Phy 402G
Dale

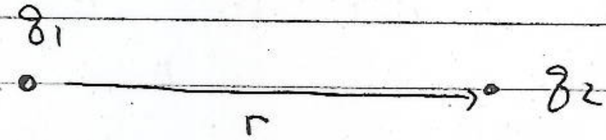
Electric charge

- Two kinds + & −
- quantized 1.6×10^{-19} C (often ignore this)
- conserved
- opposites attract, same sign repell

Electric force:

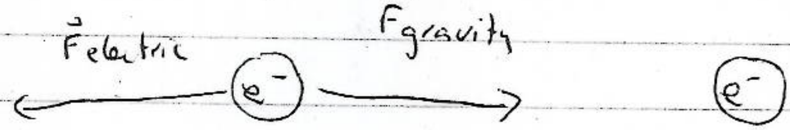
- Given by Coulomb's law

$$F = k \frac{q_1 q_2}{r^2}$$



- Electric force is strong

Two electrons

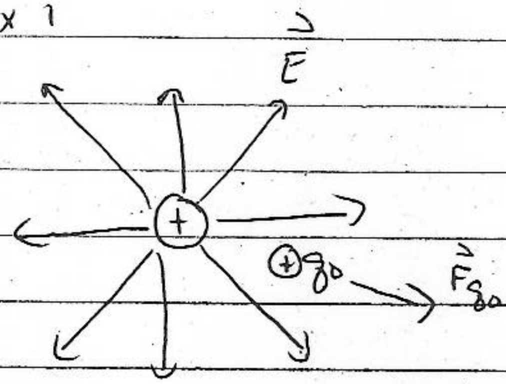


$$\frac{|\vec{F}_e|}{|\vec{F}_{gravity}|} \approx 10^{42}$$

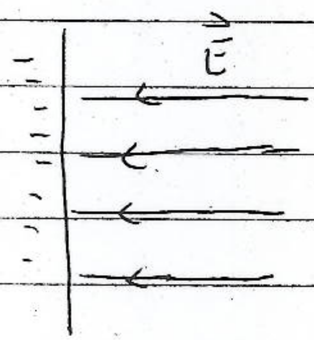
Electric Field

- magnitude $\frac{\text{Force}}{q_0}$ where q_0 is (small) test charge.
- direction: Direction a ⊕ test charge would go

Ex 1



Ex 2

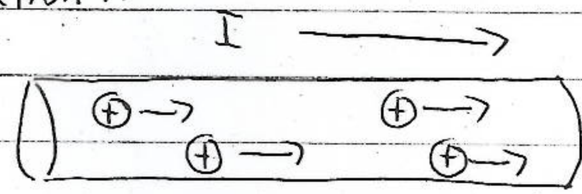


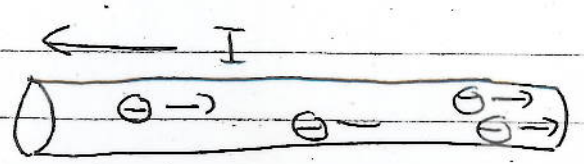
Current (C/s or Amperes)

magnitude: rate of flow of charge

$$I = \frac{dQ}{dt}$$

direction:





Drift speed in wire

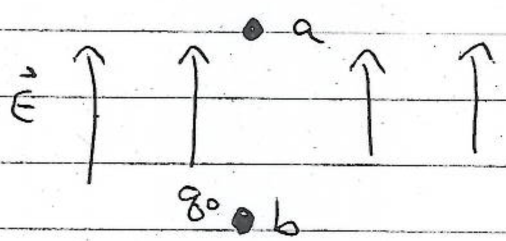
$$\frac{4}{1000}\text{"}/\text{sec}$$

Signal speed:

$$\frac{12 \text{ billion inches}}{\text{sec}}$$

Voltage (Potential difference)

$$\Delta V = \frac{\text{work}}{\text{charge}}$$

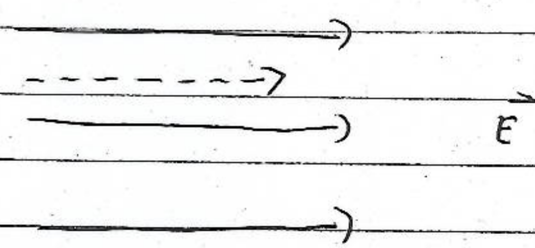


1) Take some q_0 at b.
2) Move it to a
3) measure work to do this
4) $V_b - V_a = \frac{\text{Work}}{q_0}$ → indep of q_0

Voltage is a relative quantity, describes the space

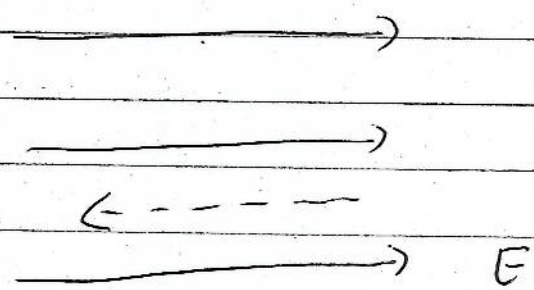
Potential energy describes a particle in space

When you go with a field line:



Voltage decreases

Against a field line

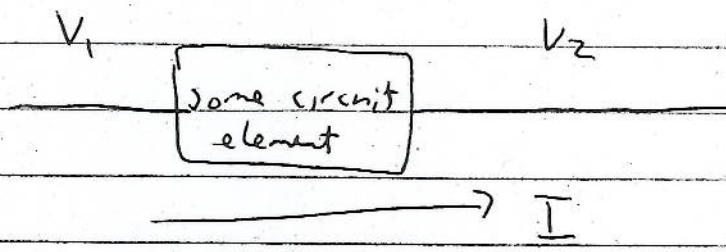


Voltage increases

+ charge "wants" to go from higher to lower voltage

− lower to higher

<u>Resistance</u> (Ohms or Ω)



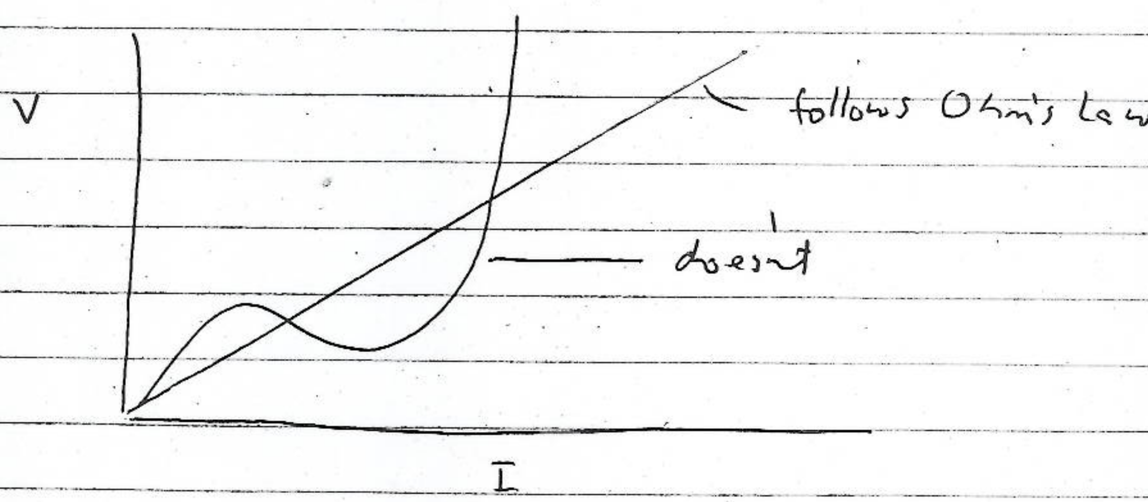
$$R = \frac{dV}{dI}$$

Often:

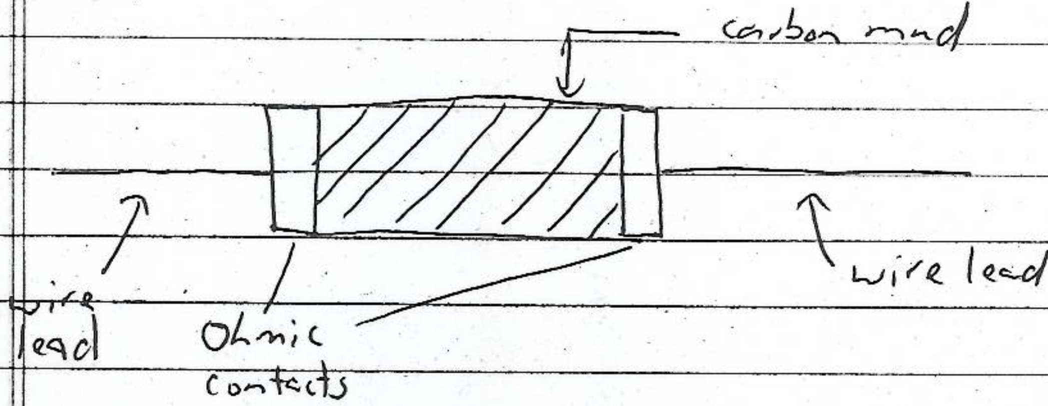
$$V = I R \qquad \text{Ohm's Law}$$

↑ constant

Ohm's Law is
- useful
- <u>Not</u> fundamental



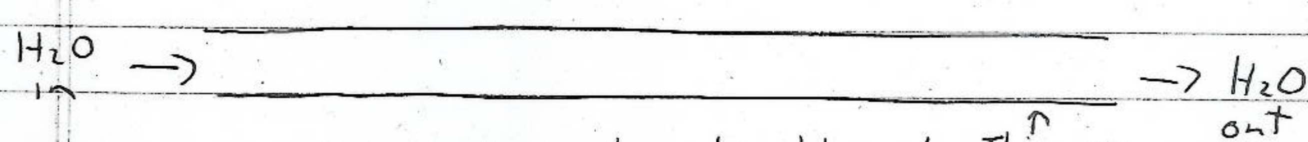
Typical resistor



Symbol:

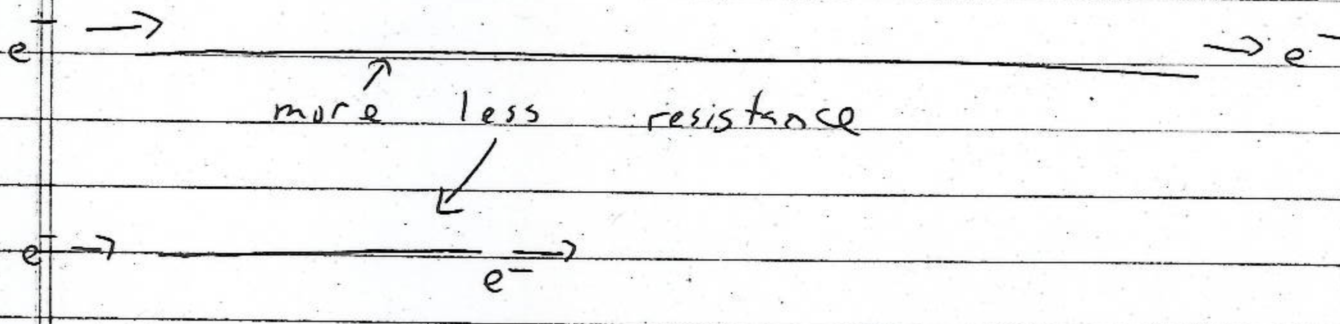
<u>Resistivity</u>

Consider water pipe



Harder to push water through This vs this

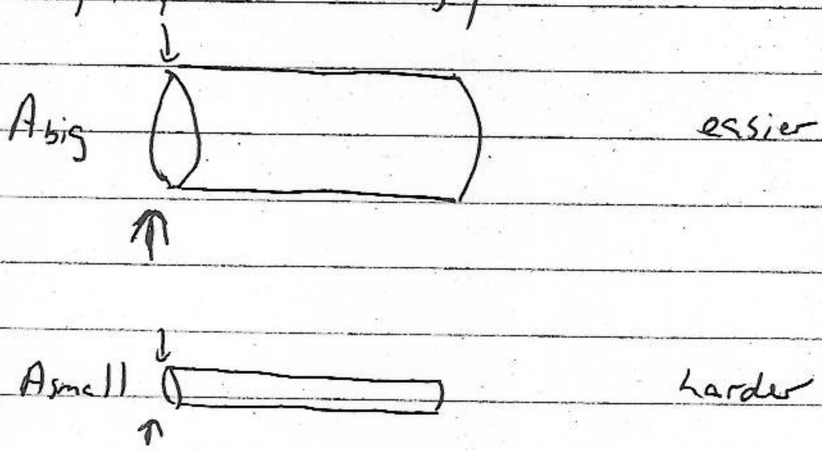
Wire



$R \propto L$

↑ length of wire

Again, water analogy



$R \propto \frac{L}{A}$ ← cross sectional area

$$R = \rho \, {}^{L}/_{A}$$

L ← length

A ← area

R ↖ resistance

ρ ↑ some constant: resistivity

ρ: bulk property

low ρ: conductor
high ρ: insulator

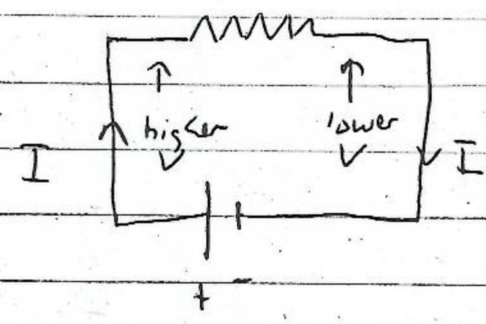
Conductors – charges move around easily
- metals
- humans
- salt solutions
- plasmas

Insulators – charges stuck to atoms/molecules
- plastic
- rubber
- glass
- quartz

(See resistor code in book.)

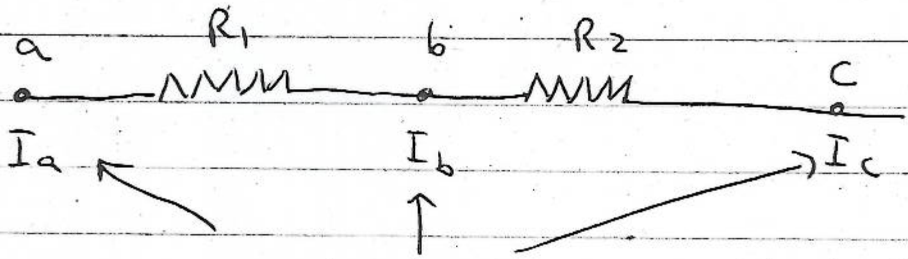
What good are resistors?
- Dissipate energy (heat, light)
- Drop voltage

To get ΔV → need I



current goes from high V to low V

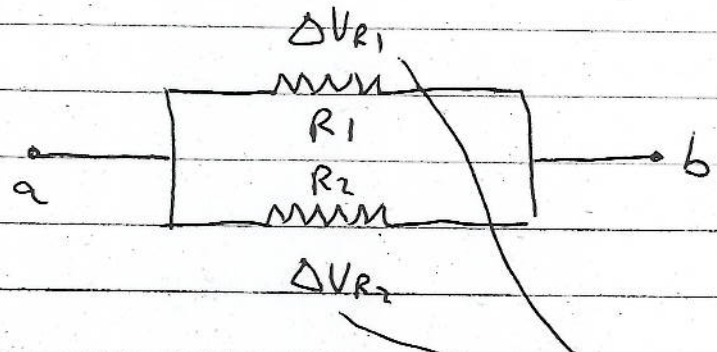
Resistors in series



same current (conservation of charge)

ΔV_{R_1} not necessarily ΔV_{R_2}

Resistors in parallel

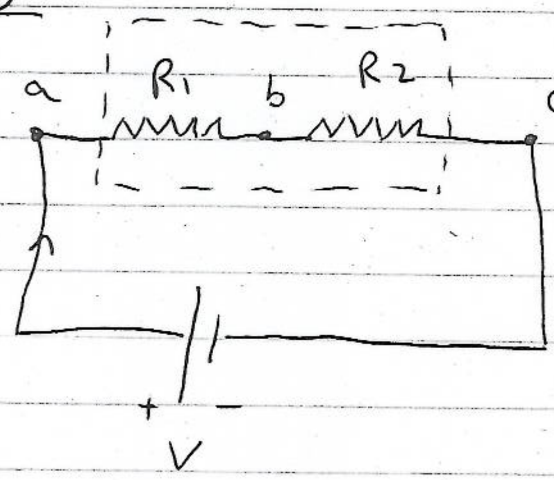


same voltages

→ conservation of energy
Potential difference btwn a & b is path independent.

I_{R_1} not necessarily I_{R_2}

Series



$$\left.\begin{aligned} V_a - V_b &= I R_1 \\ V_b - V_c &= I R_2 \end{aligned}\right\} \quad V_a - V_c = I(R_1 + R_2)$$

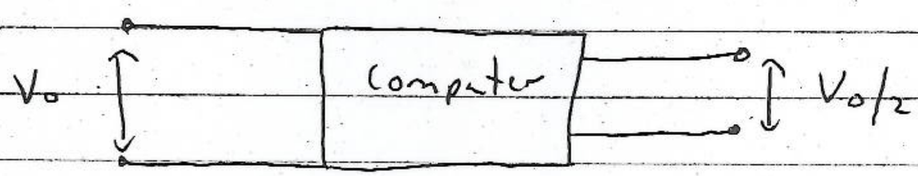
Replace with R_{eq}

General

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots}$$ series

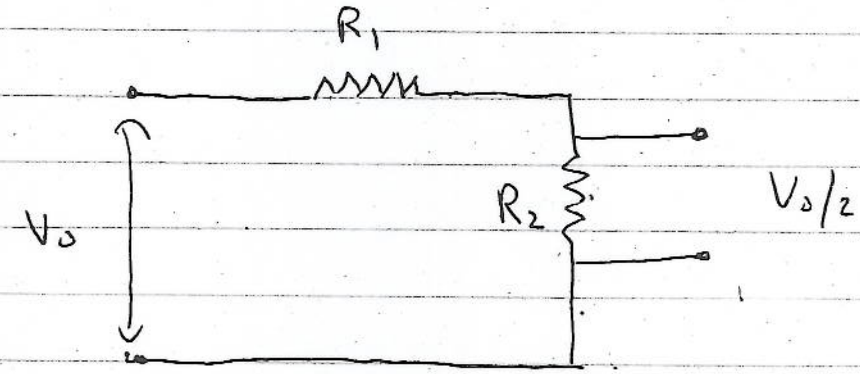
Exercise:

Analog computer $\div 2$



Input # in form of voltage V_0

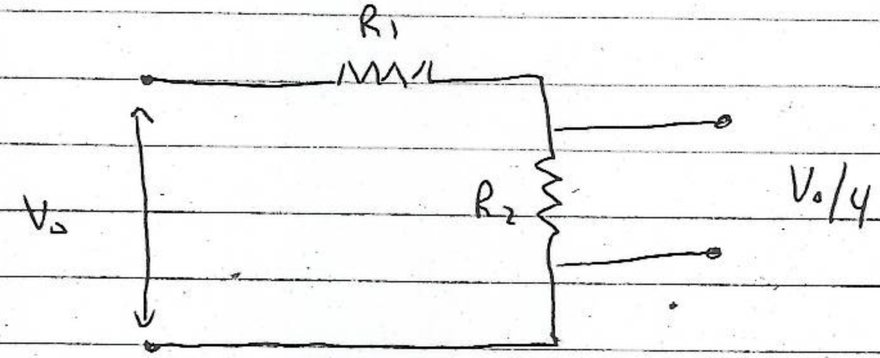
output answer $V_0/2$



If $R_1 = R_2 = R$ $\qquad V_{R_1} = V_{R_2} = V_0/2$

$IR \qquad IR$

Computer to ÷ by 4



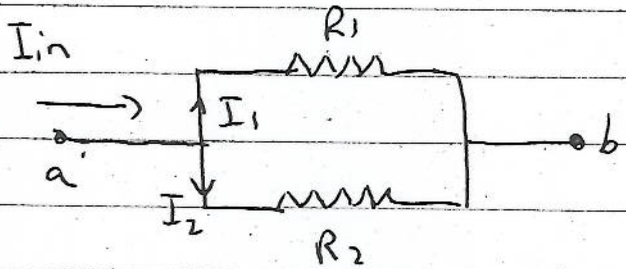
$$V_0 = IR_1 + IR_2$$

want $IR_2 = V_0/4$

$$IR_1 = 3/4\, V_0$$

$$\frac{R_2}{R_1} = \frac{1}{3}$$

Resistors in Parallel



- I_{in} splits into I_1 & I_2
- More goes to path of less resistance

$$I_{in} = I_1 + I_2 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Replace w/ R_{eq}

$$I_{in} = \frac{V_{ab}}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

General

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$ Resistors in parallel

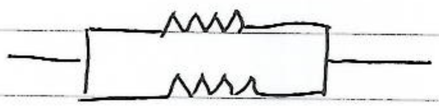
Note:

series

$$R_{eq} = R_1 + R_2$$

R_{eq} <u>bigger</u> than R_1 & R_2

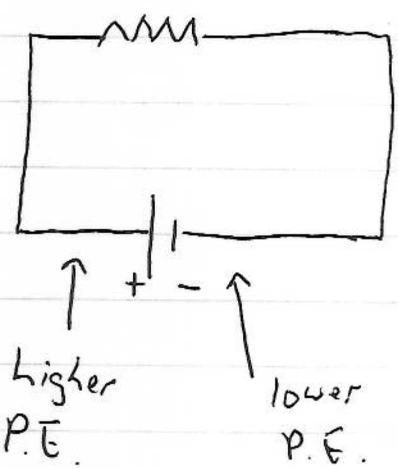
Parallel



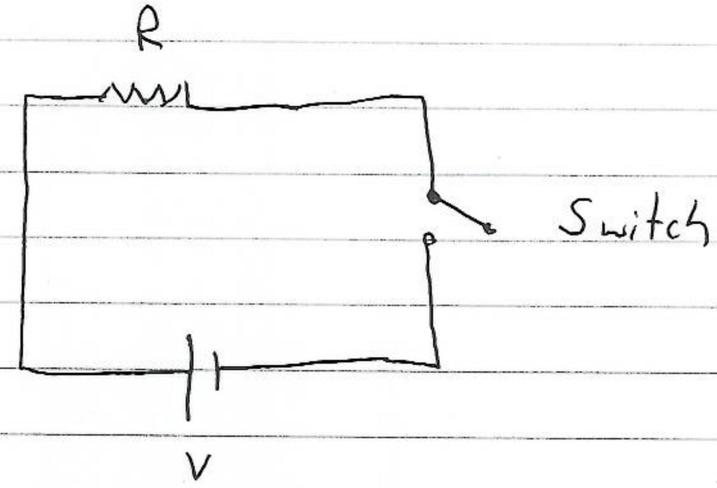
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

R_{eq} less than both R_1 & R_2
"extra path"

<u>Battery</u> - increases P.E. of charges in the circuit



Consider



1) Close switch

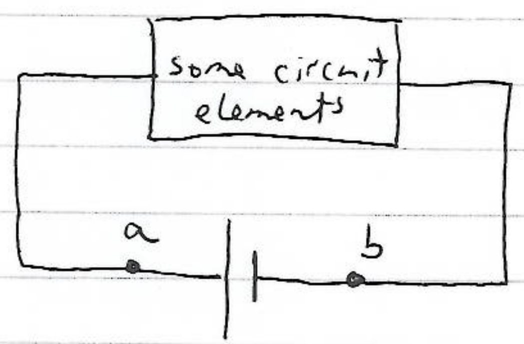
→ something complicated happens

2) Wait a while

→ circuit attains steady state

$I = \text{const}$.

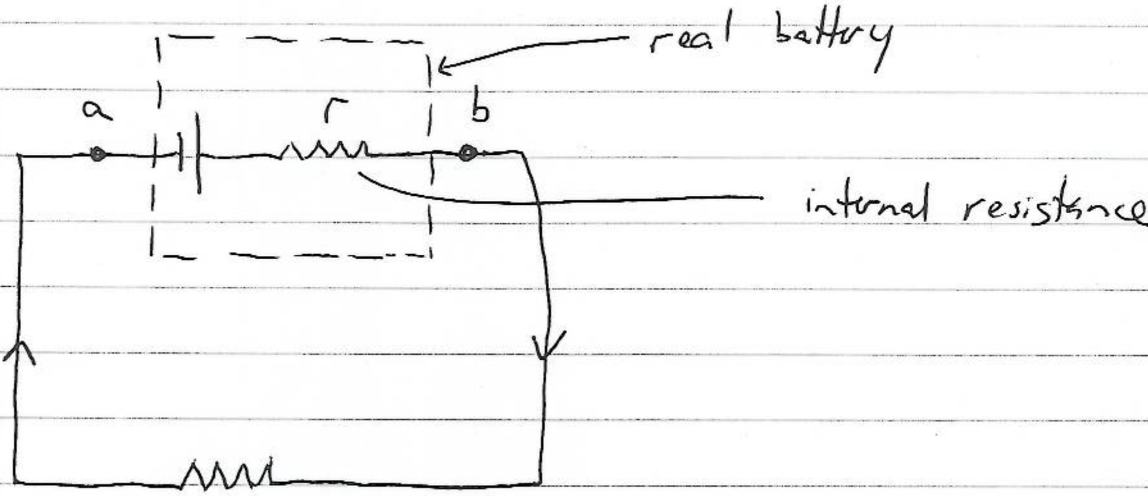
<u>Ideal Battery</u>



No matter what you do $V_a - V_b = \text{const}$.

Ideal battery is <u>not</u> a constant current source.
It <u>is</u> a constant voltage source.

Real battery (Expt 1)



Define

$\mathcal{E}$: open circuit voltage

→ terminal voltage of battery when $I = 0$

$$V_{ab} = \mathcal{E} - Ir$$

potential across battery

terminal voltage when $I=0$

Voltage drop due to internal resistance of battery

r will be measured in Lab #1

Power:

$$\text{Energy} = \text{charge} \times \text{Voltage}$$

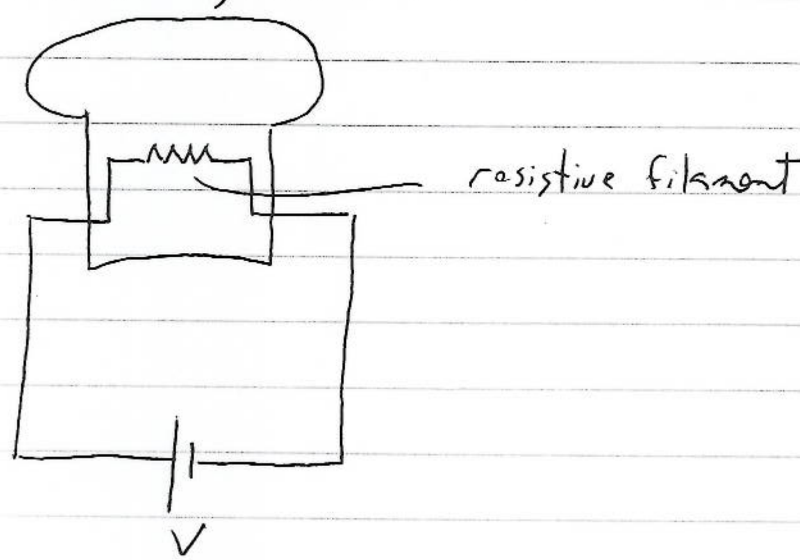
$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{\text{charge}}{\text{time}} \times \text{Voltage}$$

$$P = IV$$

$$V = IR$$

$$P = I^2R = V^2/R$$

The incandescent light bulb

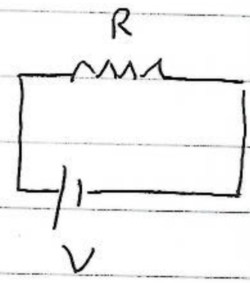


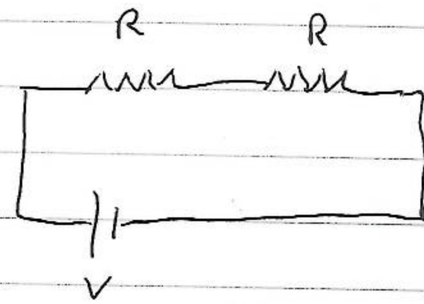
- $P = I^2R = V^2/R$
- filament heats up
- filament glows

more current → more light

Case 1

Compare for identical resistors



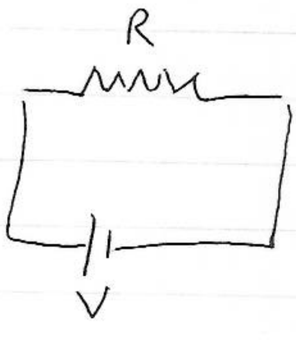


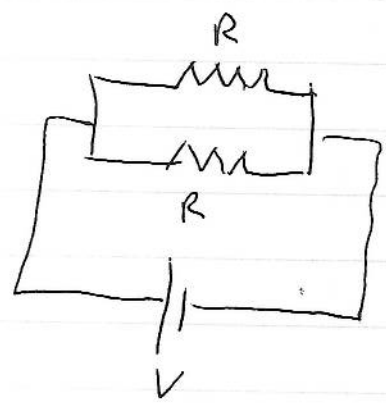
What are the relative brightnesses if each resistor is a lightbulb

$$P = V^2/R \qquad\qquad \frac{(V/2)^2}{R} \;\&\; \frac{(V/2)^2}{R}$$

Power down by 4

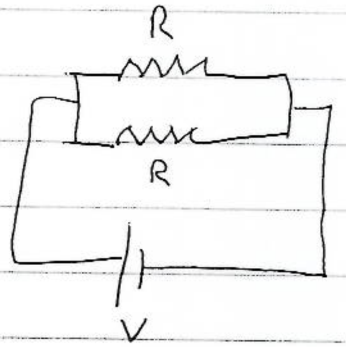
Case 2



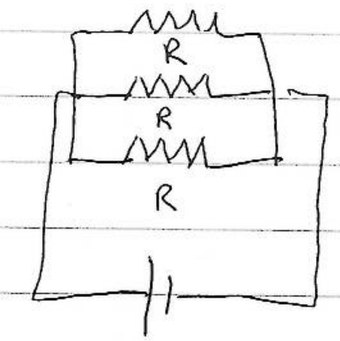


Same V, same Power, same brightnesses.

Case 3

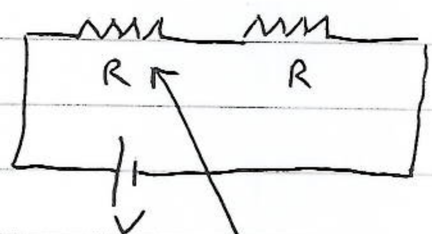


VS

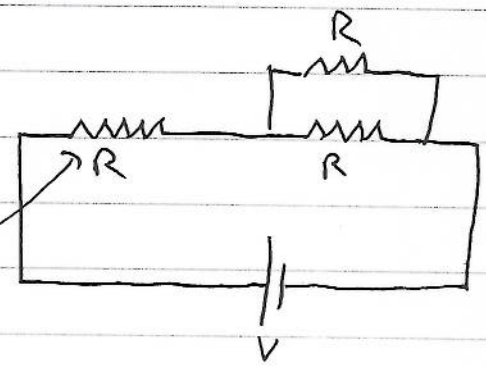


All the same brightnesses

Case 4

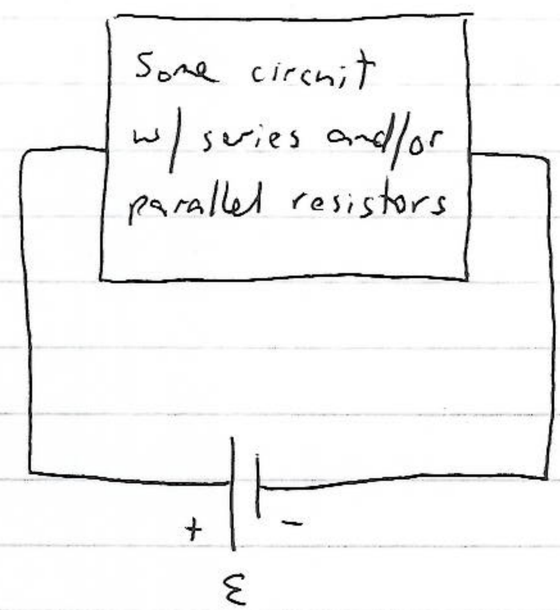


VS

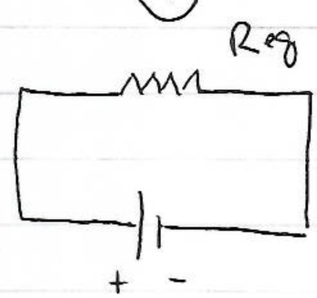


This gets brighter or dimmer?
(brighter)

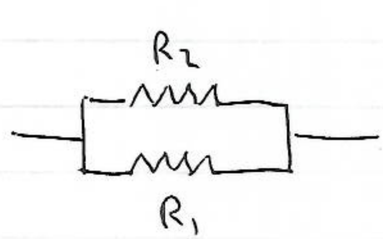
So far, we can deal with



$$\Downarrow$$

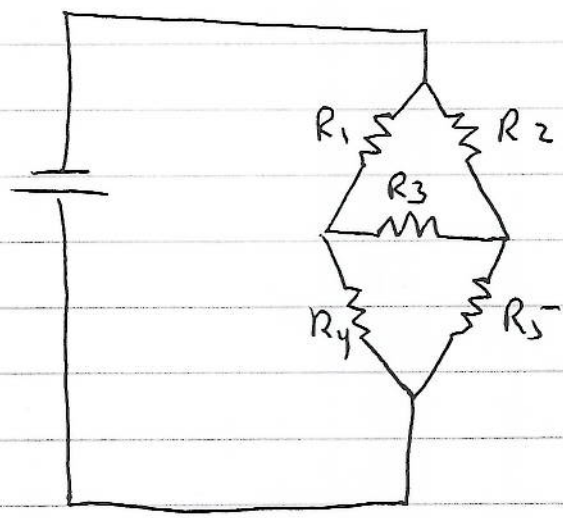


$$R_{eq} = R_1 + R_2$$

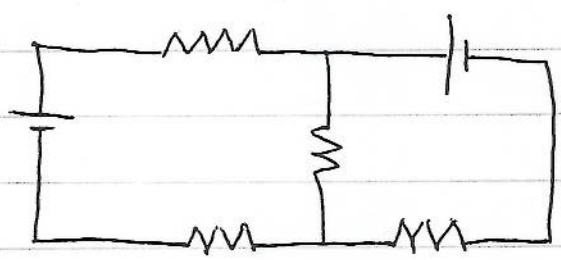


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

But what if ?



or

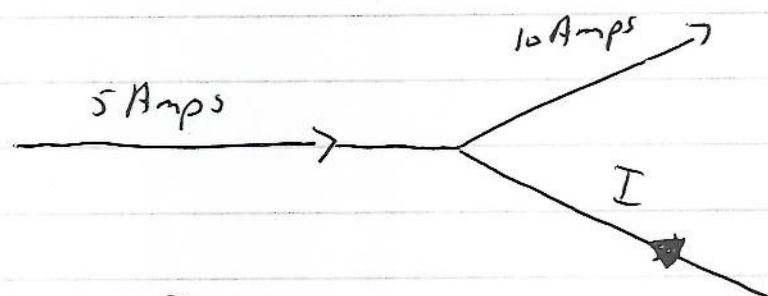


Nothing's in series, nothing's in parallel

New tools: Kirchhoff's Laws

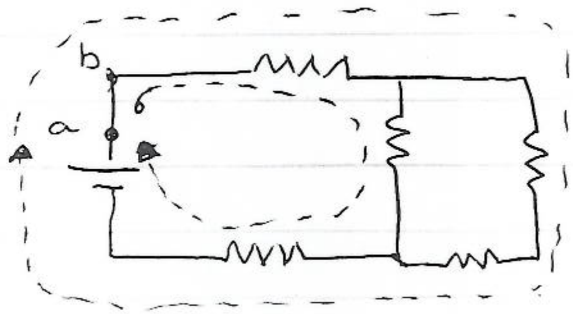
Kirchoff's Current Law (Junction Rule, node rule)
Sum of currents entering any junction = sum of those leaving. (charge conservation)

junction or node – Any point in circuit where current can split.



What is I?

Kirchhoff's Voltage Law (Loop Rule)
The algebraic sum of the changes in potential across all of the elements around any closed loop is zero. (energy conservation)

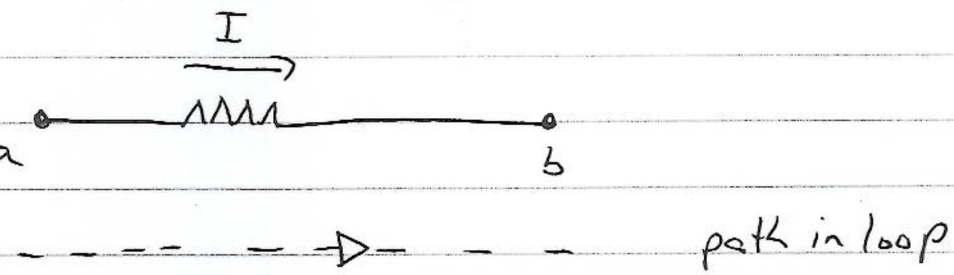


$V_a = V_b$

Stuff to know:

If you go around loop and

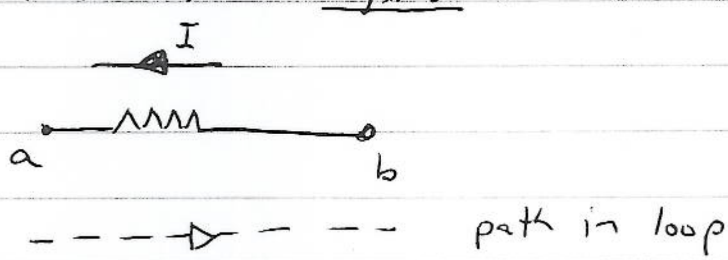
1) Cross a resistor <u>with</u> the current



Voltage (potential) drops

$$V_b - V_a = -IR$$

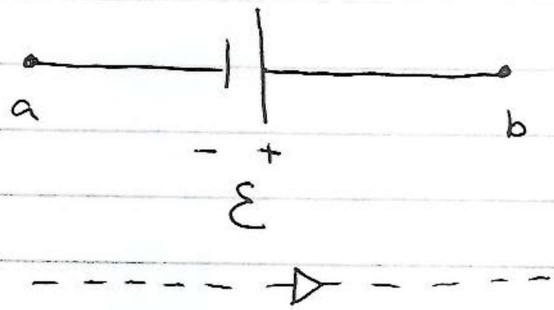
2) Cross a resistor <u>against</u> the current



Voltage increases

$$V_b - V_a = +IR$$

3) Cross a source of emf in the direction of the emf, i.e. from $-$ to $+$



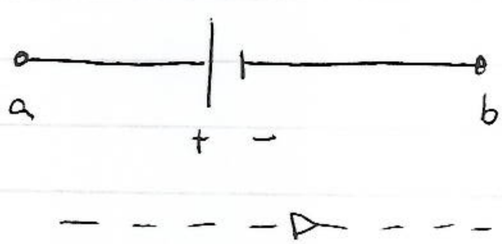
path in loop

Voltage increases

$$V_b - V_a = + \mathcal{E}$$

(For ideal emf, current direction does not matter)

4) Cross a source of emf opposite to direction of emf, i.e. from $+$ to $-$



path in loop

Voltage decreases

$$V_b - V_a = - \epsilon$$

Phil here

General approach to circuit analysis

1) Draw diagram, labeling R's & emfs

2) Simplify circuit where possible

$\left.\begin{array}{l}\text{series}\\ \text{parallel}\end{array}\right\}$ resistors $\longrightarrow R_{eq}$

3) Apply junction rule

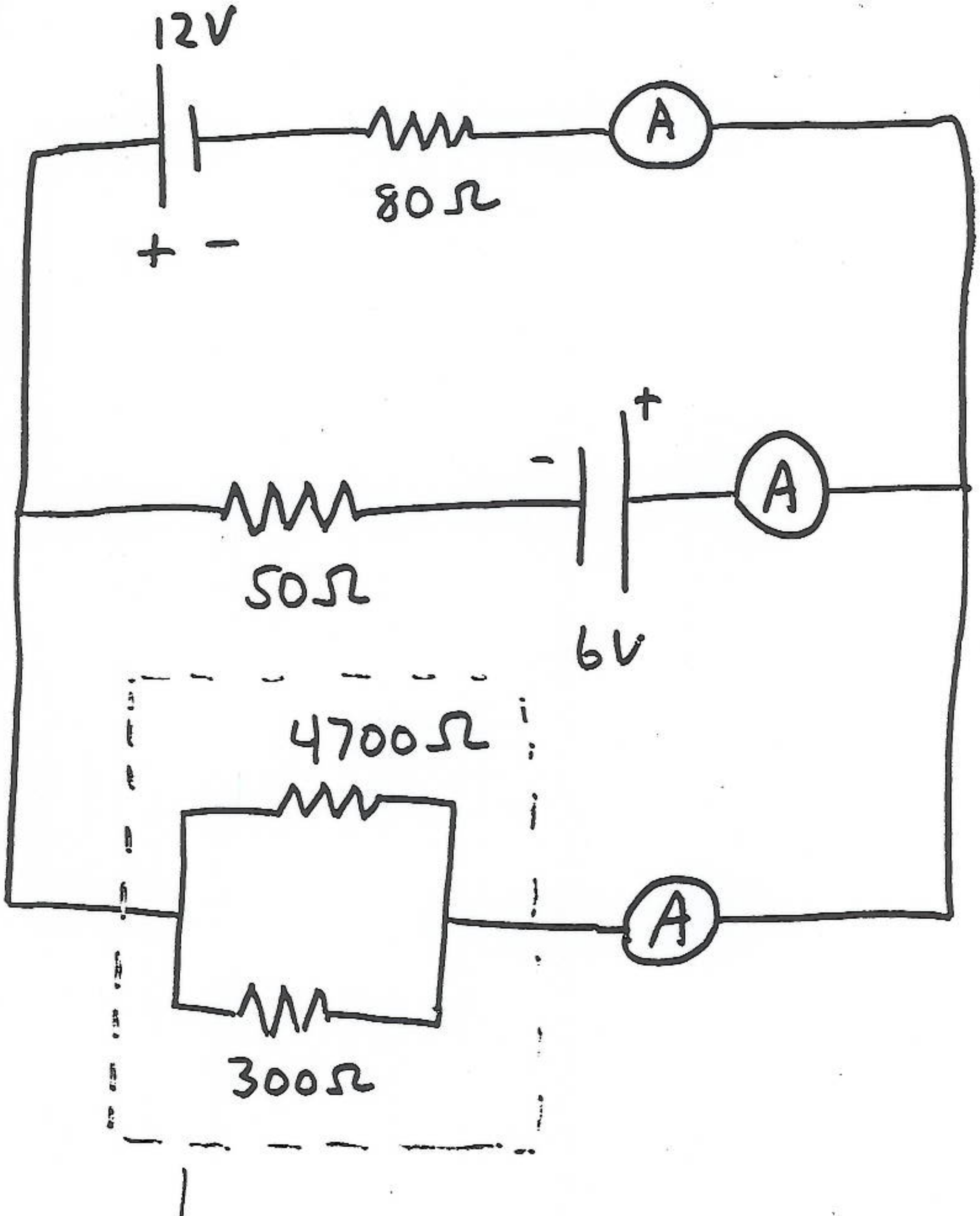
- Label currents $I_1, I_2, I_3 \ldots$
- Draw arrows for directions. If you don't know direction of a current, assume one. If final sign of current is +, you guessed correctly. If −, no problem.

4) Apply loop rule.

- Stick with assumed signs of currents
- keep applying loop rule until you have as many independent equations as unknowns.

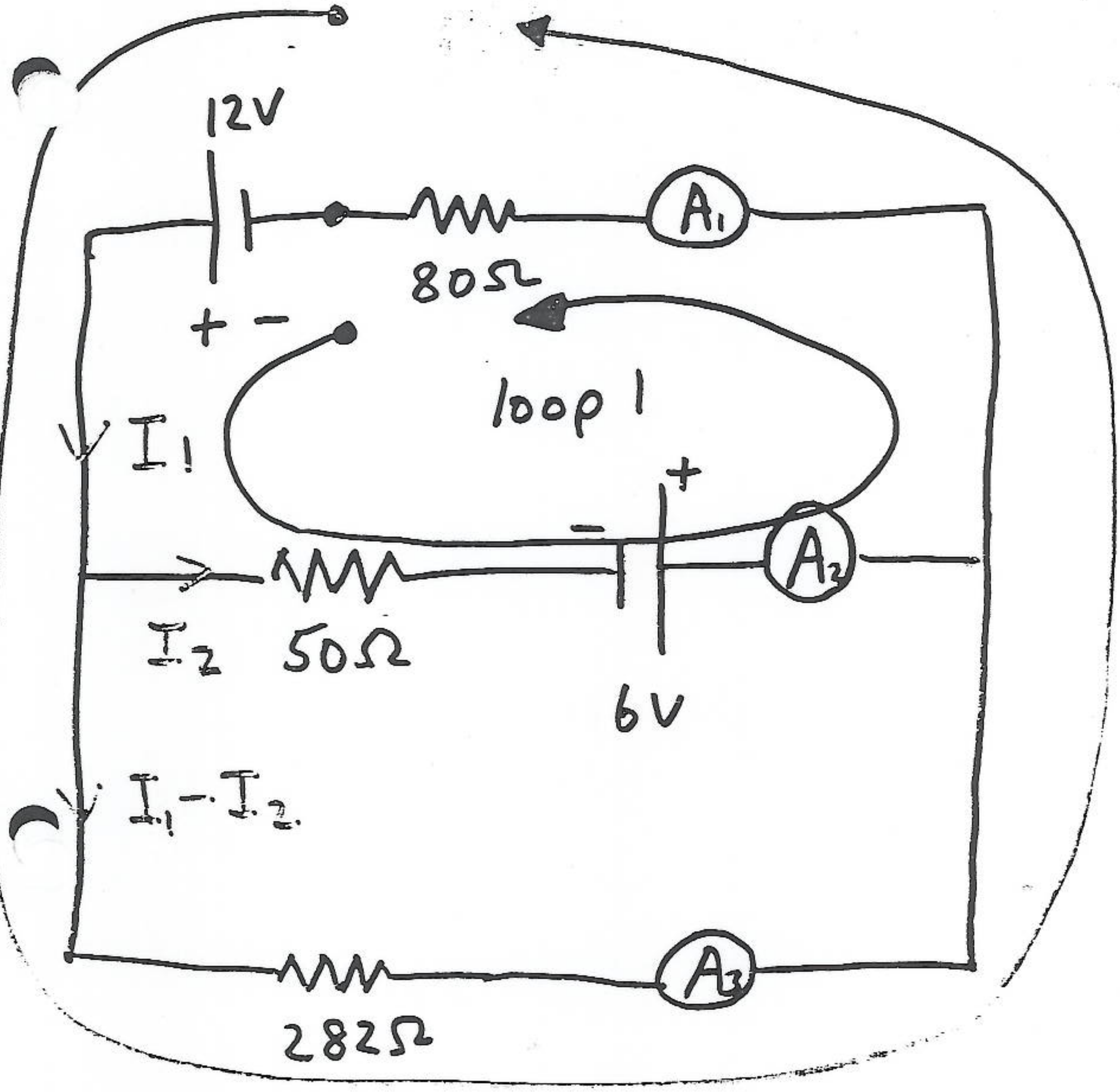
5) Solve simultaneous equations.

Example1:



$$\frac{1}{R_{eq}} = \frac{1}{4700\,\Omega} + \frac{1}{300\,\Omega}$$

$$R_{eq} = 282\,\Omega$$



Loop 1

$$12V - I_2\, 50\Omega + 6V - I_1\, 80\Omega = 0$$

$$18V - I_2\, 50\Omega - I_1\, 80\Omega = 0$$

$$\Rightarrow I_2 = \frac{1}{50\Omega}\left(18V - I_1\, 80\Omega\right)$$

Loop 2

$$12V - (I_1 - I_2)282\Omega - I_1 80\Omega = 0$$

$$12V + I_2 282\Omega - I_1 362\Omega = 0$$

$$12V + \frac{1}{50\Omega}\left(18V - I_1 80\Omega\right)282\Omega - I_1 362\Omega = 0$$

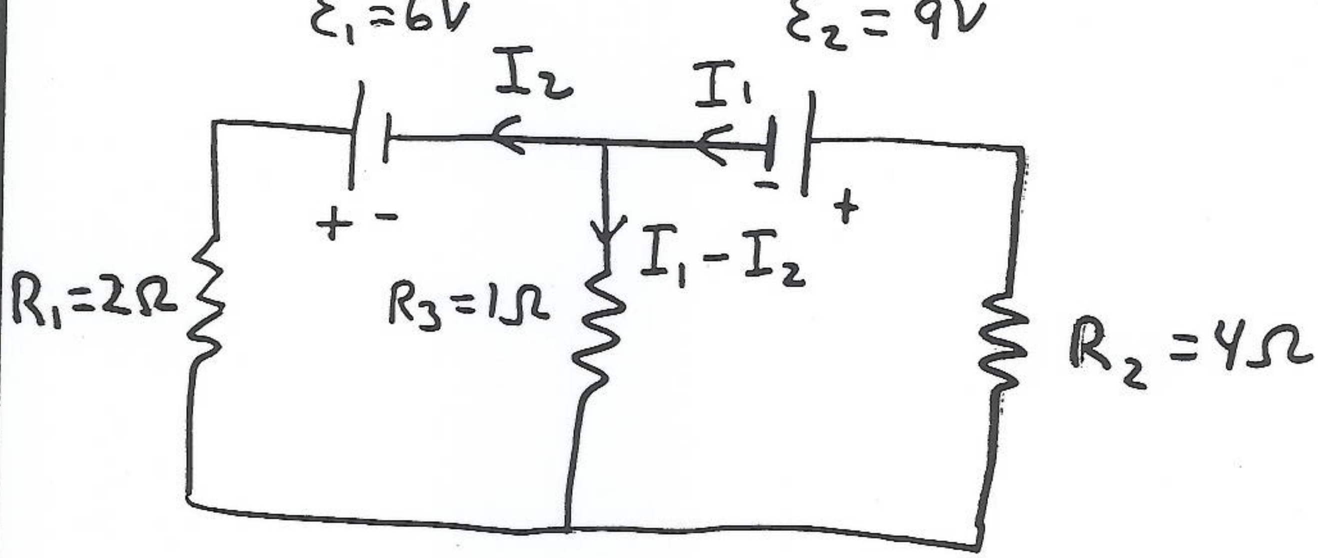
$$113.52V - I_1 813.2\Omega = 0$$

$$I_1 = 0.1396$$

$$I_2 = \frac{1}{50\Omega}\left(18V - I_1 80\Omega\right) = 0.1366$$

$$I_1 - I_2 = 0.003 \text{ Amps}$$

Ex 2: What power dissipation must R_3 be able to tolerate?



$\mathcal{E}_2$ is the biggest emf.

Deliberately pick I_1 in the "wrong" direction.

Loop 1

$$-9V + 6V - I_2 2\Omega - I_1 4\Omega = 0$$

$$I_1 = \frac{1}{4\Omega}\left(-3V - I_2 2\Omega\right)$$

Loop 2

$$6V - I_2 2\Omega + (I_1 - I_2) 1\Omega = 0$$

$$6V - I_2 3\Omega + I_1 1\Omega = 0$$

$$6V - I_2 3\Omega + \frac{1\Omega}{4\Omega}(-3V - I_2 2\Omega) = 0$$

$$I_2 (3.5\Omega) = 5.25$$

$$I_2 = 1.5 \text{ Amps}$$

$$I_1 = -1.5 \text{ Amps}$$

minus sign says we assumed "wrong" direction for current

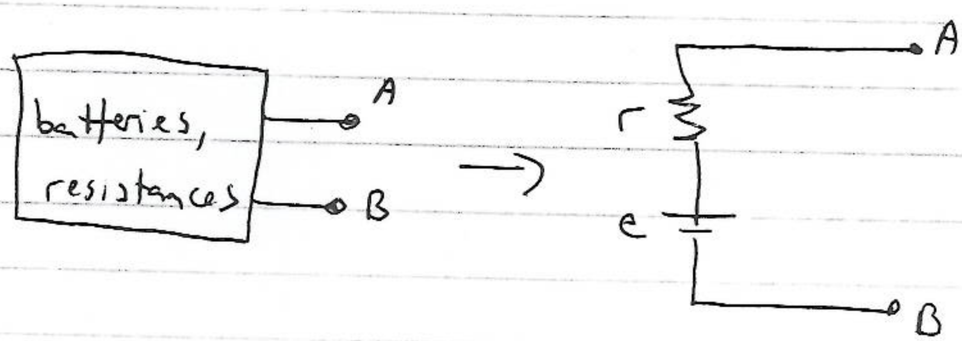
$$I \text{ in } R_3 = I_1 - I_2 = -3 \text{ Amps}$$

$$P_3 = I^2 R_3 = 9 \text{ Amps}^2 \; 1\Omega = 9W$$

Phil-
Skip Thevenin

Thevenin's Theorem (Expt 2)

Any bunch of batteries & resistances w/ two terminals
→ ideal battery (e) in series w/ one
resistance r



$V_{AB} = e - Ir$

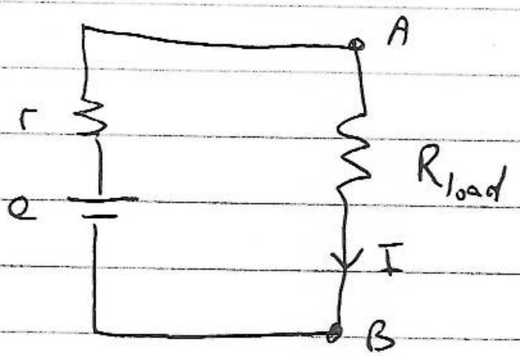
1) e is V_{AB} when $I=0$ (open circuit voltage)

2) We now know e (just a constant voltage)
In principle, can connect a wire to A & B (short them out). $V_{ab}=0$

$$r = \frac{e}{I_{\text{short circuit}}}$$

Shorting A & B can be dangerous

In practice



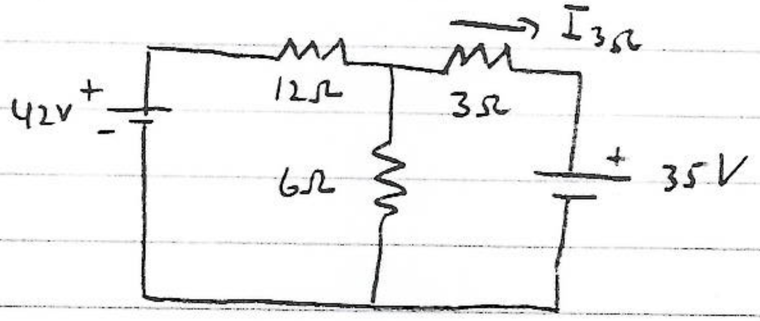
1) Measure e for open circuit (or R_{load} very large)

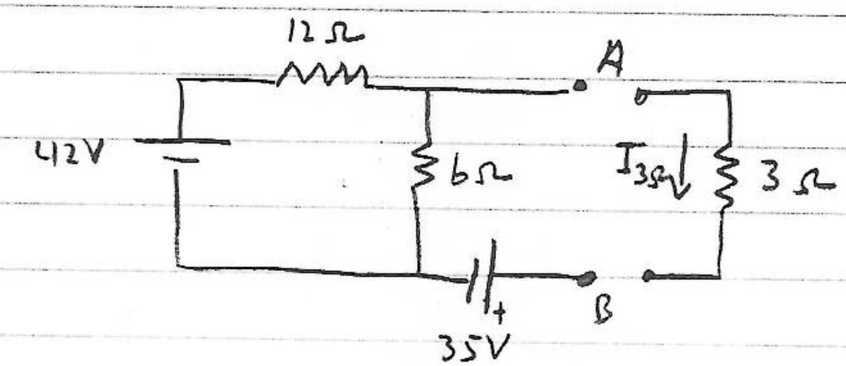
2) measure V_{AB} and I for some finite R_{load}

$$r = \frac{e - V_{AB}}{I}$$

Example:

Use Thevenin's Thm to find $I_{3\Omega}$

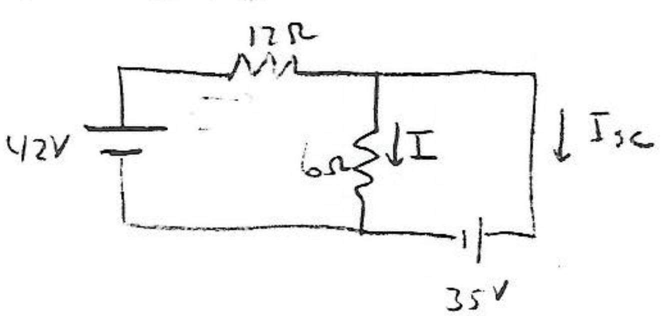




$$V_{6\Omega} = \frac{42\,V}{12\,\Omega + 6\,\Omega}\, 6\,\Omega = 14\,V$$

$$V_{oc} = 14V - 35V = -21\,V$$

Short circuit



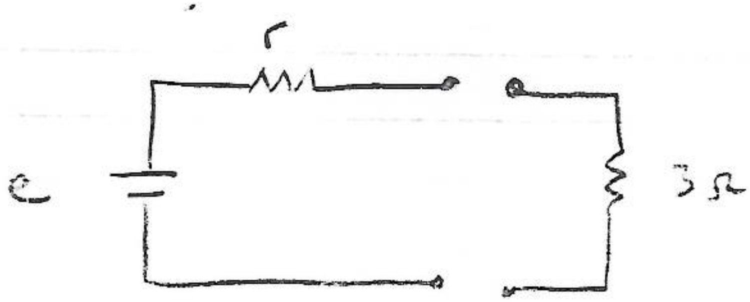
$$42V - (I + I_{sc})12\Omega - I6\Omega = 0$$
$$42V - 18I - 12I_{sc} = 0 \quad (*)$$

$$42V - (I + I_{sc})12\Omega - 35 = 0$$
$$7V - 12I - 12I_{sc} = 0$$

$$35V - 6I = 0$$
$$I = \frac{35}{6}$$

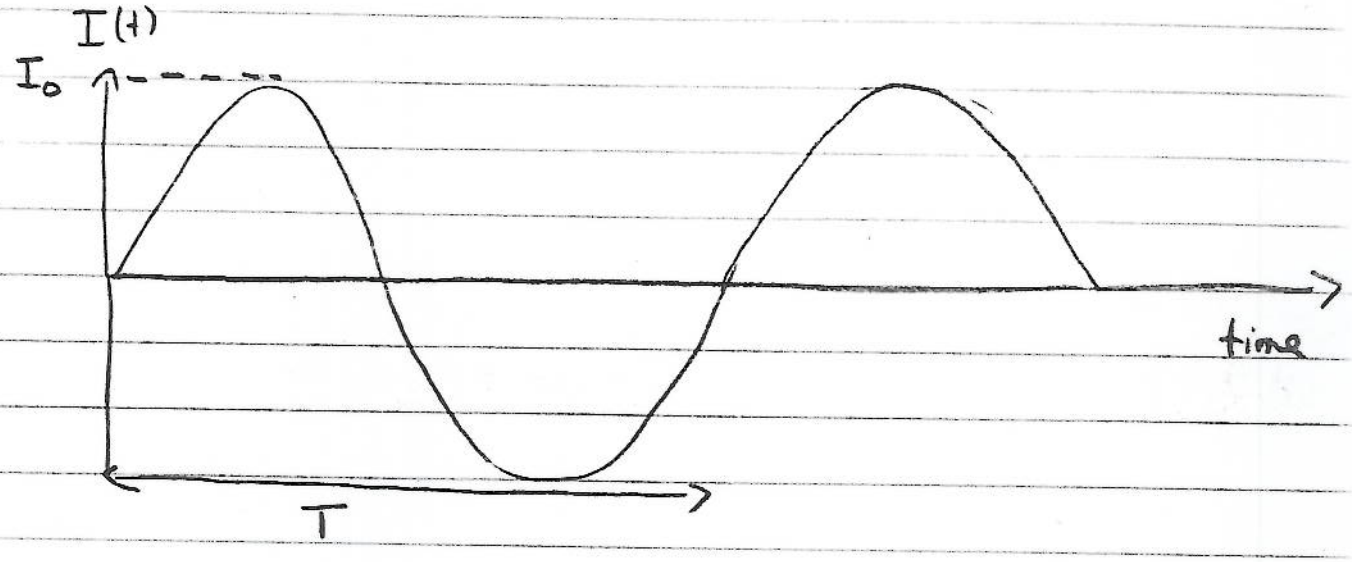
$$I_{sc} = \frac{7V - 12\frac{35}{6}}{12} = \frac{7V - 70V}{12} = \frac{-63}{12} = -\frac{21}{4}A$$

$$r = \frac{V_{oc}}{I_{sc}} = \frac{-21}{-21/4} = 4\Omega$$



$$e = I(r + 3\Omega)$$
$$-21V = I\,7\Omega$$
$$I = -3\text{ Amps}$$

AC Circuits



$I(t) = I_0 \sin \omega t$

T : period
f : frequency $\quad f = \frac{1}{T} \leftarrow$ Hz, 1/sec
ω : angular frequency $\quad \omega = 2\pi f$
$\uparrow$
$\frac{\text{radians}}{\text{sec}}$

I_0 : amplitude
$2I_0$: peak-to-peak amplitude

Average of a sine wave

$$\int_0^T \sin \omega t = 0$$

Useless.

Instead, use root-mean-square

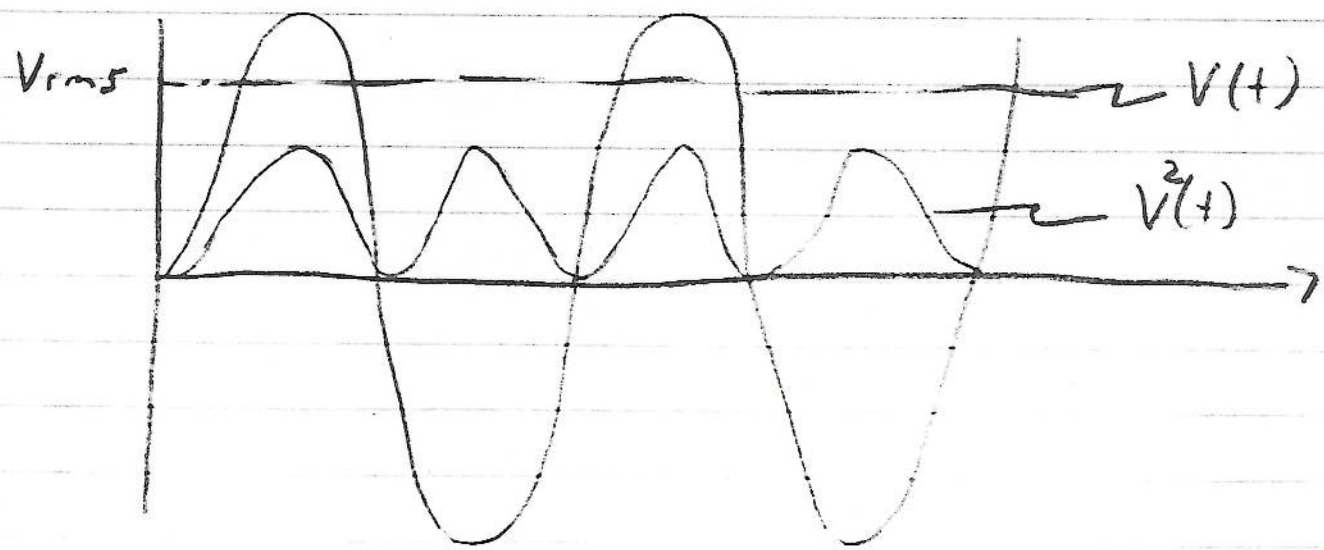
1) square it
2) average it
3) take the square root

$$V_{rms} = \left[\frac{1}{T} \int_0^T v^2(t)\, dt \right]^{1/2}$$

always +

$v(t) = V_0 \sin \omega t$

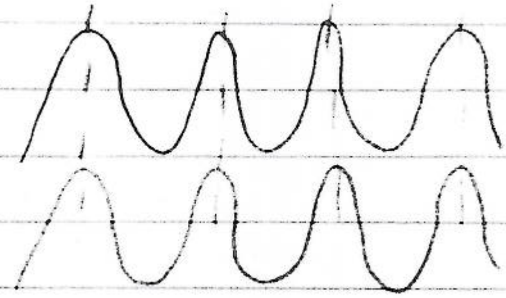
$$V_{rms} = \left[\frac{1}{T} \int_0^T V_0^2 \sin^2 \omega t \, dt \right]^{1/2} = \frac{V_0}{\sqrt{2}} = 0.707 V_0$$



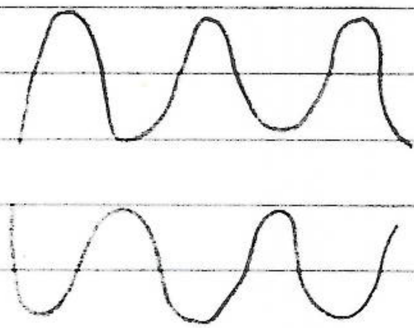
Cool thing about rms:

→ Can solve AC circuits <u>without</u> carrying around time dependences

Phase (same frequency)



in phase



180° out of phase

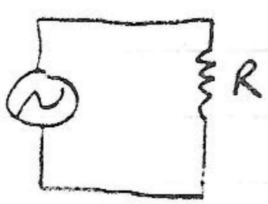
$$v(t) = V_0 \sin(2\pi f t + \varphi)$$

$\uparrow$ phase

$$V_0 \cos\omega t = V_0 \sin(\omega t + \pi/2)$$

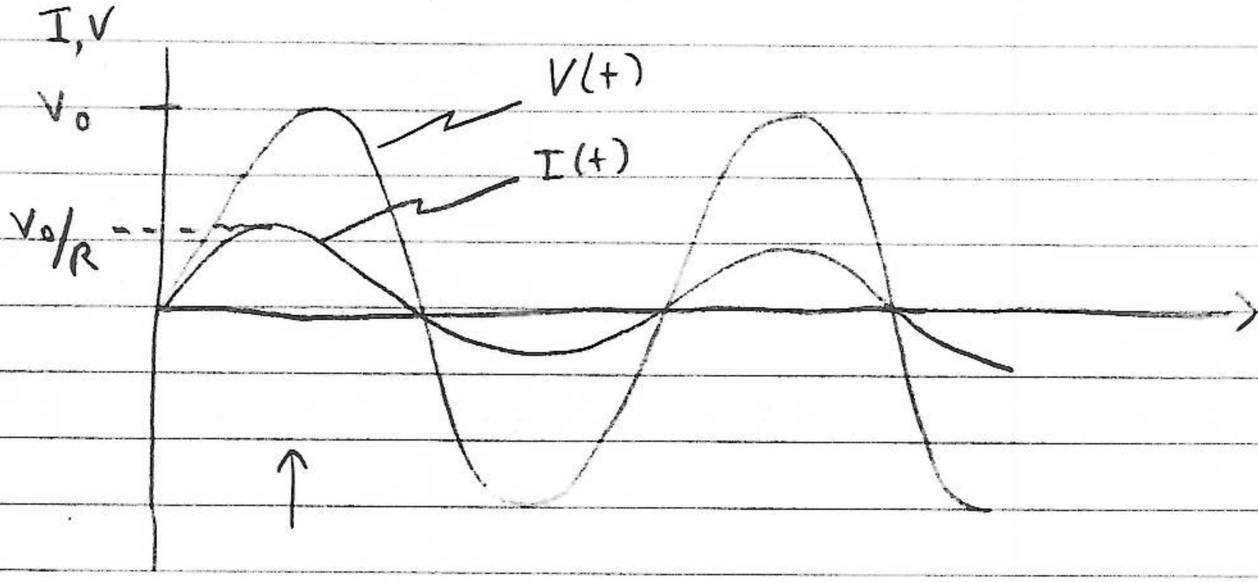
pickle

AC Power



$$i(t) = \frac{V(t)}{R} = \frac{V_0 \sin\omega t}{R} = I_0 \sin\omega t$$

I & V are in phase



Peak at the same time

$$P(t) = [i(t)]^2 R = I_0^2 R \sin^2 \omega t$$

$$P_{ave} = \frac{I_0^2 R}{2} = \frac{V_0^2}{2R}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$P_{ave} = \frac{V_{rms}^2}{R}$$ just like $\frac{V^2}{R}$ but $P \rightarrow P_{ave}$

$V \rightarrow V_{rms}$