Fission-fragment distribution for ^{238}U

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My primary goal was to simulate the primary fission fragment distribution for U-238. Fission fragments distribution is a function of mass of the fragment A, its charge Z, and its total kinetic energy TKE. I have addressed all of the three dependencies separately following the steps done by *Talou et al, 2011* [1] for neutron induced fission of Pu-239. He showed that the fragment yield is a function of all three variables as:

$$Y(A, Z, TKE) = Y(A) * P(Z|A) * P(T\ K\ E\ |A) \qquad (1)$$

1. Fragment distribution as a function of mass of the fragment Y(A)

I have found several papers where mass distribution was measured. Figure 1 shows the results of two experiments – U-238 fission caused by neutrons, *Debertin et al, 1978* [2] and photons*, Goeoek et al, 2011* [3]. Fragment mass distribution is not very different for these two cases. To use these data in futher calculations I have interpolated the photofission data by performing a least-square fit of the experimental mass yields to obtain our “best” estimate of Y (A). I also constrained the resulting yields to be symmetric about A = 120.

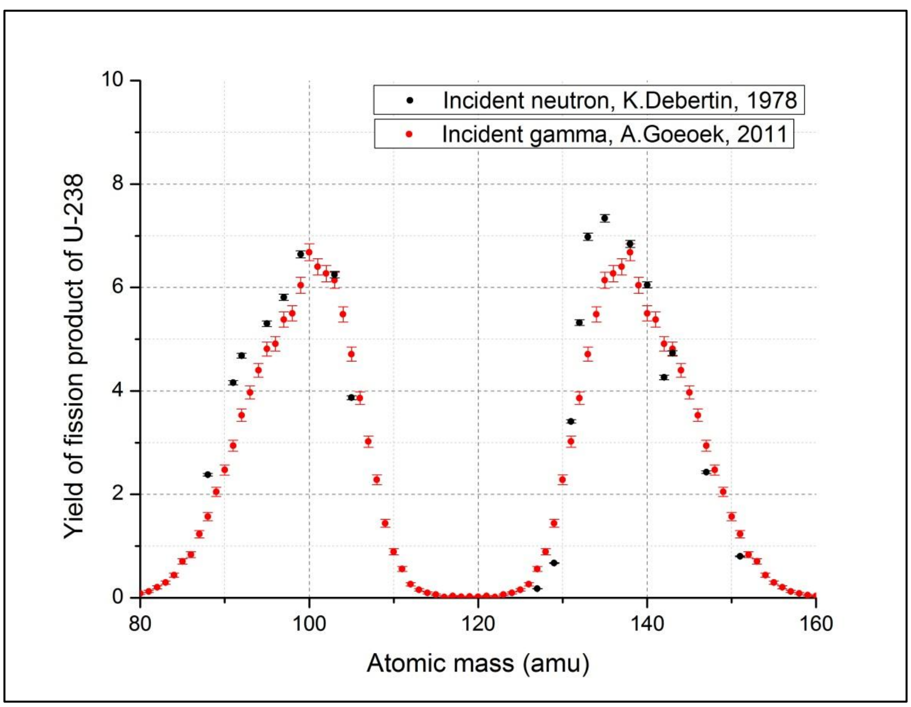

Figure 1. Yield as a function of a fragment mass.

2. <u>Fragment distribution as a function of charge of the fragment P(Z,A)</u>

The charge distribution for a given mass A was obtained using the Z_P model, described by *Wahl, 2002* [4]. The Z_P model treats dispersion of fractional independent yields of primary fission products with Z for each A. For every heavy mass, the charge deviation ΔZ, the charge width parameter σ_Z, and the odd-even factors F_Z and F_N are calculated. Then, for each heavy mass, the charge distribution was then determined using the following equation:

$$P(Z|A) = \frac{1}{2} F(A) N(A) [erf(V) - erf(W)] \tag{2}$$

where

$$V = \frac{Z - Z_p + 0.5}{\sigma_z \sqrt{2}} \tag{3}$$

and

$$W = \frac{Z - Z_p - 0.5}{\sigma_z \sqrt{2}} \tag{4}$$

The most probable charge can be found as:

$$Z_p = A_h \frac{Z_c}{A_c} + \Delta Z \tag{5}$$

where Z_c and A_c are charge and mass of the compound nucleus. In our case Z_c = 92 and A_c = 238.

N(A) is just a normalization coefficient needed to satisfy $\Sigma P(Z|A) = 1$ for any A. This normalization is required because the even-odd factors, F(A), destroy the intrinsic normalization properties of Gaussian distributions. F(A) coefficients are calculated as following:

For Z	**For N**	**F(A)**
Even	Even	$F_Z \cdot F_N$
Even	Odd	F_Z / F_N
Odd	Even	F_N / F_Z
Odd	Odd	$1/(F_Z \cdot F_N)$

Table 1. Calculating odd-even factors F(A) for different number of neutrons and protons in a fragment.

The four parameters, ΔZ, σ_Z, F_Z, and F_N were determined for each fission reaction by the method of least squares. I have adopted these parameters from the Wahl report [4], see Figure 2. Using the above parameters, Equations (2)-(5), and Table 1, I calculated the charge distribution as follows. First, I found the most probable charge as a function of A (see Figure 3). Next, I calculated the odd-even coefficients F(A) for different numbers of neutrons and protons, using formulas from Table 1 (see Figure 4). Finally, I calculated the yield P(Z,A) as a function of A and Z of the fragment (see Figure 5).

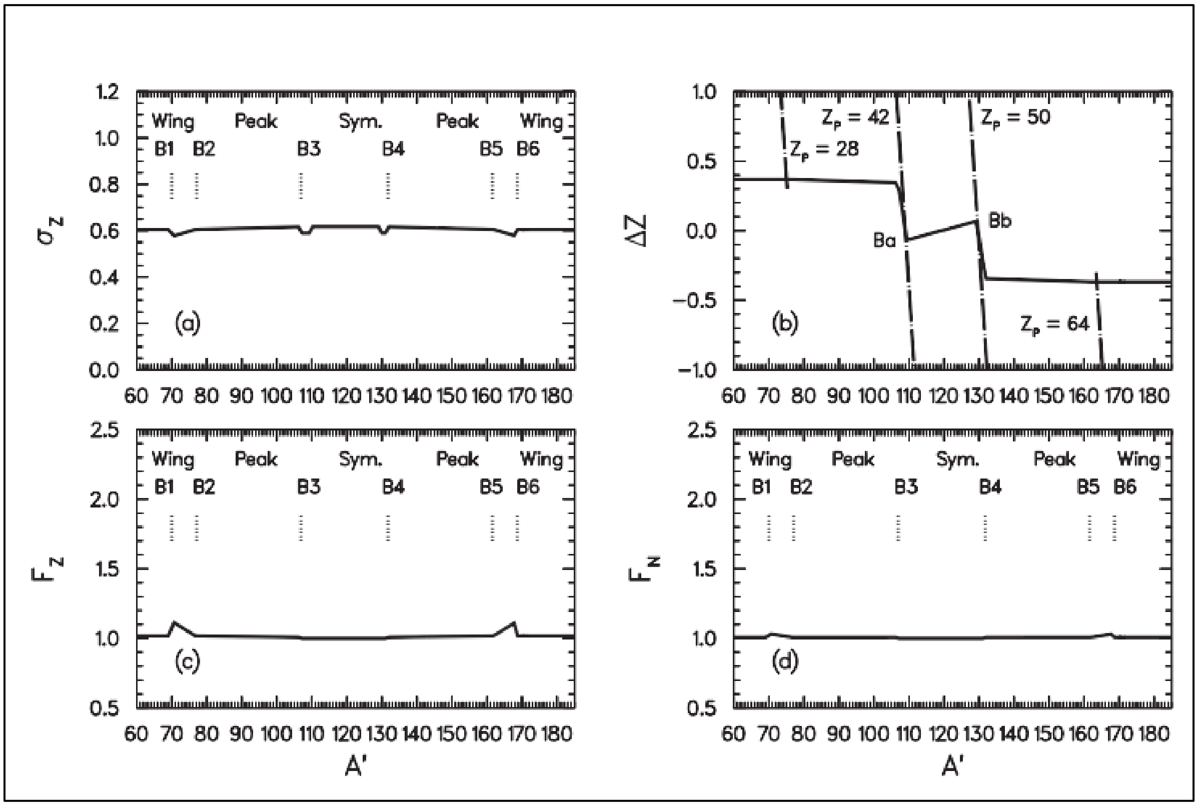

Figure 2. The four parameters, ΔZ, σ_Z, F_Z, and F_N, adapted from Wahl, 2002 [4].

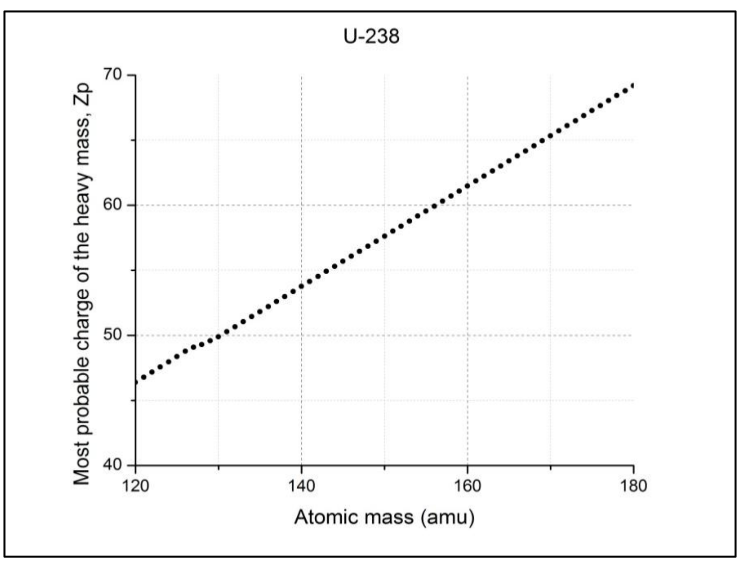

Figure 3. Most probable charge of the fragment as a function of its atomic mass.

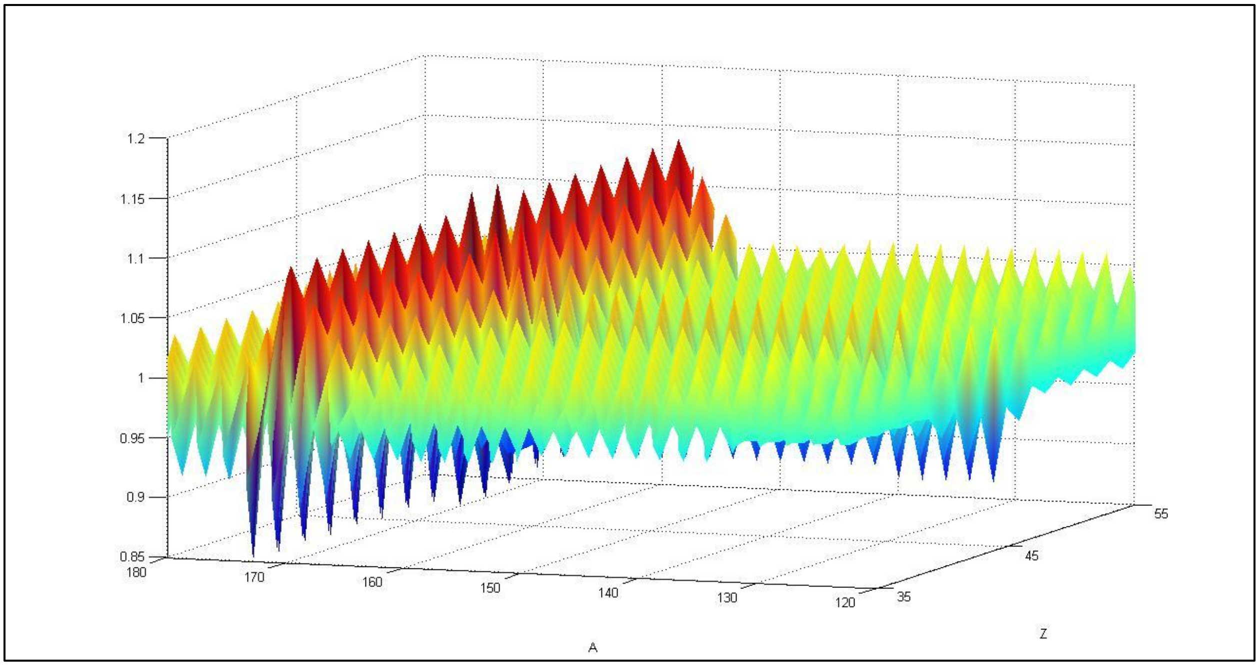

Figure 4. Odd-even factors F(A) as a function of A and Z.

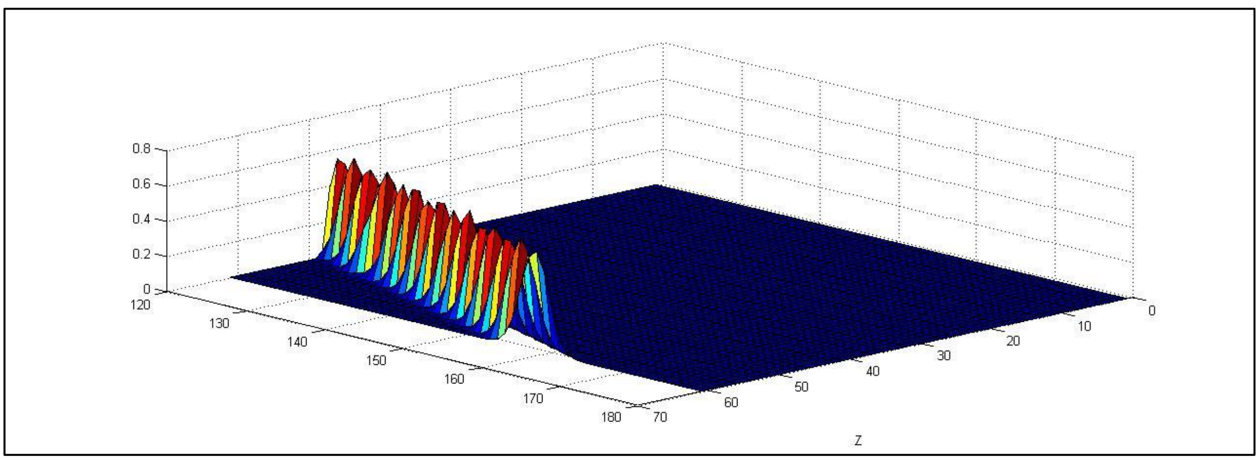

Figure 5. Yield P(Z,A) as a function of A and Z of the fragment.

3. <u>Fragment distribution as a function of kinetic energy of the fragment P(TKE,A)</u>

Measurements of the average total kinetic energy per fragment mass $\langle TKE(A)\rangle$ and the width of the (assumed) Gaussian distribution $\sigma^2_{TKE}(A)$ were used to reconstruct the total kinetic energy distribution *Y* (*TKE*) according to *Talou et al, 2011*:

$$\mathrm{P(TKE|A)} = \frac{1}{\sqrt{2\pi\sigma^2_{TKE}(A)}} * \exp\{-\frac{[TKE - \{TKE\}(A)]^2}{2\sigma^2_{TKE}(A)}\} \quad (6)$$

Figures 6 and 7 show the measured data sets on $\langle TKE(A)\rangle$ from *Goeoek et al.* [2] and *Jacobs et al., 1979* [4] correspondingly.

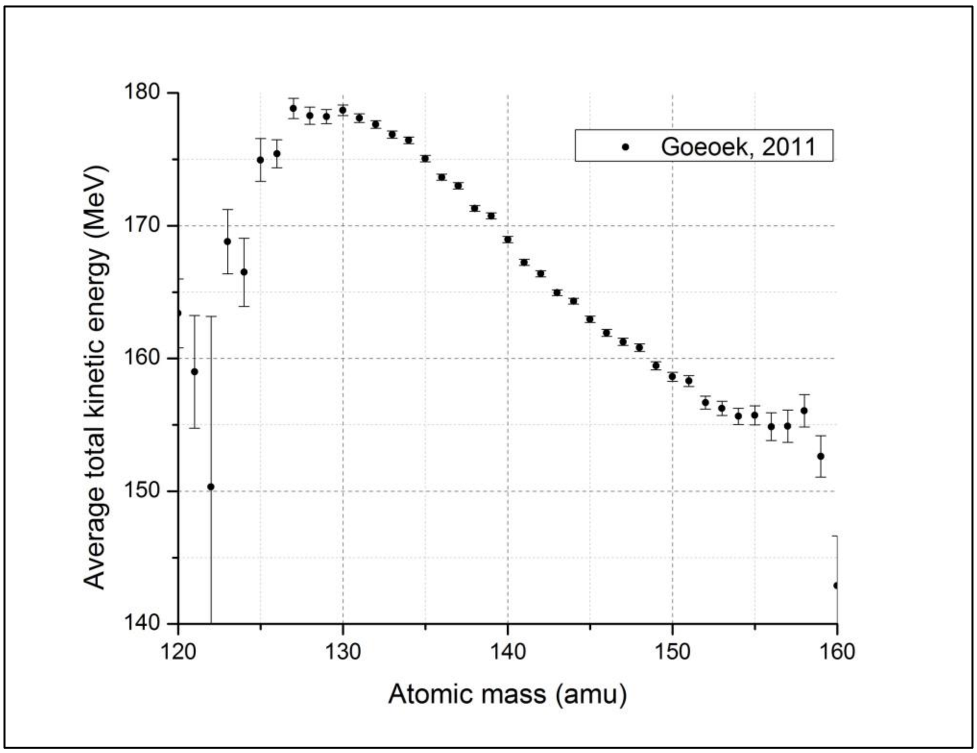

Figure 6. Average total kinetic energy for U-238, adapted from Goeoek et al., 2011 [4]

Using this data I reconstructed the total kinetic energy distribution term, see Figure 8.

The final step was to put all three terms together and thus obtain the overall fragment distribution, which is a function of three variables, A, Z, and TKE, and is a 3D matrix. Different projections of the matrix can be obtained by integrating over one of the variables. To compare our results with Talou's work, I have plotted the yield as a function of the total kinetic energy and atomic mass of the fragment, integrating over the charge.

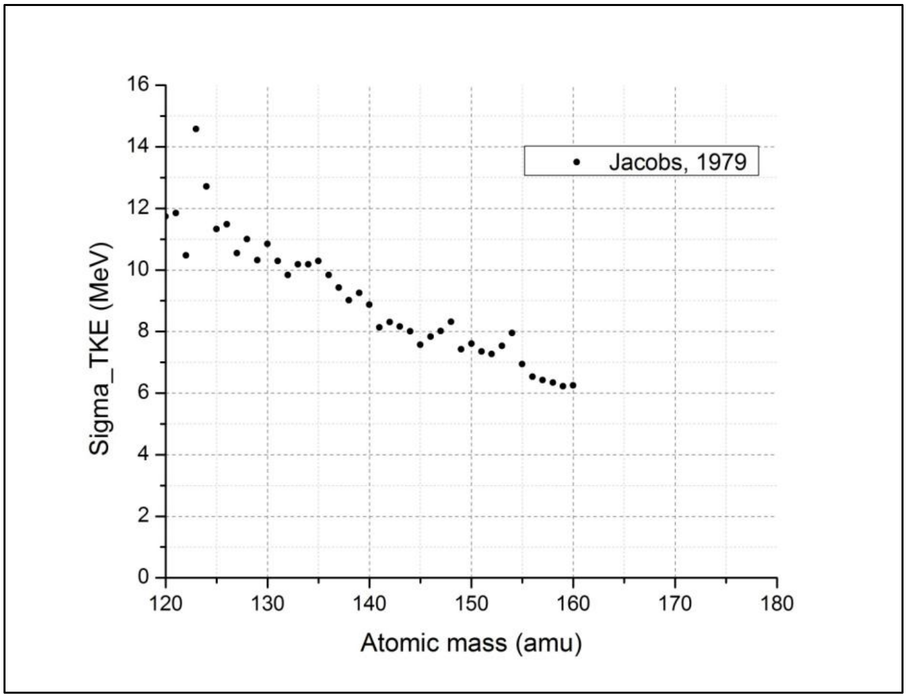

Figure 7. Sigma of total kinetic energy for photofission of U-238, E_γ = 12 MeV, adopted from Jacobs et al., 1979 [5]

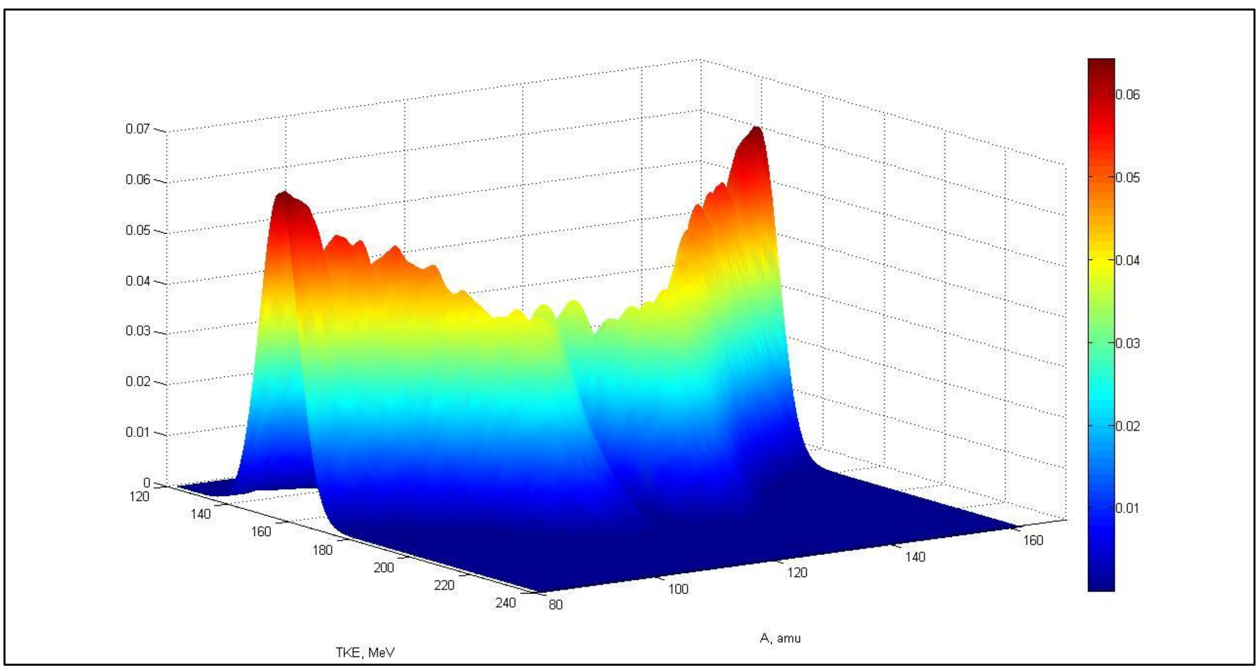

Figure 8. Y(TKE,A) - Yield as a function of the total kinetic energy and atomic number. This is not the final result, this is just the third term in the overall expression (Eq. 1).

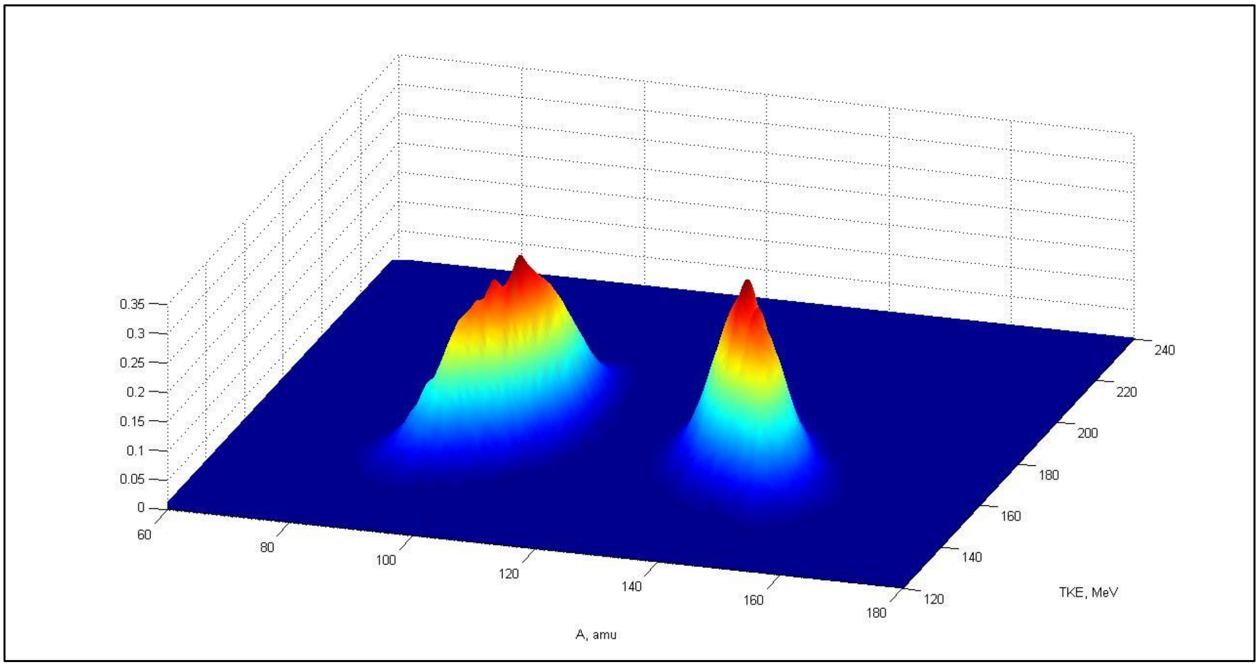

Figure 9. A projection of the final distribution, Y(A,Z,TKE).

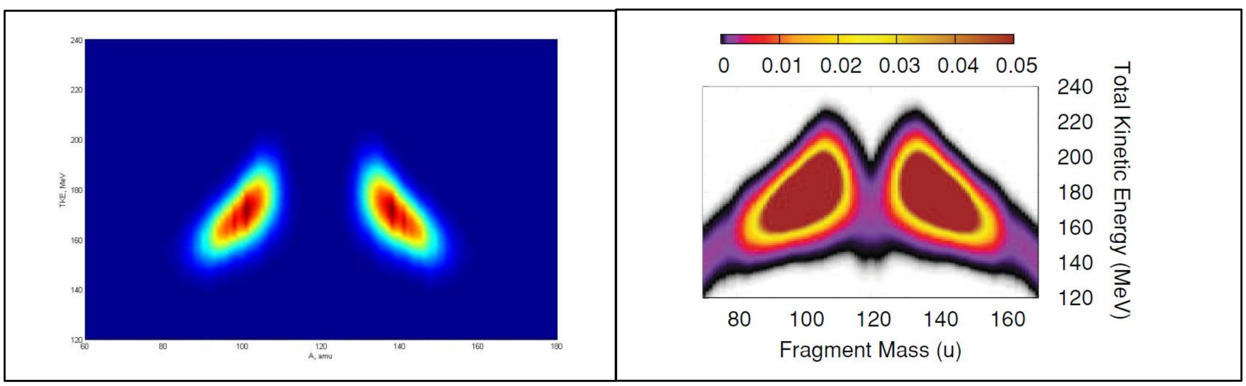

Figure 10. Comparison of my result for photofission of U-238 (left) and Talou's results (right) for neutron induced fission of Pu-239.

References

[1] Talou P et al., Advanced Monte Carlo modeling of prompt fission neutrons for thermal and fast neutron-induced fission reaction on Pu-239, PHYSICAL REVIEW C 83, 064612 (2011)

[2]Debertin Et Al. ,fission product yields in U-238 fission by Cf-252 neutrons, *Int.Conf.on Neutr.Phys.and Nucl.Data,Harwell 1978, page 229, 1978/09*

[3]Goeoek et al., Fragment characteristics from fission of 238U and 234U induced by 6.5-9.0 MeV bremsstrahlung, *Nuclear Physics, Section A, volume 851, page 1, 2011*

[4] Wahl, Arthur C., Systematics of Fission-Product Yields, Los Alamos National Laboratory, LA-13928, April 25,2002.

[5] Jacobs, E., Fragment mass and kinetic energy distributions for the photofission of ^{238}U with 12-, 15-, 20-, 30-, and 70-MeV bremsstrahlung, Physical Review C (Nuclear Physics), Volume 20, Issue 6, December 1979, pp.2249-2256