THE ART OF PROBABILITY ASSIGNMENT

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Abstract

The problem of assingning probabilities when very little is known is analized in the case where the quantity of interest is a physical observable, i.e. can be measured and its value expressed by a number. It is pointed out that the assignment of probabilities based on observations is a process of inference and an essential link between observable sample frequencies and probabilities is the Bayes' theorem where a prior in probability space enters the game. When very many measurements are performed the inferred probabilities are increasingly insensitive to that prior, but this is not the case when we try to assign "non-informative" probabilities. It is suggested that this assignment should be done such that the resulting probability distribution is the least sensitive to specific variations in the prior. In the discrete case an assignment rule is obtained which calls for the minimization of the Renyi's distance of degree 2

$$D_2(p) = \ln \sum_{k=1}^{n-1} p_k \frac{p_k}{p_{k+1}}$$

between the original and the shifted distribution, whereas in the continuous case the corresponding quantity to be minimized is the Fisher information

$$F[p] = \int dx p(x) \left[\frac{\partial \ln p(x)}{\partial x}\right]^2$$

The role of the constraints representing incomplete information is also elucidated and arguments against probability updating schemes are proposed.

Key Words: Maximum Entropy, non-informative prior