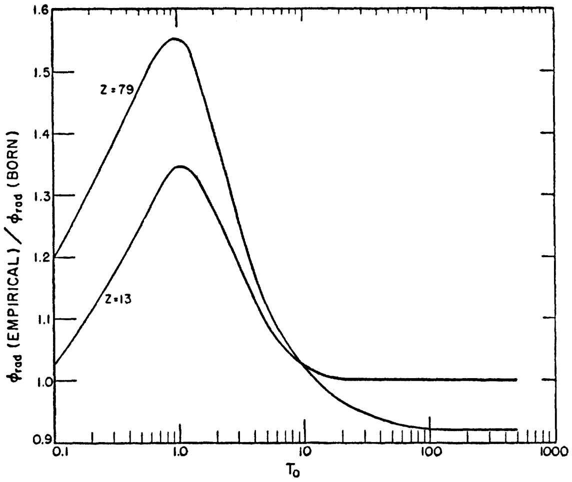

FIG. 23. Approximate correction factors for the Born-approximation ϕ_{rad} values with screening shown in Fig. 22. These factors have been estimated from the ratios of the empirical (dashed and dot-dashed) curves to the Born-approximation curves with screening in Fig. 22.

dipole moment. Therefore the electron-electron bremsstrahlung cross section becomes zero for calculations based only on the nonrelativistic dipole approximation. Garibyan[40] has made calculations beyond the dipole approximation and has obtained the following nonvanishing result for the cross-section differential in photon energy:

$$d\sigma_k'=\frac{r_0^2Z}{137}\left(\frac{8}{15}\frac{\beta}{\beta_0^3}\frac{dk}{k}\right)\left[17-\frac{3(\beta_0^2-\beta^2)^2}{(\beta_0^2+\beta^2)^2}+\left(\frac{\beta_0^2+\beta^2}{\beta_0\beta}+26\frac{\beta_0\beta}{\beta_0^2+\beta^2}-\frac{24\beta_0^3\beta^3}{(\beta_0^2+\beta^2)^3}\right)\times\ln\left(\frac{\beta_0+\beta}{\beta_0-\beta}\right)\right], \quad \text{(III-2)}$$

which for $k\rightarrow 0$ becomes[39]

$$d\sigma_{k\rightarrow 0}'=\frac{r_0^2Z}{137}\left[\frac{32}{5}\frac{dk}{k}\frac{1}{\beta_0^2}\left(\ln\frac{4\beta_0^2}{k}+\frac{17}{12}\right)\right]. \quad \text{(III-3)}$$

These results are only valid for $T_0\ll 1$.

TABLE VI. Maximum photon energy for electron-electron bremsstrahlung.

T_0	k_{max} $\theta_0=0°$	$\theta_0=90°$
100	99.	0.98
10	9.7	0.83
1.0	0.79	0.33
0.1	0.069	0.048
0.01	0.0054	0.0050

[40] G. M. Garibyan, Zhur. Eksptl. i Teoret. Fiz. 24, 617 (1953).

(2) Extreme-Relativistic Energies

Several calculations[39] based on the extreme-relativistic approximations give the following approximate formula for the cross-section differential in photon energy:

$$d\sigma_k'=\frac{4r_0^2Z}{137}\frac{dk}{k}\left[\left(1+\left(\frac{E}{E_0}\right)^2-\frac{2}{3}\frac{E}{E_0}\right)\times\left(\ln\frac{2E_0E}{k}-\frac{3}{2}\right)-\frac{E_0E}{9}\right], \quad \text{(III-4)}$$

which is similar in form to the electron-nuclear cross-section Formula 3BN(b).

The total radiation cross section obtained from For-

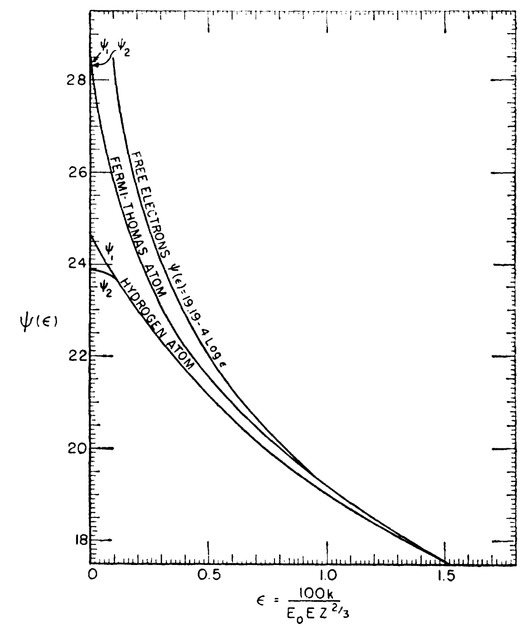

FIG. 24. Screening factors,[41] ψ_1 and ψ_2, for electron-electron bremsstrahlung. The curve marked "Hydrogen atom" was calculated[41] with exact wave functions. For free electrons, $\psi_1=\psi_2=\psi$.

mula (III-4) is given as

$$\phi_{rad}'=\frac{4r_0^2Z}{137}[\ln(2E_0)-\tfrac{4}{3}]. \quad \text{(III-5)}$$

C. Cross-Section Formulas with Binding Corrections

The influence of atomic binding on the electron-electron bremsstrahlung cross section has been calculated only in the extreme-relativistic approximation. With the Thomas-Fermi model, the corrected formula for the cross-section differential in photon energy is[39]

$$d\sigma_k'=\frac{4r_0^2Z}{137}\frac{dk}{k}\left[\left(1+\left(\frac{E}{E_0}\right)^2\right)(\tfrac{1}{4}\psi_1(\epsilon)-1-\ln Z^{\frac{2}{3}})-\frac{2}{3}\frac{E}{E_0}(\tfrac{1}{4}\psi_2(\epsilon)-\tfrac{5}{6}-\ln Z^{\frac{2}{3}})\right], \quad \text{(III-6)}$$

where ϵ is equal to $100k(E_0EZ^{1/3})^{-1}$, and ψ_1 and ψ_2 are given[41] by the data in Fig. 24. For complete screening where $\epsilon \sim 0$, the cross section becomes

$$d\sigma_k' = \frac{4r_0^2Z}{137}\frac{dk}{k}\left(1+\left(\frac{E}{E_0}\right)^2 - \frac{2}{3}\frac{E}{E_0}\right)\ln\frac{530}{Z^{2/3}}. \quad \text{(III-7)}$$

The total radiation cross section which is obtained for the complete screening case from (III-7) is given by

$$\phi_{\mathrm{rad}}' = (4r_0^2Z/137)\ln(530/Z^{2/3}). \quad \text{(III-8)}$$

A comparison of this Formula (III-8) with the electron-nuclear bremsstrahlung cross-section Formula 4BS shows that the Z electrons in an atom increase the electron-nuclear cross section by the factor η so that the total cross section becomes

$$\phi_{\mathrm{rad}}^{\mathrm{total}} = Z(Z+\eta)(\phi_{\mathrm{rad}}^{\mathrm{4BS}}/Z^2). \quad \text{(III-9)}$$

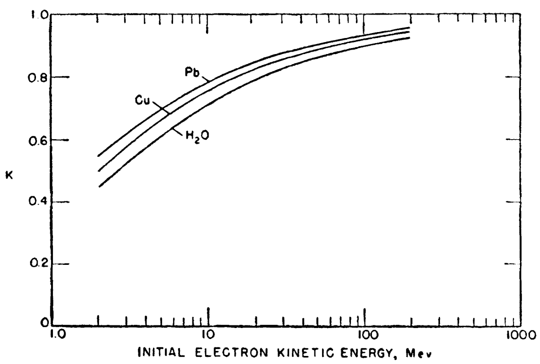

FIG. 25. Dependence of the radiation probability correction factor, $K(=\phi_{\mathrm{rad}}/\phi_{\mathrm{rad}}^*)$, on the initial electron kinetic energy and the target atomic number.

For complete screening, η is given by

$$\eta = \ln\frac{530}{Z^{2/3}} \bigg/ \left(\ln\frac{183}{Z^{1/3}}+\frac{1}{18}\right), \quad \text{(III-10)}$$

which varies from 1.04 for magnesium to 0.88 for lead. For most cases, a value of η equal to unity is sufficiently accurate.

IV. THICK-TARGET BREMSSTRAHLUNG PRODUCTION

Bremsstrahlung is produced in *thick* targets for most practical cases. In this discussion, a target is defined to be *thick* if the scattering and energy loss processes that occur as the electrons traverse the target have an appreciable influence on the bremsstrahlung production. In principle, a complete description of the bremsstrahlung emitted from a given target can be obtained from the cross sections for the pertinent elementary processes. For example, the angular distribution of the total bremsstrahlung power, the shape of the bremsstrahlung spectrum from an x-ray tube, or the efficiency of bremsstrahlung production can be calculated if detailed data are available with regard to the bremsstrahlung and electron scattering (elastic and inelastic) processes. However, any such analysis is necessarily a complicated procedure, since the calculations for the energy loss and scattering of the primary electrons and the absorption of the x-rays in the target must be included with the cross-section information of Sec. II. Also, the analysis depends on the characteristics of a given experimental situation. For these reasons, this paper does not give a complete, systematic treatment of thick-target bremsstrahlung production; instead it is confined to the presentation of pertinent experimental data as well as useful analytical results and procedures. Also, emphasis is placed on thick-target results that give absolute data on photon intensities and bremsstrahlung production efficiencies.

Some of the analytical results for thick-target bremsstrahlung are most conveniently expressed in terms of certain quantities which are defined in the following discussion. When an electron traverses a target, the average energy lost in the path length element dx by radiation can be written as

$$-dE_0 = NE_0(K\phi_{\mathrm{rad}}^*)dx = KE_0dt, \quad \text{(IV-1)}$$

where N is the number of target atoms per cm^3 and $K\phi_{\mathrm{rad}}^*$ is equal to the cross section ϕ_{rad} defined in Sec. IIC. ϕ_{rad}^* is equal to $(4Z^2r_0^2/137)\ln(183Z^{-1/3})\ \mathrm{cm}^2$, which is approximately the same as the expression for ϕ_{rad} at extreme-relativistic energies (see Formula 4BS). K is defined as the radiation probability correction factor and is plotted in Fig. 25 for various values of the target atomic number and the electron kinetic energy. The length t is given in units of the radiation length, t_0, which is defined as

$$t_0 = 1/N\phi_{\mathrm{rad}}^*\ \mathrm{cm}. \quad \text{(IV-2)}$$

Values for t_0 in units of $\mathrm{g/cm}^2$ as a function of the target atomic number are plotted in Fig. 26.

A. Thick-Target Bremsstrahlung Angular Distributions

(1) *Nonrelativistic and Intermediate Energies*[42]

For electron energies that are small or comparable to the electron rest energy, no analytical or empirical formulas have been derived for estimating the bremsstrahlung angular distribution from thick targets, and only a few experimental results are available.

In contrast to the extreme-relativistic region, the radiation intensity produced at these low energies is

[41] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939), and **101**, 1836 (1956).

[42] The results that are presented for the nonrelativistic and intermediate energy region where $T_0 \leq 1$ apply only to targets that are thick enough to stop the electrons. For the relativistic region where $T_0 \gg 1$, there is no such restriction on the target thickness.