

Qweak Technical Note:

A Somewhat General Look at Background Contributions to the Qweak Asymmetry

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Abstract

Backgrounds were budgeted to contribute about 0.5% to the error for the $e + p$ elastic asymmetry in the Qweak experiment. This determines how well we need to measure both the asymmetry and dilution of the backgrounds. In this report, three interesting background asymmetry cases are considered relative to the $e + p$ elastic asymmetry: zero (Moeller scattering-like), -10x larger (window-like), and +10x larger (pion electroproduction-like). With the expected dilution magnitudes taken from Myers [1], and assuming a single background should not contribute more than 0.25% to the Qweak asymmetry, I find the systematic error from the target window dilution, df (systematic), to be the greatest concern. For the zero asymmetry case the contribution goes like df , hence with the expected Moeller dilution $f \simeq 0.01$, it is sufficient to determine $df/f \simeq 25\%$. For the large asymmetry cases, the contribution is magnified by the asymmetry ratio, in these examples $10df$. For the case of pion electroproduction, the expected dilution is small, $\simeq 0.001$, hence we again only require $df/f \simeq 25\%$. For the case of relatively large window backgrounds with expected dilution $f \simeq 0.015$ in the hybrid Be-Al scenario, we require df/f to be an aggressive 1.7%. To do this, it is important we be able to count/track scattered events separately from the upstream Be and the downstream Al windows with an emptied target at low luminosity. The asymmetries will be determined in separate measurements with thick dummy targets, and the measurements combined to yield the window background correction $f_{Be}A_{Be} + f_{Al}A_{Al}$. It will also be important to have a current monitor which is highly linear, has a well-determined zero offset, and is stable over the day or so that it takes to complete the measurements during the target full/empty cycle.

1 Introduction

Backgrounds are nominally allowed to contribute about 0.5% to the error budget for the $e + p$ elastic asymmetry in the Qweak experiment. (For reference, Figure 1 is a copy of the error table from the final Qweak proposal.[2]) Now that it's time to start writing run plans, we have to be precise about what this means in terms of statistical errors and allocating beam time (see Myers in reference [3]), as well as in terms of systematic errors. Although my original intention was to merely derive accurate expressions for the background error, the emphasis in this report quickly became the contribution from the systematic error in the dilution, df . I ignored this contribution during the proposal writing stage, and indeed the statistical contribution from df will likely be negligible. However, the fact that we are measuring a Standard Model-suppressed asymmetry means that the contribution from the systematic error in df is magnified. If df (systematic) is not kept under control, its contribution can easily exceed our entire error budget for backgrounds even if the asymmetry of the background is known exactly. The purpose of this report is to make this latter point (obvious in hindsight), specify how well we have to measure f for each of the major backgrounds, identify the highest nail, and marshal resources to pound it down.

Table 4: *Total error estimate for the Q_{weak} experiment. The contributions to both the physics asymmetry and the extracted Q_W^p are given. In most cases, the error magnification due to the 33% hadronic dilution is a factor of 1.49. The enhancement for the Q^2 term is somewhat larger.*

Source of error	Contribution to $\Delta A_{\text{phys}}/A_{\text{phys}}$	Contribution to $\Delta Q_W^p/Q_W^p$
Counting Statistics	2.1%	3.2%
Hadronic structure	—	1.5 %
Beam polarimetry	1.0 %	1.5%
Absolute Q^2	0.5%	1.0%
Backgrounds	0.5%	0.7%
Helicity-correlated beam properties	0.5%	0.7%
TOTAL:	2.5%	4.1%

Figure 1: Copy of the error table from the final Qweak proposal.

1.1 Equations

As a starting point, we write the “measured” or “experimental” asymmetry as

$$A_M = \frac{R_E P A_E + R_B P A_B}{R_E + R_B} \quad (1)$$

where R_E and A_E are the $e + p$ total elastic rate and average physics asymmetry, respectively, R_B and A_B are the total background rate and average background physics asymmetry¹, respectively, and P is the beam polarization during production running on a full target. Defining the usual dilution factor, $f \equiv R_B/(R_E + R_B)$, the above equation can be rewritten as $A_M/P = (1 - f)A_E + fA_B$ which after solving for A_E yields

$$A_E = \frac{A_M/P - fA_B}{1 - f} \quad (2)$$

In order to calculate error derivatives, we have to know what we’re going to measure. Clearly, we’re going to measure A_M/P using the majority of our beam time on the full target. We also have to determine the f appropriate for production running. For the target window backgrounds, this can be done by running at very low beam current, measuring the target full and target empty event rates, correcting for tracking efficiencies, correcting the target empty event rate for the missing beam energy/angle straggling which would have been caused by the 4% radiation length of LH_2 , and correcting the target full rate for the significant density reduction that may be caused by 180 μA beam. (Current mode measurements of f might also be possible, but because the target empty measurements require low beam currents, I suspect nonlinearities would limit the precision to 100%.) We’ll also need a current monitor which is linear, has a negligible zero offset, and has a gain which is stable over the day or so that it takes to complete the measurements during the target full/empty cycle. An absolute beam current should not be necessary, and might be extremely difficult to implement in any case at the nA scale. Thick dummy windows might be helpful for checking radiative corrections to f , but aren’t practical for determining f since the relative thicknesses of the dummy and true windows would have to be known at the percent level, and the true windows will be only 5-10 mils (127-254 microns) thick.

¹This simple expression is sufficient for my purposes here. It is straightforward to generalize to multiple backgrounds. Light-weighting can be incorporated by replacing R_i with $R_i L_i$.

There doesn't appear to be a practical way to make a direct determination of fA_B , so the final independent variable for our error analysis is A_B . Because the thin target windows yield low rate and may be damaged by high currents when the target is emptied, window asymmetries A_B must be measured with thick dummy windows. The separate measurements of f_{Be} , f_{Al} , A_{Be} , and A_{Al} must then be combined to yield the window background correction $f_{Be}A_{Be} + f_{Al}A_{Al}$.

Getting back to error derivatives, taking the natural log of both sides of Eqn. 2 and differentiating with respect to A_M/P , A_B , and f , the following error terms result:

$$\left(\frac{dA_E}{A_E}\right)_{A_M/P} = \frac{d(A_M/P)}{A_E} \cdot \frac{1}{1-f} \quad (3)$$

$$\left(\frac{dA_E}{A_E}\right)_{A_B} = \frac{dA_B}{A_E} \cdot \frac{f}{1-f} \quad (4)$$

$$\left(\frac{dA_E}{A_E}\right)_f = \frac{df}{(1-f)^2} \cdot \frac{A_M/P - A_B}{A_E} \quad (5)$$

These expressions may look unfamiliar, but if the first two terms are rewritten to leading order, they match what one might have written down by inspection. The third term looks particularly nonintuitive, but gives reasonable results if rewritten to leading order for the three limiting cases:

$$for A_B = A_E, \quad \left(\frac{dA_E}{A_E}\right)_f = 0 \quad (6)$$

$$for A_B = 0, \quad \left(\frac{dA_E}{A_E}\right)_f \simeq df \quad (7)$$

$$for A_B \gg A_E, \quad \left(\frac{dA_E}{A_E}\right)_f \simeq df \frac{A_B}{A_E} \quad (8)$$

Note the error magnification in the last term/scenario.

To get a feeling for the required relative error on A_B , it is helpful to rewrite Eqn. 4 for the cases of non-zero asymmetry as

$$\left(\frac{dA_E}{A_E}\right)_{A_B} = \frac{A_B}{A_E} \cdot \frac{f}{1-f} \cdot \frac{dA_B}{A_B} \simeq 10f \cdot \frac{dA_B}{A_B} \quad (9)$$

For the window-like case with $f = 0.015$, this means $\left(\frac{dA_E}{A_E}\right)_{A_B} = 0.15 \frac{dA_B}{A_B}$. Note the error de-magnification. Thus, if the background measurement errors were allowed to match the production measurement, the errors on the background could be about $2\%/0.15 \simeq 13\%$ which would be a piece of cake. However, we are only allowing the error contribution from a single background to be 0.25%. Thus the allowed error on a background measurement is $0.25\%/0.15 = 1.7\%$. Neglecting systematic contributions, my crude estimate is of the same order of magnitude as the more accurate results in Table 3 of reference [3] which indicates the need for $dA_B/A_B(\text{statistics}) \simeq 2\%$.

1.2 Inputs

The above equations were put into a computer program which calculates the individual errors (statistical and systematic), as well as the quadrature sum. The input assumptions are listed in Table 1, but a few additional words of explanation are needed. Depending on the details of how the data are taken, errors in polarimetry or Q^2 may be strongly correlated between the measurements of A_M and A_B . I'm treating them as uncorrelated errors for now, but the results here don't depend strongly on this assumption. For the statistical error on f , I assumed that a counting experiment could quickly determine $df(\text{statistical})$ to 2 parts in 10^4 which then has a negligible effect on the total error.² Finally, the contribution which may be most important, and whose magnitude about which reasonable people may disagree, is the systematic error on f . For the figures, I assumed that we will usually only be able to determine a given background dilution to a relative error of 10%. We'll probably be able to do better than this for backgrounds which can be directly measured (like electrons which have elastically scattered from the target windows), but we'll be lucky to do this well for multi-bounce backgrounds which may not be trackable (like low energy electrons and photons from multi-bounce Moeller events).

²7 hours at 1 KHz DAQ rate

Table 1: *Input parameters for the calculations.*

Parameter	Value
<i>from the final proposal:</i>	
A_E physics asymmetry	-234 ppb
R_E	6.5 GHz
Total Production Time	106 days
<i>reasonable assumptions:</i>	
Full Target Time	$0.9 * 106 = 95.4$ days
Dummy Target Time	$0.1 * 106 = 10.6$ days
Thickness Dummy/Thickness Windows	x10
$d(A_M/P)/(A_M/P)$ (systematic)	1% (mostly polarimetry)
dA_B/A_B (systematic)	1% (mostly polarimetry)
<i>background dependent assumptions:</i>	
df/f (statistics)	0.02%
df/f (systematic)	10%

1.3 Results and Discussion

For the case $A_B = 0$ (a Moeller-like background), Figure 2 plots dA_E/A_E vs f . This zero asymmetry case resembles that for multi-bounce soft backgrounds originating from Moeller scattering in the target. The direct Moeller scattering process has a relatively small asymmetry since the Q^2 is only several percent that of $e+p$ scattering in our acceptance. The resulting total error on A_E is about 2.3% in the limit of no background and does not rise strongly with increasing background fraction. The background error budget would be saturated if this background were as large as $f = 0.05$ under the assumption that $df/f = 10\%$. In reference [1], Myers predicts Moeller backgrounds of $f \simeq 0.01$, thus our 0.25% maximum contribution to dA_E/A_E requires only that df/f (systematic) = 25%.

For the case $A_B = -10A_E$ (a window-like background), Figure 3 plots dA_E/A_E vs f . This case resembles that of elastic scattering from Al or Be windows, although the average window asymmetry for Be is actually predicted to be about 20% higher.[1] In sharp contrast to the previous figure, here the total error on the asymmetry increases dramatically with increasing background fraction due to the df (systematic) contribution. For the target window backgrounds, assuming the predicted dilution

of $f = 0.015$ from reference [1], then our goal should be $df/f(\text{systematic}) = 1.7\%$. The exact value may differ give or take a $\sqrt{2}$, but clearly this will be challenging. Fortunately, backgrounds make a small contribution to the overall error on A_E , so even doubling the total background contributions from 0.5% to 1.0% only increases the error on the $e + p$ statistics-dominated A_E error from 2.5% to 2.6%.

One thing in Figure 3 that may require explanation is why the systematic error on the measured asymmetry *decreases* with increasing dilution. After all, how could anything get better with increasing background? The reason is that because A_E and A_B have opposite signs in this scenario, A_M is rapidly decreasing, so a 1% systematic uncertainty on A_M leads to a decreasing dA_M . In fact, the measured asymmetry actually vanishes as $f \rightarrow 0.10$. Clearly, some systematic floor should be added for beam false asymmetry corrections, for example, but this does not significantly affect the main conclusions.

For the case $A_B = +10A_E$ (an inelastic-like background), Figure 4 plots dA_E/A_E vs f . This case resembles that of pion electroproduction on Hydrogen. As with the window backgrounds examined above, here the total error on the asymmetry increases rapidly with increasing background fraction due to the $df(\text{systematic})$ contribution. From reference [1], $f \simeq 0.001$ dominated by showering of inelastics in the shield wall windows, so we only need to determine df/f to 25%. This is fortunate given the difficulty in tracking showers on the edges of the Region III VDC acceptance.

The required df/f are summarized in Table 2. Numerical values for all asymmetry scenarios are given in the Appendix.

Table 2: *Summary of how well we have to determine the estimated dilution factors from reference [1] assuming that the contribution to dA_E/A_E should not exceed 0.25%. As discussed in the text, we have to determine the dA_B/A_B for the large asymmetry cases to about 2%.*

Background	A_B/A_E	f	df/f	Comments
Moeller-like	0	0.01	25%	see Eqn. 7
Window-like	-10	0.015	1.7%	see Eqn. 8
Inelastic-like	+10	0.001	25%	see Eqn. 8

2 Summary

The requirements on how well we have to determine df/f for Moeller-like and inelastic-like backgrounds look very reasonable, $\simeq 25\%$. The requirement for the target window backgrounds of $df/f = 1.7\%$ is more aggressive. Should the target window background end up being twice the background error budget in the proposal however, the final error on A_E would only increase from 2.5% to 2.6%. Of course, if every working group (Q^2 , polarimetry, etc) were to blow their error budget by a factor of 2, then the final error on A_E would balloon from 2.5% to 3.4% and the error on Qweak(proton) would be approximately 5.4%. We want to avoid this.

It looks like the experiment remains do-able without a preradiiator. The window background is uncomfortably large, but a pre-radiator would not reduce the the (window elastic)/($e + p$ elastic) ratio since the scattered electrons have similar energies.

Finally, if anyone would like these figures made under a different set of assumptions, I can change the program and generate new plots in 20 minutes.

Acknowledgements

I'd like to acknowledge helpful discussions with K. Myers, G. Smith, and S. Covrig.

References

- [1] K. Myers, "Background Simulations and Measurement", talk at Qweak collaboration meeting , March 6-7, 2009
- [2] Qweak Collaboration, "The Qweak Experiment: a Search for New Physics via a Measurement of the Proton's Weak Charge", TJNAF proposal, December 10, 2007.
- [3] K. Myers, "Target Window Background Runtime Requirements", Qweak technical note, June 1, 2009, and references within.

A Tabulated Errors for Three Scenarios

Terms dAm, dAb, and df are shorthand for Eqns. 3-5 in the main text.

!Input parameters and constraints are as follows:										
!Ael (ppb), Rel (GHz), Tmsr (days) = -234.000 6.500 95.400										
!Relative dummy thickness, Tbkg (days) = 10. 10.600004										
!Relative sys. error on Amsr, Abkg (%) 0.01 0.01										
!df_stat_rel, df_sys_rel = 0.000 0.100										
!Abkg(ppb) Rb/Rs dAm dAb df dAE/AE dAm dAb df dAE/AE dAE/AE										
! <---statistical----> <---systematic----> TOTAL										
0.00	0.001	2.17	0.06	0.00	2.17	1.00	0.00	0.01	1.00	2.39
0.00	0.002	2.17	0.08	0.00	2.18	1.00	0.00	0.02	1.00	2.39
0.00	0.003	2.18	0.10	0.00	2.18	1.00	0.00	0.03	1.00	2.40
0.00	0.004	2.18	0.11	0.00	2.18	1.00	0.00	0.04	1.00	2.40
0.00	0.005	2.18	0.12	0.00	2.18	1.00	0.00	0.05	1.00	2.40
0.00	0.006	2.18	0.14	0.00	2.18	1.00	0.00	0.06	1.00	2.40
0.00	0.007	2.18	0.15	0.00	2.18	1.00	0.00	0.07	1.00	2.40
0.00	0.008	2.18	0.16	0.00	2.19	1.00	0.00	0.08	1.00	2.41
0.00	0.009	2.18	0.17	0.00	2.19	1.00	0.00	0.09	1.00	2.41
0.00	0.010	2.18	0.18	0.00	2.19	1.00	0.00	0.10	1.00	2.41
0.00	0.011	2.18	0.18	0.00	2.19	1.00	0.00	0.11	1.01	2.41
0.00	0.012	2.19	0.19	0.00	2.19	1.00	0.00	0.12	1.01	2.41
0.00	0.013	2.19	0.20	0.00	2.20	1.00	0.00	0.13	1.01	2.42
0.00	0.014	2.19	0.21	0.00	2.20	1.00	0.00	0.14	1.01	2.42
0.00	0.015	2.19	0.21	0.00	2.20	1.00	0.00	0.15	1.01	2.42
0.00	0.016	2.19	0.22	0.00	2.20	1.00	0.00	0.16	1.01	2.42
0.00	0.017	2.19	0.23	0.00	2.20	1.00	0.00	0.17	1.01	2.42
0.00	0.018	2.19	0.23	0.00	2.20	1.00	0.00	0.18	1.02	2.43
0.00	0.019	2.19	0.24	0.00	2.21	1.00	0.00	0.19	1.02	2.43
0.00	0.020	2.19	0.25	0.00	2.21	1.00	0.00	0.20	1.02	2.43
0.00	0.021	2.19	0.25	0.00	2.21	1.00	0.00	0.21	1.02	2.43
0.00	0.022	2.20	0.26	0.00	2.21	1.00	0.00	0.22	1.02	2.44
0.00	0.023	2.20	0.27	0.00	2.21	1.00	0.00	0.23	1.03	2.44
0.00	0.024	2.20	0.27	0.00	2.21	1.00	0.00	0.24	1.03	2.44
0.00	0.025	2.20	0.28	0.00	2.22	1.00	0.00	0.25	1.03	2.44
0.00	0.026	2.20	0.28	0.00	2.22	1.00	0.00	0.26	1.03	2.45
0.00	0.027	2.20	0.29	0.00	2.22	1.00	0.00	0.27	1.04	2.45
0.00	0.028	2.20	0.29	0.00	2.22	1.00	0.00	0.28	1.04	2.45
0.00	0.029	2.20	0.30	0.00	2.22	1.00	0.00	0.29	1.04	2.46
0.00	0.030	2.20	0.30	0.00	2.23	1.00	0.00	0.30	1.04	2.46
0.00	0.031	2.21	0.31	0.00	2.23	1.00	0.00	0.31	1.05	2.46
0.00	0.032	2.21	0.31	0.00	2.23	1.00	0.00	0.32	1.05	2.46
0.00	0.033	2.21	0.32	0.00	2.23	1.00	0.00	0.33	1.05	2.47
0.00	0.034	2.21	0.32	0.00	2.23	1.00	0.00	0.34	1.06	2.47
0.00	0.035	2.21	0.33	0.00	2.23	1.00	0.00	0.35	1.06	2.47
0.00	0.036	2.21	0.33	0.00	2.24	1.00	0.00	0.36	1.06	2.48
0.00	0.037	2.21	0.34	0.00	2.24	1.00	0.00	0.37	1.07	2.48
0.00	0.038	2.21	0.34	0.00	2.24	1.00	0.00	0.38	1.07	2.48
0.00	0.039	2.21	0.35	0.00	2.24	1.00	0.00	0.39	1.07	2.48
0.00	0.040	2.22	0.35	0.00	2.24	1.00	0.00	0.40	1.08	2.49
0.00	0.041	2.22	0.35	0.00	2.24	1.00	0.00	0.41	1.08	2.49
0.00	0.042	2.22	0.36	0.00	2.25	1.00	0.00	0.42	1.08	2.49
0.00	0.043	2.22	0.36	0.00	2.25	1.00	0.00	0.43	1.09	2.50
0.00	0.044	2.22	0.37	0.00	2.25	1.00	0.00	0.44	1.09	2.50
0.00	0.045	2.22	0.37	0.00	2.25	1.00	0.00	0.45	1.10	2.50
0.00	0.046	2.22	0.38	0.00	2.25	1.00	0.00	0.46	1.10	2.51
0.00	0.047	2.22	0.38	0.00	2.25	1.00	0.00	0.47	1.10	2.51
0.00	0.048	2.22	0.38	0.00	2.26	1.00	0.00	0.48	1.11	2.51
0.00	0.049	2.22	0.39	0.00	2.26	1.00	0.00	0.49	1.11	2.52
0.00	0.050	2.23	0.39	0.00	2.26	1.00	0.00	0.50	1.12	2.52
0.00	0.051	2.23	0.40	0.00	2.26	1.00	0.00	0.51	1.12	2.52
0.00	0.052	2.23	0.40	0.00	2.26	1.00	0.00	0.52	1.13	2.53
0.00	0.053	2.23	0.40	0.00	2.27	1.00	0.00	0.53	1.13	2.53
0.00	0.054	2.23	0.41	0.00	2.27	1.00	0.00	0.54	1.14	2.54
0.00	0.055	2.23	0.41	0.00	2.27	1.00	0.00	0.55	1.14	2.54
0.00	0.056	2.23	0.41	0.00	2.27	1.00	0.00	0.56	1.15	2.54
0.00	0.057	2.23	0.42	0.00	2.27	1.00	0.00	0.57	1.15	2.55
0.00	0.058	2.23	0.42	0.00	2.27	1.00	0.00	0.58	1.16	2.55
0.00	0.059	2.24	0.43	0.00	2.28	1.00	0.00	0.59	1.16	2.55
0.00	0.060	2.24	0.43	0.00	2.28	1.00	0.00	0.60	1.17	2.56
0.00	0.061	2.24	0.43	0.00	2.28	1.00	0.00	0.61	1.17	2.56
0.00	0.062	2.24	0.44	0.00	2.28	1.00	0.00	0.62	1.18	2.57

0.00	0.063	2.24	0.44	0.00	2.28	1.00	0.00	0.63	1.18	2.57
0.00	0.064	2.24	0.44	0.00	2.28	1.00	0.00	0.64	1.19	2.57
0.00	0.065	2.24	0.45	0.00	2.29	1.00	0.00	0.65	1.19	2.58
0.00	0.066	2.24	0.45	0.00	2.29	1.00	0.00	0.66	1.20	2.58
0.00	0.067	2.24	0.45	0.00	2.29	1.00	0.00	0.67	1.20	2.59
0.00	0.068	2.24	0.46	0.00	2.29	1.00	0.00	0.68	1.21	2.59
0.00	0.069	2.25	0.46	0.00	2.29	1.00	0.00	0.69	1.21	2.59
0.00	0.070	2.25	0.46	0.00	2.29	1.00	0.00	0.70	1.22	2.60
0.00	0.071	2.25	0.47	0.00	2.30	1.00	0.00	0.71	1.23	2.60
0.00	0.072	2.25	0.47	0.00	2.30	1.00	0.00	0.72	1.23	2.61
0.00	0.073	2.25	0.47	0.00	2.30	1.00	0.00	0.73	1.24	2.61
0.00	0.074	2.25	0.48	0.00	2.30	1.00	0.00	0.74	1.24	2.62
0.00	0.075	2.25	0.48	0.00	2.30	1.00	0.00	0.75	1.25	2.62
0.00	0.076	2.25	0.48	0.00	2.30	1.00	0.00	0.76	1.26	2.62
0.00	0.077	2.25	0.49	0.00	2.31	1.00	0.00	0.77	1.26	2.63
0.00	0.078	2.26	0.49	0.00	2.31	1.00	0.00	0.78	1.27	2.63
0.00	0.079	2.26	0.49	0.00	2.31	1.00	0.00	0.79	1.27	2.64
0.00	0.080	2.26	0.50	0.00	2.31	1.00	0.00	0.80	1.28	2.64
0.00	0.081	2.26	0.50	0.00	2.31	1.00	0.00	0.81	1.29	2.65
0.00	0.082	2.26	0.50	0.00	2.31	1.00	0.00	0.82	1.29	2.65
0.00	0.083	2.26	0.50	0.00	2.32	1.00	0.00	0.83	1.30	2.66
0.00	0.084	2.26	0.51	0.00	2.32	1.00	0.00	0.84	1.31	2.66
0.00	0.085	2.26	0.51	0.00	2.32	1.00	0.00	0.85	1.31	2.67
0.00	0.086	2.26	0.51	0.00	2.32	1.00	0.00	0.86	1.32	2.67
0.00	0.087	2.26	0.52	0.00	2.32	1.00	0.00	0.87	1.33	2.67
0.00	0.088	2.27	0.52	0.00	2.32	1.00	0.00	0.88	1.33	2.68
0.00	0.089	2.27	0.52	0.00	2.33	1.00	0.00	0.89	1.34	2.68
0.00	0.090	2.27	0.53	0.00	2.33	1.00	0.00	0.90	1.35	2.69
0.00	0.091	2.27	0.53	0.00	2.33	1.00	0.00	0.91	1.35	2.69
0.00	0.092	2.27	0.53	0.00	2.33	1.00	0.00	0.92	1.36	2.70
0.00	0.093	2.27	0.53	0.00	2.33	1.00	0.00	0.93	1.37	2.70
0.00	0.094	2.27	0.54	0.00	2.33	1.00	0.00	0.94	1.37	2.71
0.00	0.095	2.27	0.54	0.00	2.34	1.00	0.00	0.95	1.38	2.71
0.00	0.096	2.27	0.54	0.00	2.34	1.00	0.00	0.96	1.39	2.72
0.00	0.097	2.27	0.55	0.00	2.34	1.00	0.00	0.97	1.39	2.72
0.00	0.098	2.28	0.55	0.00	2.34	1.00	0.00	0.98	1.40	2.73
0.00	0.099	2.28	0.55	0.00	2.34	1.00	0.00	0.99	1.41	2.73
0.00	0.100	2.28	0.55	0.00	2.34	1.00	0.00	1.00	1.41	2.74

```

!Input parameters and constraints are as follows:
!Ael (ppb), Rel (GHz), Tmsr (days) = -234.000    6.500    95.400
!Relative dummy thickness, Tbkg (days) = 10. 10.600004
!Relative sys. error on Amsr, Abkg (%) 0.01 0.01
!df_stat_rel, df_sys_rel = 0.000 0.100

!Abkg(ppb) Rb/Rs dAm dAb df dAE/AE dAm dAb df dAE/AE dAE/AE
!          <----statistical----> <----systematic----> TOTAL
-2340.00  0.001  2.17  0.06  0.00  2.17  1.01  0.01  0.09  1.01  2.40
-2340.00  0.002  2.17  0.08  0.00  2.18  1.02  0.02  0.18  1.04  2.41
-2340.00  0.003  2.18  0.10  0.00  2.18  1.03  0.03  0.27  1.07  2.42
-2340.00  0.004  2.18  0.11  0.00  2.18  1.04  0.04  0.36  1.10  2.44
-2340.00  0.005  2.18  0.12  0.00  2.18  1.05  0.05  0.45  1.14  2.46
-2340.00  0.006  2.18  0.14  0.00  2.18  1.06  0.06  0.54  1.19  2.49
-2340.00  0.007  2.18  0.15  0.00  2.18  1.07  0.07  0.63  1.24  2.51
-2340.00  0.008  2.18  0.16  0.00  2.19  1.08  0.08  0.72  1.30  2.54
-2340.00  0.009  2.18  0.17  0.00  2.19  1.09  0.09  0.81  1.36  2.58
-2340.00  0.010  2.18  0.18  0.00  2.19  1.10  0.10  0.90  1.42  2.61
-2340.00  0.011  2.18  0.18  0.00  2.19  1.11  0.11  0.99  1.49  2.65
-2340.00  0.012  2.19  0.19  0.00  2.19  1.12  0.12  1.08  1.56  2.69
-2340.00  0.013  2.19  0.20  0.00  2.20  1.13  0.13  1.17  1.63  2.74
-2340.00  0.014  2.19  0.21  0.00  2.20  1.14  0.14  1.26  1.70  2.78
-2340.00  0.015  2.19  0.21  0.00  2.20  1.15  0.15  1.35  1.78  2.83
-2340.00  0.016  2.19  0.22  0.00  2.20  1.16  0.16  1.44  1.86  2.88
-2340.00  0.017  2.19  0.23  0.00  2.20  1.17  0.17  1.53  1.93  2.93
-2340.00  0.018  2.19  0.23  0.00  2.20  1.18  0.18  1.62  2.01  2.98
-2340.00  0.019  2.19  0.24  0.00  2.21  1.19  0.19  1.71  2.09  3.04
-2340.00  0.020  2.19  0.25  0.00  2.21  1.20  0.20  1.80  2.17  3.10
-2340.00  0.021  2.19  0.25  0.00  2.21  1.21  0.21  1.89  2.25  3.16
-2340.00  0.022  2.20  0.26  0.00  2.21  1.22  0.22  1.98  2.34  3.22
-2340.00  0.023  2.20  0.27  0.00  2.21  1.23  0.23  2.07  2.42  3.28
-2340.00  0.024  2.20  0.27  0.00  2.21  1.24  0.24  2.16  2.50  3.34
-2340.00  0.025  2.20  0.28  0.00  2.22  1.25  0.25  2.25  2.59  3.41
-2340.00  0.026  2.20  0.28  0.00  2.22  1.26  0.26  2.34  2.67  3.47
-2340.00  0.027  2.20  0.29  0.00  2.22  1.27  0.27  2.43  2.76  3.54
-2340.00  0.028  2.20  0.29  0.01  2.22  1.28  0.28  2.52  2.84  3.61
-2340.00  0.029  2.20  0.30  0.01  2.22  1.29  0.29  2.61  2.93  3.67
-2340.00  0.030  2.20  0.30  0.01  2.23  1.30  0.30  2.70  3.01  3.74
-2340.00  0.031  2.21  0.31  0.01  2.23  1.31  0.31  2.79  3.10  3.82
-2340.00  0.032  2.21  0.31  0.01  2.23  1.32  0.32  2.88  3.18  3.89
-2340.00  0.033  2.21  0.32  0.01  2.23  1.33  0.33  2.97  3.27  3.96
-2340.00  0.034  2.21  0.32  0.01  2.23  1.34  0.34  3.06  3.36  4.03
-2340.00  0.035  2.21  0.33  0.01  2.23  1.35  0.35  3.15  3.44  4.11
-2340.00  0.036  2.21  0.33  0.01  2.24  1.36  0.36  3.24  3.53  4.18
-2340.00  0.037  2.21  0.34  0.01  2.24  1.37  0.37  3.33  3.62  4.26
-2340.00  0.038  2.21  0.34  0.01  2.24  1.38  0.38  3.42  3.71  4.33
-2340.00  0.039  2.21  0.35  0.01  2.24  1.39  0.39  3.51  3.80  4.41
-2340.00  0.040  2.22  0.35  0.01  2.24  1.40  0.40  3.60  3.88  4.48
-2340.00  0.041  2.22  0.35  0.01  2.24  1.41  0.41  3.69  3.97  4.56
-2340.00  0.042  2.22  0.36  0.01  2.25  1.42  0.42  3.78  4.06  4.64
-2340.00  0.043  2.22  0.36  0.01  2.25  1.43  0.43  3.87  4.15  4.72
-2340.00  0.044  2.22  0.37  0.01  2.25  1.44  0.44  3.96  4.24  4.80
-2340.00  0.045  2.22  0.37  0.01  2.25  1.45  0.45  4.05  4.33  4.88
-2340.00  0.046  2.22  0.38  0.01  2.25  1.46  0.46  4.14  4.41  4.96
-2340.00  0.047  2.22  0.38  0.01  2.25  1.47  0.47  4.23  4.50  5.04
-2340.00  0.048  2.22  0.38  0.01  2.26  1.48  0.48  4.32  4.59  5.12
-2340.00  0.049  2.22  0.39  0.01  2.26  1.49  0.49  4.41  4.68  5.20
-2340.00  0.050  2.23  0.39  0.01  2.26  1.50  0.50  4.50  4.77  5.28
-2340.00  0.051  2.23  0.40  0.01  2.26  1.51  0.51  4.59  4.86  5.36
-2340.00  0.052  2.23  0.40  0.01  2.26  1.52  0.52  4.68  4.95  5.44
-2340.00  0.053  2.23  0.40  0.01  2.27  1.53  0.53  4.77  5.04  5.52
-2340.00  0.054  2.23  0.41  0.01  2.27  1.54  0.54  4.86  5.13  5.61
-2340.00  0.055  2.23  0.41  0.01  2.27  1.55  0.55  4.95  5.22  5.69
-2340.00  0.056  2.23  0.41  0.01  2.27  1.56  0.56  5.04  5.31  5.77
-2340.00  0.057  2.23  0.42  0.01  2.27  1.57  0.57  5.13  5.40  5.85
-2340.00  0.058  2.23  0.42  0.01  2.27  1.58  0.58  5.22  5.48  5.94
-2340.00  0.059  2.24  0.43  0.01  2.28  1.59  0.59  5.31  5.57  6.02
-2340.00  0.060  2.24  0.43  0.01  2.28  1.60  0.60  5.40  5.66  6.10
-2340.00  0.061  2.24  0.43  0.01  2.28  1.61  0.61  5.49  5.75  6.19
-2340.00  0.062  2.24  0.44  0.01  2.28  1.62  0.62  5.58  5.84  6.27
-2340.00  0.063  2.24  0.44  0.01  2.28  1.63  0.63  5.67  5.93  6.36
-2340.00  0.064  2.24  0.44  0.01  2.28  1.64  0.64  5.76  6.02  6.44
-2340.00  0.065  2.24  0.45  0.01  2.29  1.65  0.65  5.85  6.11  6.53
-2340.00  0.066  2.24  0.45  0.01  2.29  1.66  0.66  5.94  6.20  6.61
-2340.00  0.067  2.24  0.45  0.01  2.29  1.67  0.67  6.03  6.29  6.70
-2340.00  0.068  2.24  0.46  0.01  2.29  1.68  0.68  6.12  6.38  6.78
-2340.00  0.069  2.25  0.46  0.01  2.29  1.69  0.69  6.21  6.47  6.87

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-2340.00	0.070	2.25	0.46	0.01	2.29	1.70	0.70	6.30	6.56	6.95
-2340.00	0.071	2.25	0.47	0.01	2.30	1.71	0.71	6.39	6.65	7.04
-2340.00	0.072	2.25	0.47	0.01	2.30	1.72	0.72	6.48	6.74	7.12
-2340.00	0.073	2.25	0.47	0.01	2.30	1.73	0.73	6.57	6.83	7.21
-2340.00	0.074	2.25	0.48	0.01	2.30	1.74	0.74	6.66	6.92	7.30
-2340.00	0.075	2.25	0.48	0.01	2.30	1.75	0.75	6.75	7.01	7.38
-2340.00	0.076	2.25	0.48	0.01	2.30	1.76	0.76	6.84	7.10	7.47
-2340.00	0.077	2.25	0.49	0.01	2.31	1.77	0.77	6.93	7.19	7.55
-2340.00	0.078	2.26	0.49	0.01	2.31	1.78	0.78	7.02	7.28	7.64
-2340.00	0.079	2.26	0.49	0.01	2.31	1.79	0.79	7.11	7.37	7.73
-2340.00	0.080	2.26	0.50	0.01	2.31	1.80	0.80	7.20	7.46	7.81
-2340.00	0.081	2.26	0.50	0.01	2.31	1.81	0.81	7.29	7.55	7.90
-2340.00	0.082	2.26	0.50	0.01	2.31	1.82	0.82	7.38	7.65	7.99
-2340.00	0.083	2.26	0.50	0.01	2.32	1.83	0.83	7.47	7.74	8.07
-2340.00	0.084	2.26	0.51	0.02	2.32	1.84	0.84	7.56	7.83	8.16
-2340.00	0.085	2.26	0.51	0.02	2.32	1.85	0.85	7.65	7.92	8.25
-2340.00	0.086	2.26	0.51	0.02	2.32	1.86	0.86	7.74	8.01	8.34
-2340.00	0.087	2.26	0.52	0.02	2.32	1.87	0.87	7.83	8.10	8.42
-2340.00	0.088	2.27	0.52	0.02	2.32	1.88	0.88	7.92	8.19	8.51
-2340.00	0.089	2.27	0.52	0.02	2.33	1.89	0.89	8.01	8.28	8.60
-2340.00	0.090	2.27	0.53	0.02	2.33	1.90	0.90	8.10	8.37	8.69
-2340.00	0.091	2.27	0.53	0.02	2.33	1.91	0.91	8.19	8.46	8.77
-2340.00	0.092	2.27	0.53	0.02	2.33	1.92	0.92	8.28	8.55	8.86
-2340.00	0.093	2.27	0.53	0.02	2.33	1.93	0.93	8.37	8.64	8.95
-2340.00	0.094	2.27	0.54	0.02	2.33	1.94	0.94	8.46	8.73	9.04
-2340.00	0.095	2.27	0.54	0.02	2.34	1.95	0.95	8.55	8.82	9.12
-2340.00	0.096	2.27	0.54	0.02	2.34	1.96	0.96	8.64	8.91	9.21
-2340.00	0.097	2.27	0.55	0.02	2.34	1.97	0.97	8.73	9.00	9.30
-2340.00	0.098	2.28	0.55	0.02	2.34	1.98	0.98	8.82	9.09	9.39
-2340.00	0.099	2.28	0.55	0.02	2.34	1.99	0.99	8.91	9.18	9.48
-2340.00	0.100	2.28	0.55	0.02	2.34	2.00	1.00	9.00	9.27	9.57

```

!Input parameters and constraints are as follows:
!Ael (ppb), Rel (GHz), Tmsr (days) = -234.000    6.500    95.400
!Relative dummy thickness, Tbkg (days) = 10. 10.600004
!Relative sys. error on Amsr, Abkg (%) 0.01 0.01
!df_stat_rel, df_sys_rel = 0.000 0.100

!Abkg(ppb) Rb/Rs dAm dAb df dAE/AE dAm dAb df dAE/AE dAE/AE
!           <----statistical----> <----systematic----> TOTAL
2340.00  0.001  2.17  0.06  0.00  2.17  0.99  0.01  0.11  1.00  2.39
2340.00  0.002  2.17  0.08  0.00  2.18  0.98  0.02  0.22  1.00  2.40
2340.00  0.003  2.18  0.10  0.00  2.18  0.97  0.03  0.33  1.03  2.41
2340.00  0.004  2.18  0.11  0.00  2.18  0.96  0.04  0.44  1.06  2.42
2340.00  0.005  2.18  0.12  0.00  2.18  0.95  0.05  0.55  1.10  2.44
2340.00  0.006  2.18  0.14  0.00  2.18  0.94  0.06  0.66  1.15  2.47
2340.00  0.007  2.18  0.15  0.00  2.18  0.93  0.07  0.77  1.21  2.50
2340.00  0.008  2.18  0.16  0.00  2.19  0.92  0.08  0.88  1.28  2.53
2340.00  0.009  2.18  0.17  0.00  2.19  0.91  0.09  0.99  1.35  2.57
2340.00  0.010  2.18  0.18  0.00  2.19  0.90  0.10  1.10  1.42  2.61
2340.00  0.011  2.18  0.18  0.00  2.19  0.89  0.11  1.21  1.51  2.66
2340.00  0.012  2.19  0.19  0.00  2.19  0.88  0.12  1.32  1.59  2.71
2340.00  0.013  2.19  0.20  0.00  2.20  0.87  0.13  1.43  1.68  2.76
2340.00  0.014  2.19  0.21  0.00  2.20  0.86  0.14  1.54  1.77  2.82
2340.00  0.015  2.19  0.21  0.00  2.20  0.85  0.15  1.65  1.86  2.88
2340.00  0.016  2.19  0.22  0.00  2.20  0.84  0.16  1.76  1.96  2.94
2340.00  0.017  2.19  0.23  0.00  2.20  0.83  0.17  1.87  2.05  3.01
2340.00  0.018  2.19  0.23  0.00  2.20  0.82  0.18  1.98  2.15  3.08
2340.00  0.019  2.19  0.24  0.00  2.21  0.81  0.19  2.09  2.25  3.15
2340.00  0.020  2.19  0.25  0.00  2.21  0.80  0.20  2.20  2.35  3.22
2340.00  0.021  2.19  0.25  0.00  2.21  0.79  0.21  2.31  2.45  3.30
2340.00  0.022  2.20  0.26  0.00  2.21  0.78  0.22  2.42  2.55  3.38
2340.00  0.023  2.20  0.27  0.01  2.21  0.77  0.23  2.53  2.65  3.46
2340.00  0.024  2.20  0.27  0.01  2.21  0.76  0.24  2.64  2.76  3.54
2340.00  0.025  2.20  0.28  0.01  2.22  0.75  0.25  2.75  2.86  3.62
2340.00  0.026  2.20  0.28  0.01  2.22  0.74  0.26  2.86  2.97  3.70
2340.00  0.027  2.20  0.29  0.01  2.22  0.73  0.27  2.97  3.07  3.79
2340.00  0.028  2.20  0.29  0.01  2.22  0.72  0.28  3.08  3.18  3.88
2340.00  0.029  2.20  0.30  0.01  2.22  0.71  0.29  3.19  3.28  3.96
2340.00  0.030  2.20  0.30  0.01  2.23  0.70  0.30  3.30  3.39  4.05
2340.00  0.031  2.21  0.31  0.01  2.23  0.69  0.31  3.41  3.49  4.14
2340.00  0.032  2.21  0.31  0.01  2.23  0.68  0.32  3.52  3.60  4.23
2340.00  0.033  2.21  0.32  0.01  2.23  0.67  0.33  3.63  3.71  4.33
2340.00  0.034  2.21  0.32  0.01  2.23  0.66  0.34  3.74  3.81  4.42
2340.00  0.035  2.21  0.33  0.01  2.23  0.65  0.35  3.85  3.92  4.51
2340.00  0.036  2.21  0.33  0.01  2.24  0.64  0.36  3.96  4.03  4.61
2340.00  0.037  2.21  0.34  0.01  2.24  0.63  0.37  4.07  4.14  4.70
2340.00  0.038  2.21  0.34  0.01  2.24  0.62  0.38  4.18  4.24  4.80
2340.00  0.039  2.21  0.35  0.01  2.24  0.61  0.39  4.29  4.35  4.89
2340.00  0.040  2.22  0.35  0.01  2.24  0.60  0.40  4.40  4.46  4.99
2340.00  0.041  2.22  0.35  0.01  2.24  0.59  0.41  4.51  4.57  5.09
2340.00  0.042  2.22  0.36  0.01  2.25  0.58  0.42  4.62  4.68  5.19
2340.00  0.043  2.22  0.36  0.01  2.25  0.57  0.43  4.73  4.78  5.29
2340.00  0.044  2.22  0.37  0.01  2.25  0.56  0.44  4.84  4.89  5.38
2340.00  0.045  2.22  0.37  0.01  2.25  0.55  0.45  4.95  5.00  5.48
2340.00  0.046  2.22  0.38  0.01  2.25  0.54  0.46  5.06  5.11  5.58
2340.00  0.047  2.22  0.38  0.01  2.25  0.53  0.47  5.17  5.22  5.68
2340.00  0.048  2.22  0.38  0.01  2.26  0.52  0.48  5.28  5.33  5.79
2340.00  0.049  2.22  0.39  0.01  2.26  0.51  0.49  5.39  5.44  5.89
2340.00  0.050  2.23  0.39  0.01  2.26  0.50  0.50  5.50  5.55  5.99
2340.00  0.051  2.23  0.40  0.01  2.26  0.49  0.51  5.61  5.65  6.09
2340.00  0.052  2.23  0.40  0.01  2.26  0.48  0.52  5.72  5.76  6.19
2340.00  0.053  2.23  0.40  0.01  2.27  0.47  0.53  5.83  5.87  6.29
2340.00  0.054  2.23  0.41  0.01  2.27  0.46  0.54  5.94  5.98  6.40
2340.00  0.055  2.23  0.41  0.01  2.27  0.45  0.55  6.05  6.09  6.50
2340.00  0.056  2.23  0.41  0.01  2.27  0.44  0.56  6.16  6.20  6.60
2340.00  0.057  2.23  0.42  0.01  2.27  0.43  0.57  6.27  6.31  6.71
2340.00  0.058  2.23  0.42  0.01  2.27  0.42  0.58  6.38  6.42  6.81
2340.00  0.059  2.24  0.43  0.01  2.28  0.41  0.59  6.49  6.53  6.91
2340.00  0.060  2.24  0.43  0.01  2.28  0.40  0.60  6.60  6.64  7.02
2340.00  0.061  2.24  0.43  0.01  2.28  0.39  0.61  6.71  6.75  7.12
2340.00  0.062  2.24  0.44  0.01  2.28  0.38  0.62  6.82  6.86  7.23
2340.00  0.063  2.24  0.44  0.01  2.28  0.37  0.63  6.93  6.97  7.33
2340.00  0.064  2.24  0.44  0.01  2.28  0.36  0.64  7.04  7.08  7.44
2340.00  0.065  2.24  0.45  0.01  2.29  0.35  0.65  7.15  7.19  7.54
2340.00  0.066  2.24  0.45  0.01  2.29  0.34  0.66  7.26  7.30  7.65
2340.00  0.067  2.24  0.45  0.01  2.29  0.33  0.67  7.37  7.41  7.75
2340.00  0.068  2.24  0.46  0.01  2.29  0.32  0.68  7.48  7.52  7.86
2340.00  0.069  2.25  0.46  0.02  2.29  0.31  0.69  7.59  7.63  7.96

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2340.00	0.070	2.25	0.46	0.02	2.29	0.30	0.70	7.70	7.74	8.07
2340.00	0.071	2.25	0.47	0.02	2.30	0.29	0.71	7.81	7.85	8.18
2340.00	0.072	2.25	0.47	0.02	2.30	0.28	0.72	7.92	7.96	8.28
2340.00	0.073	2.25	0.47	0.02	2.30	0.27	0.73	8.03	8.07	8.39
2340.00	0.074	2.25	0.48	0.02	2.30	0.26	0.74	8.14	8.18	8.50
2340.00	0.075	2.25	0.48	0.02	2.30	0.25	0.75	8.25	8.29	8.60
2340.00	0.076	2.25	0.48	0.02	2.30	0.24	0.76	8.36	8.40	8.71
2340.00	0.077	2.25	0.49	0.02	2.31	0.23	0.77	8.47	8.51	8.82
2340.00	0.078	2.26	0.49	0.02	2.31	0.22	0.78	8.58	8.62	8.92
2340.00	0.079	2.26	0.49	0.02	2.31	0.21	0.79	8.69	8.73	9.03
2340.00	0.080	2.26	0.50	0.02	2.31	0.20	0.80	8.80	8.84	9.14
2340.00	0.081	2.26	0.50	0.02	2.31	0.19	0.81	8.91	8.95	9.24
2340.00	0.082	2.26	0.50	0.02	2.31	0.18	0.82	9.02	9.06	9.35
2340.00	0.083	2.26	0.50	0.02	2.32	0.17	0.83	9.13	9.17	9.46
2340.00	0.084	2.26	0.51	0.02	2.32	0.16	0.84	9.24	9.28	9.56
2340.00	0.085	2.26	0.51	0.02	2.32	0.15	0.85	9.35	9.39	9.67
2340.00	0.086	2.26	0.51	0.02	2.32	0.14	0.86	9.46	9.50	9.78
2340.00	0.087	2.26	0.52	0.02	2.32	0.13	0.87	9.57	9.61	9.89
2340.00	0.088	2.27	0.52	0.02	2.32	0.12	0.88	9.68	9.72	9.99
2340.00	0.089	2.27	0.52	0.02	2.33	0.11	0.89	9.79	9.83	10.10
2340.00	0.090	2.27	0.53	0.02	2.33	0.10	0.90	9.90	9.94	10.21
2340.00	0.091	2.27	0.53	0.02	2.33	0.09	0.91	10.01	10.05	10.32
2340.00	0.092	2.27	0.53	0.02	2.33	0.08	0.92	10.12	10.16	10.43
2340.00	0.093	2.27	0.53	0.02	2.33	0.07	0.93	10.23	10.27	10.53
2340.00	0.094	2.27	0.54	0.02	2.33	0.06	0.94	10.34	10.38	10.64
2340.00	0.095	2.27	0.54	0.02	2.34	0.05	0.95	10.45	10.49	10.75
2340.00	0.096	2.27	0.54	0.02	2.34	0.04	0.96	10.56	10.60	10.86
2340.00	0.097	2.27	0.55	0.02	2.34	0.03	0.97	10.67	10.71	10.97
2340.00	0.098	2.28	0.55	0.02	2.34	0.02	0.98	10.78	10.82	11.07
2340.00	0.099	2.28	0.55	0.02	2.34	0.01	0.99	10.89	10.93	11.18
2340.00	0.100	2.28	0.55	0.02	2.34	0.00	1.00	11.00	11.05	11.29

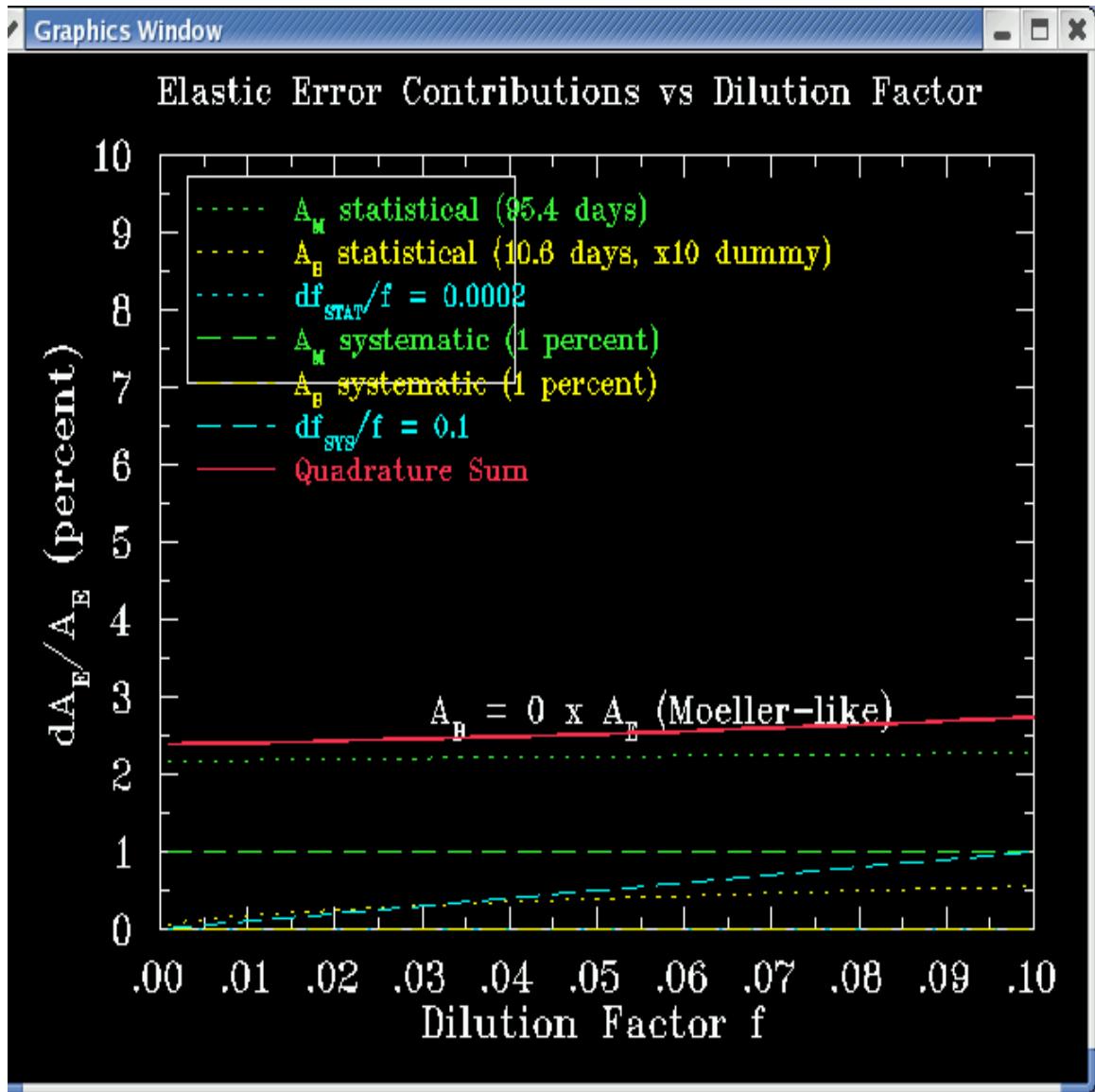


Figure 2: Contributions to the final error on A_E under the assumptions listed in the legend. The background assumed here, $A_B = 0$, resembles that for multi-bounce soft backgrounds originating from Moeller scattering in the target. In this case, the systematic error from df is almost negligible for a dilution of $O(0.01)$ and $df/f(\text{systematic}) = 10\%$.

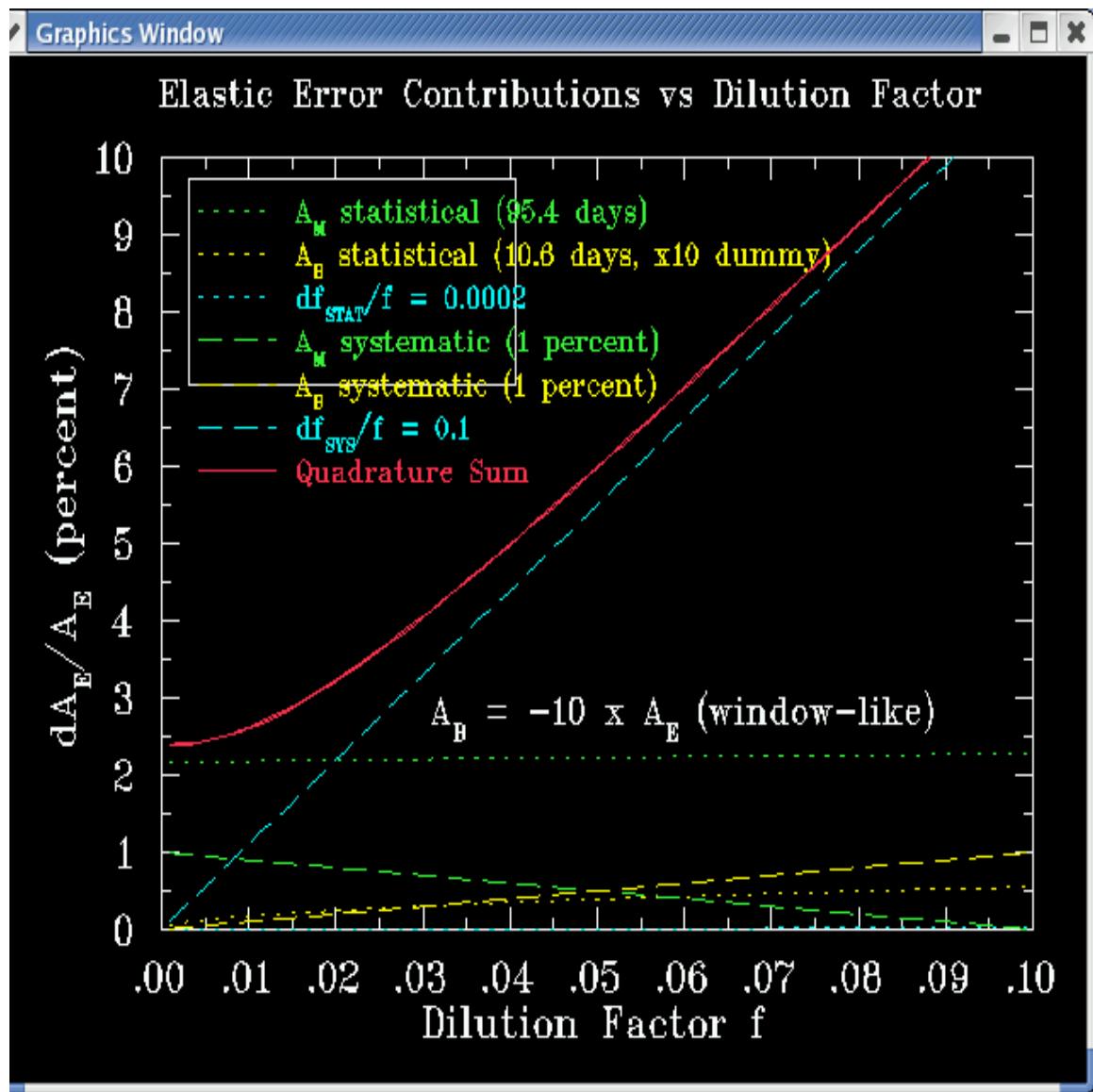


Figure 3: Contributions to the final error on A_E under the assumptions listed in the legend. The background assumed here, $A_B = -10A_E$, resembles elastic scattering from Al or Be target windows. The blue long-dashed line shows that the systematic error from df can be important for a dilution of $O(0.01)$ and $df/f(\text{systematic}) = 10\%$.

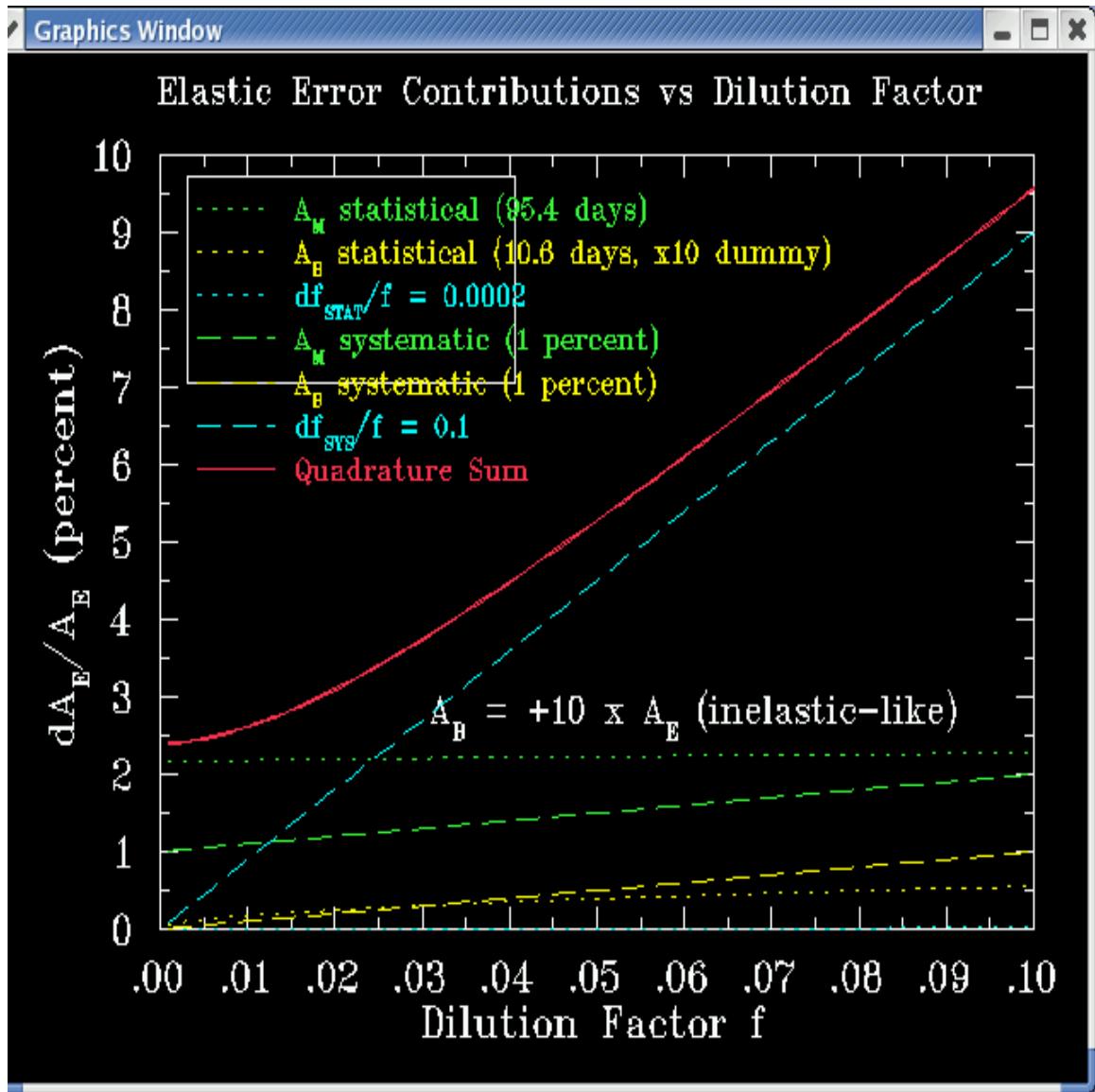


Figure 4: Contributions to the final error on A_E under the assumptions listed in the legend. The background assumed here, $A_B = +10A_E$, resembles that for scattered electrons from pion production on Hydrogen. The blue long-dashed line shows that the systematic error from df can be important for a dilution of $O(0.01)$ and $df/f(\text{systematic}) = 10\%$.