A Positron Production Measurement Using The IAC's High Repetition Rate Linac

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Outline

- Motivation
- Positron Production
- Experiment Setup Overview
- HRRL Emittance Measurement
- Positron Transportation
- Positron Detection
- Positron Production Simulation: G4beamline

Motivation

- **Polarized positron beam - a tool for intermediate energy nuclear physics at Jefferson Lab.**
- **A joint project between IAC (production efficiency) and JLab (polarization).**

IDAHO STATE UNIVERSITY

Idaho Accelerator Center

Jefferson Lab

Motivation

$G_E(Q^2)$: electric form factors.

$G_M(Q^2)$: magnetic form factors.

μ: nucleon magnetic moment

the ratio:

$R = \mu\, G_E(Q^2)/G_M(Q^2)$

Discrepancy?

Failure of the one–photon approximation [7-12] in Rosenbluth.

Larger Q^2 -> Larger two-photon exchange [13].

Direct experimental evidence for the contribution of TPE can be obtained by comparing the ratio of e^+p to e^-p cross sections.

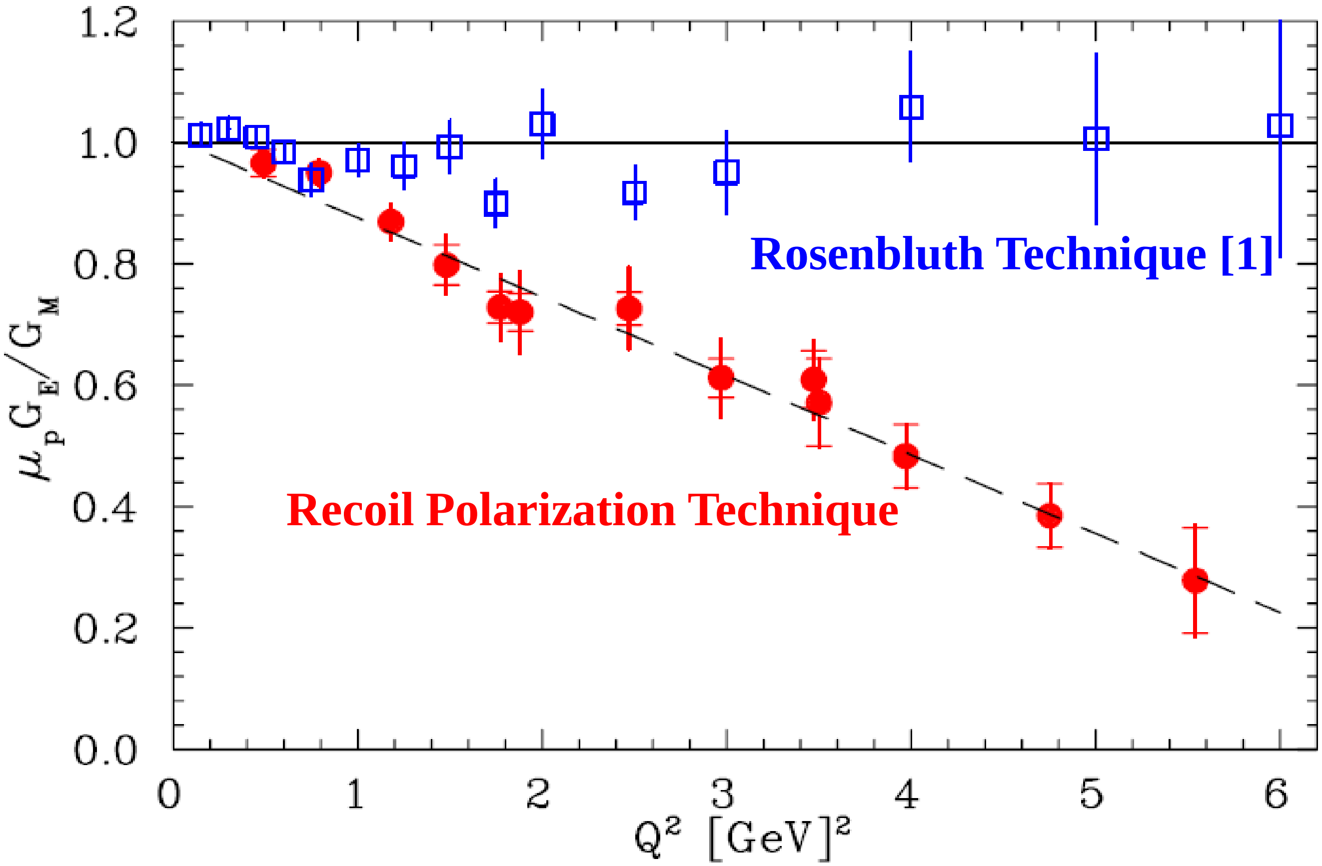
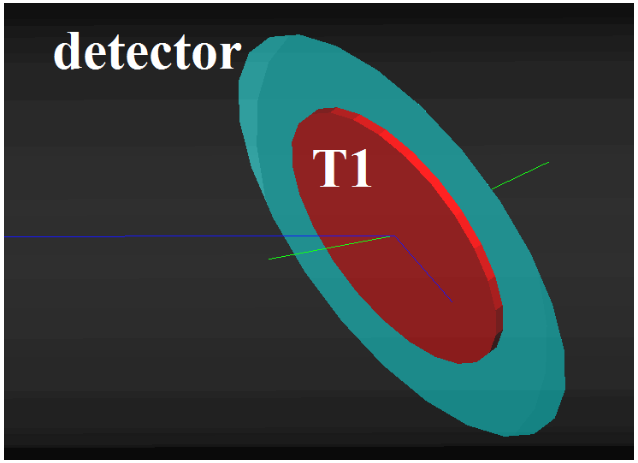


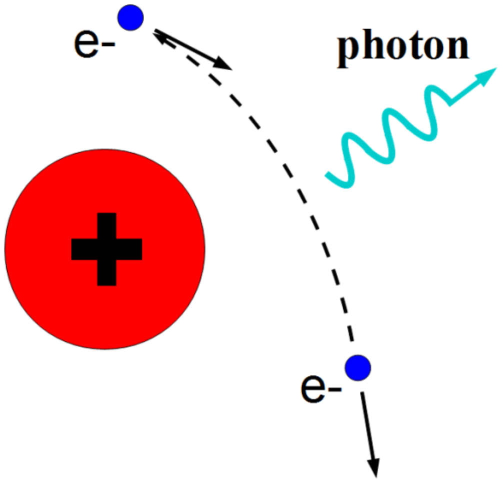
FIG. 1: Ratio of electric to magnetic form factor as extracted by Rosenbluth measurements (hollow squares) [2, 3] and from the JLab measurements of recoil polarization (solid circles). The dashed line is the fit to the polarization transfer data [4,5, 6].

Positron Production

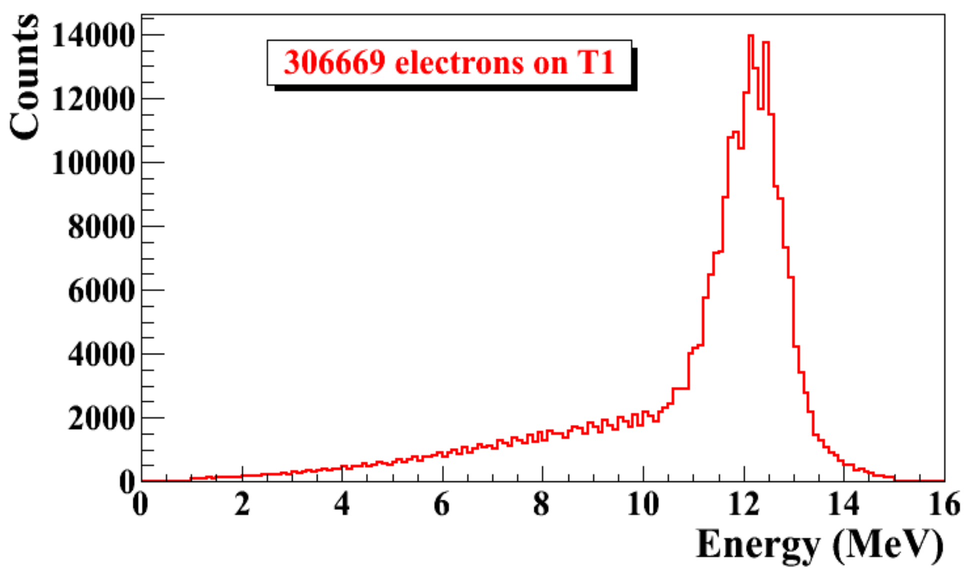
Electron beam on T1 ⇨ bremsstrahlung



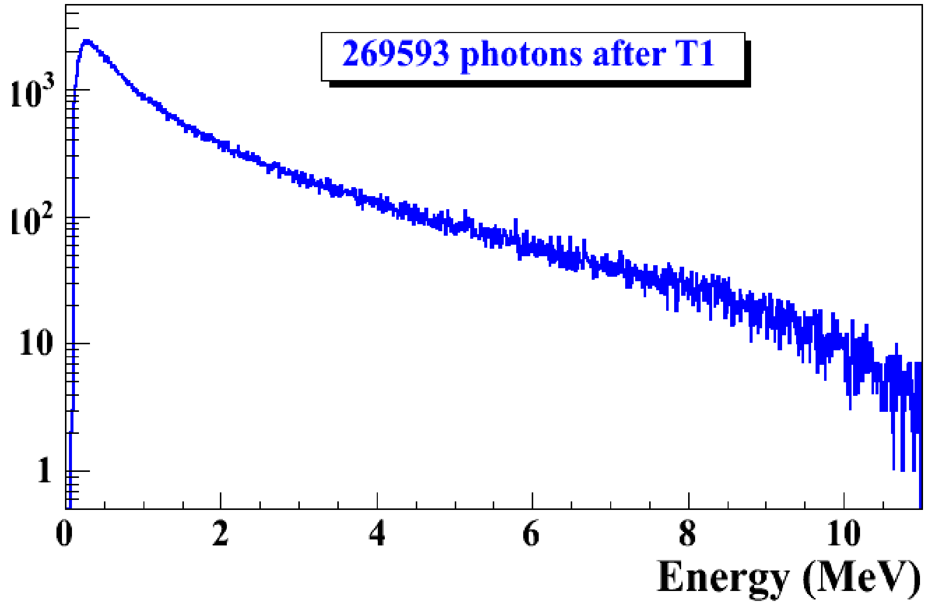
Electron on tungsten foil creates Bremsstrahlung photons. LABELS



Bremsstrahlung process.



Simulated Bremsstrahlung electron energy distribution right before a tungsten foil.



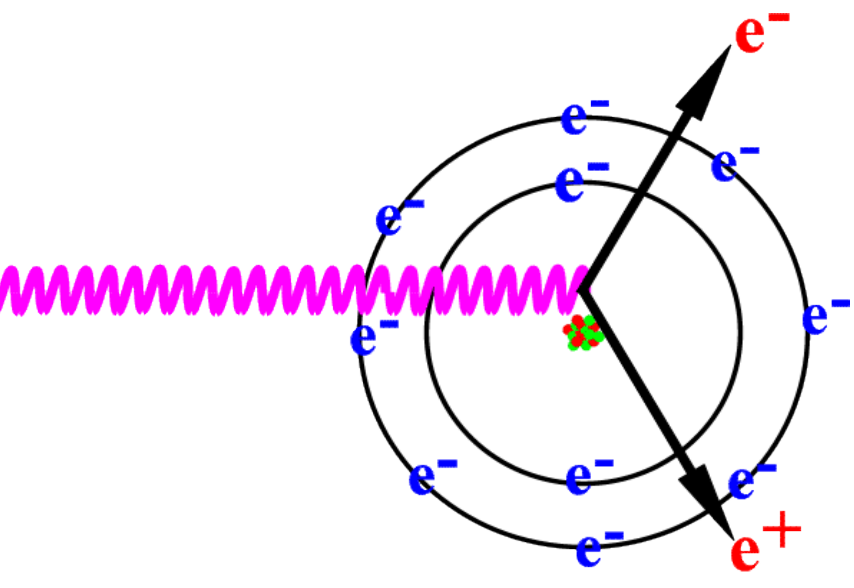
Simulated Bremsstrahlung photon energy right after a tungsten foil.

Positron Production

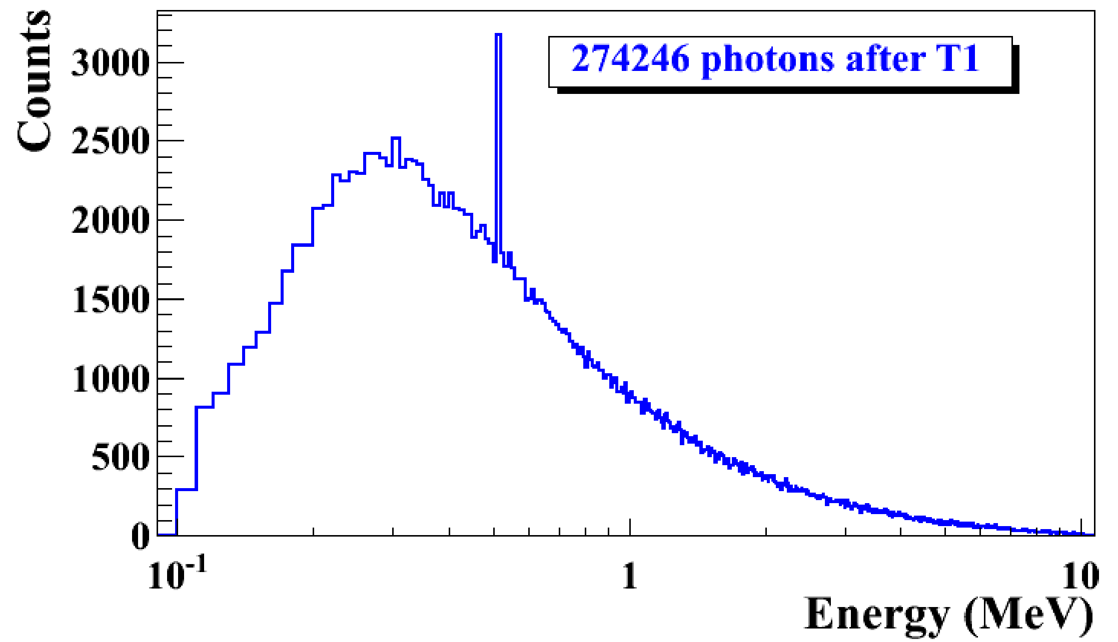
3 ways photon interacts with matter:

- Photoelectric effect
- Compton scattering
- Pair production

$E_\gamma > 1.022$ MeV ⇨ Pair production.



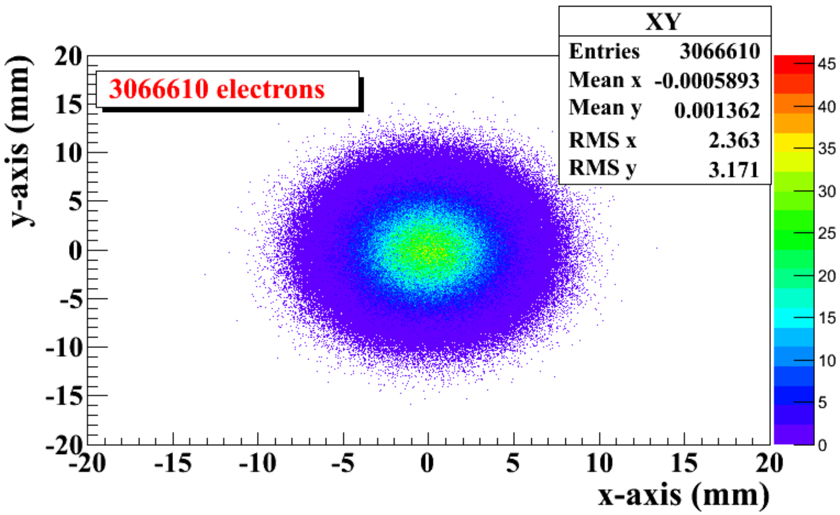
Pair production [1].

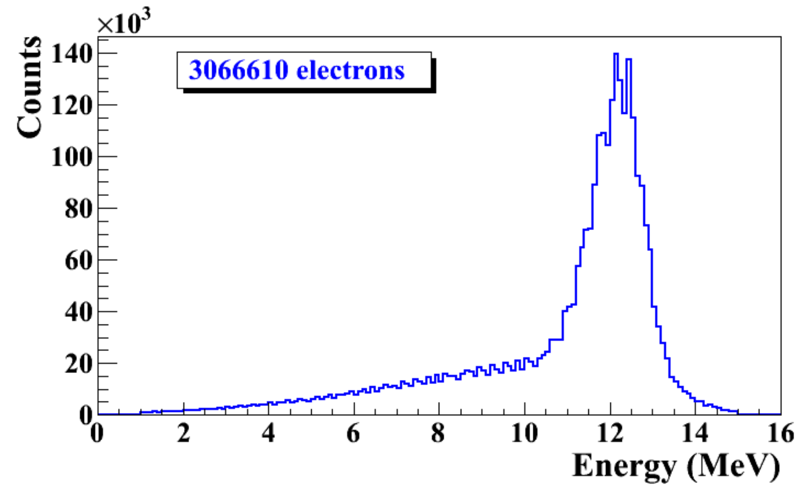


Simulated Bremsstrahlung photon energy right after a tungsten foil.

Positron Production

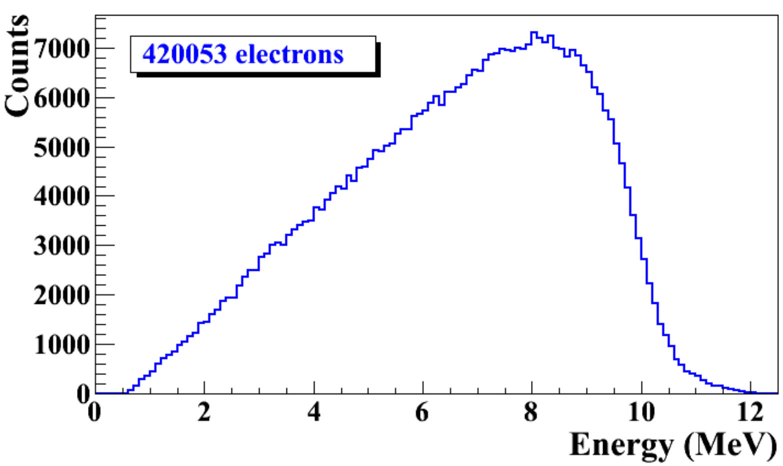
G4beamline simulation: $e^- \rightarrow T1$

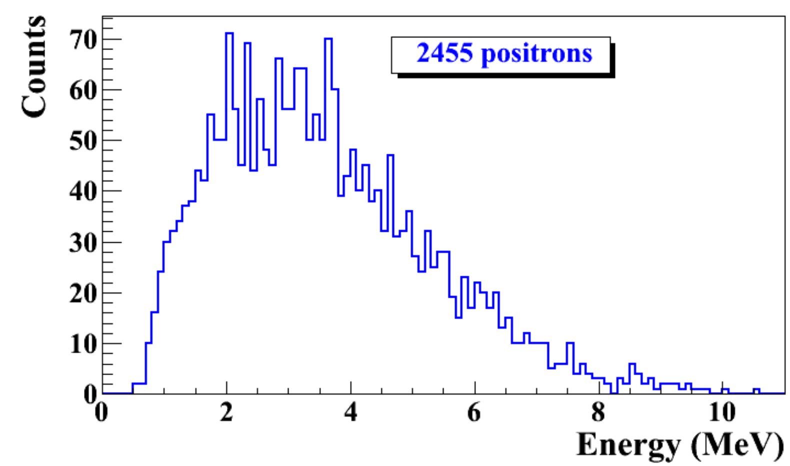




e^- beam on T1: transverse distribution.

e^- beam on T1: energy distribution.





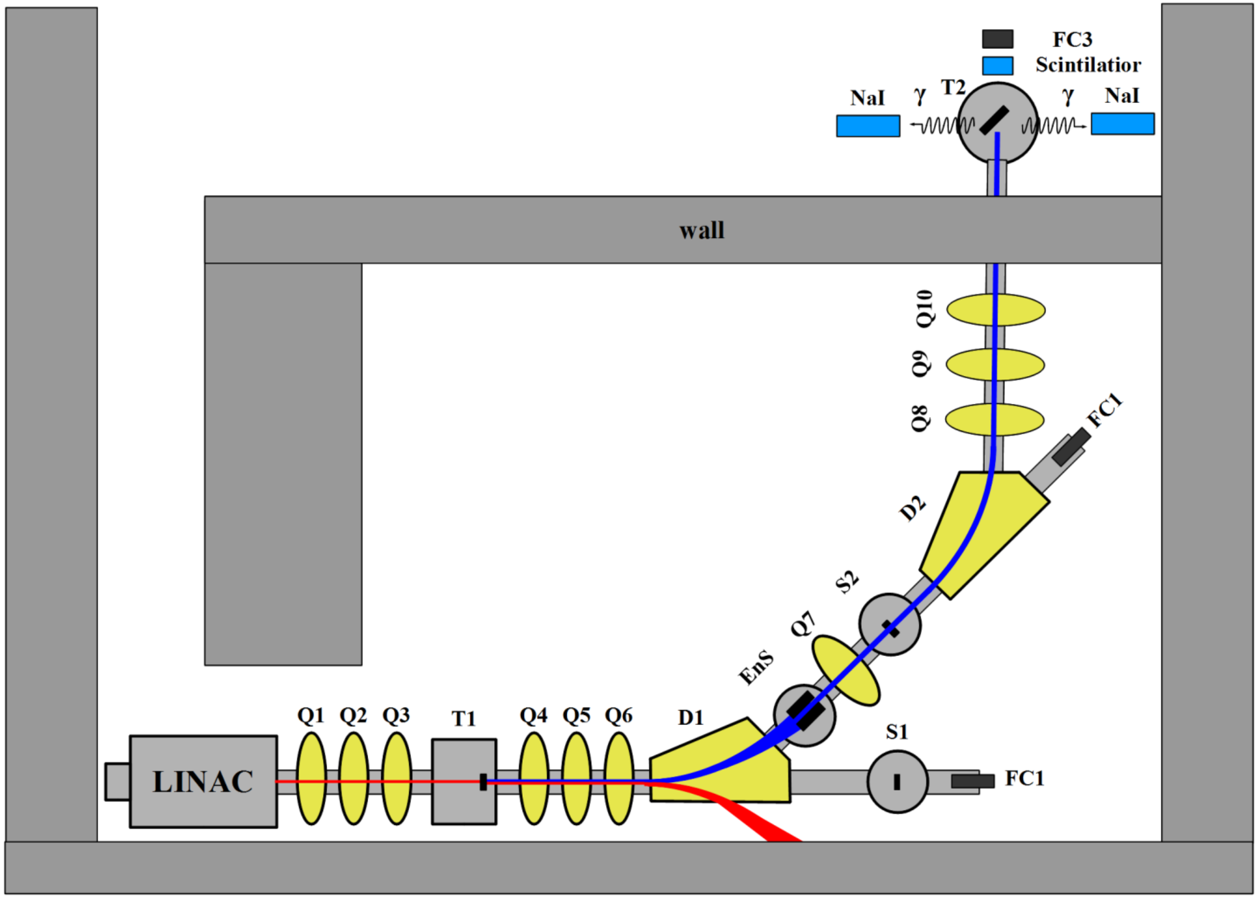
e^- beam after T1: energy distribution.

e^+ beam after T1: energy distribution.

Experiment Setup Overview

- Production.
- Transportation.
- Detection and estimation.

Item	Description
T1	Positrons target
T2	Annihilation target
EnS	Energy Slit
FC1, FC2	Faraday Cups
Q1,...Q10	Quadrupoles
D1, D2	Dipoles
NaI	NaI Detectors
OTR	Optical Transition Radiation screen
YAG	Yttrium Aluminium Garnet screen

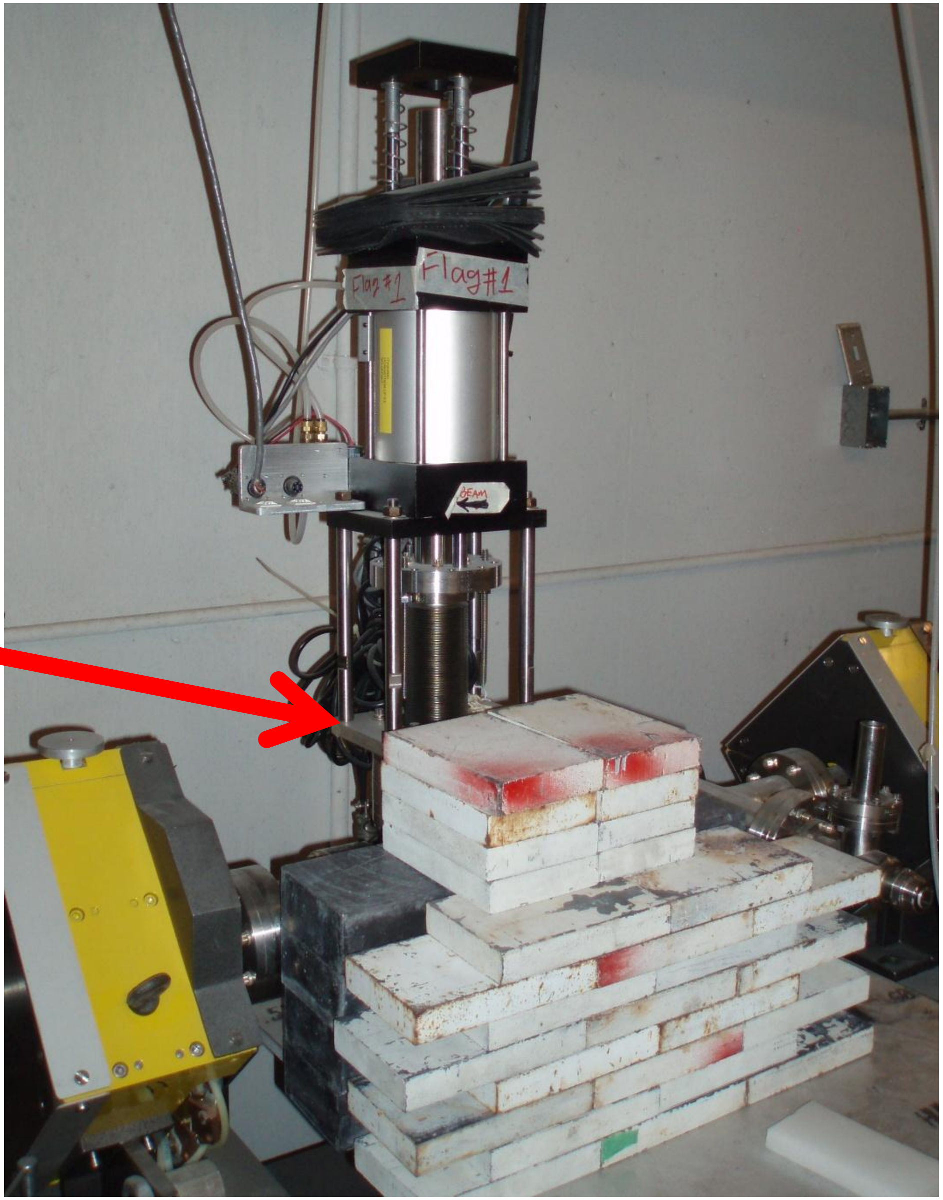


HRRL beamline layout.

Experiment Setup Overview

Electron beam with 12 MeV peak energy from High Repetition Rate Linac (HRRL) on T1.

T1: tungsten foil with 0.04 inch (1.016 mm) thickness, 1.25 inch (31.75 mm) diameter. Placed with 45 degree angle regarding to the beam direction.



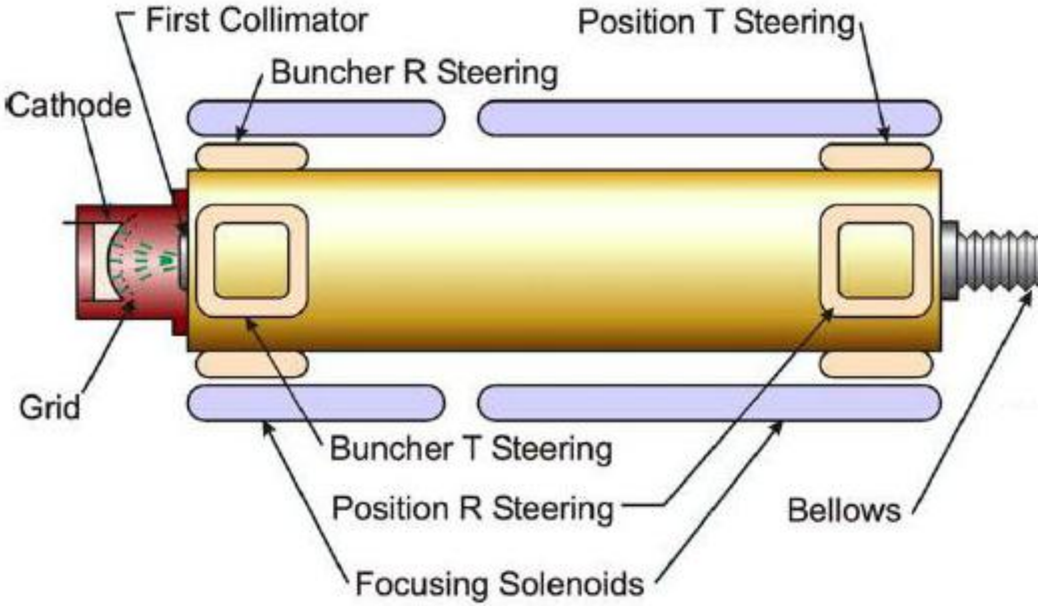
Experiment Setup Overview

HRRL is 16 MeV S-band (2856 MHz) Linac.

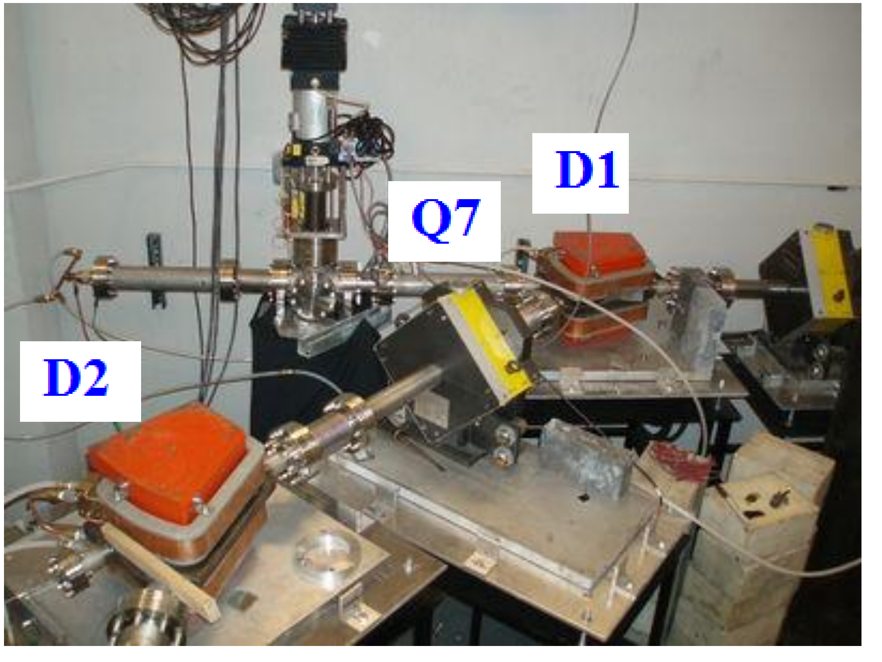
HRRL Located in the Beam Lab of the Department of Physics, ISU.

HRRL Parameters.

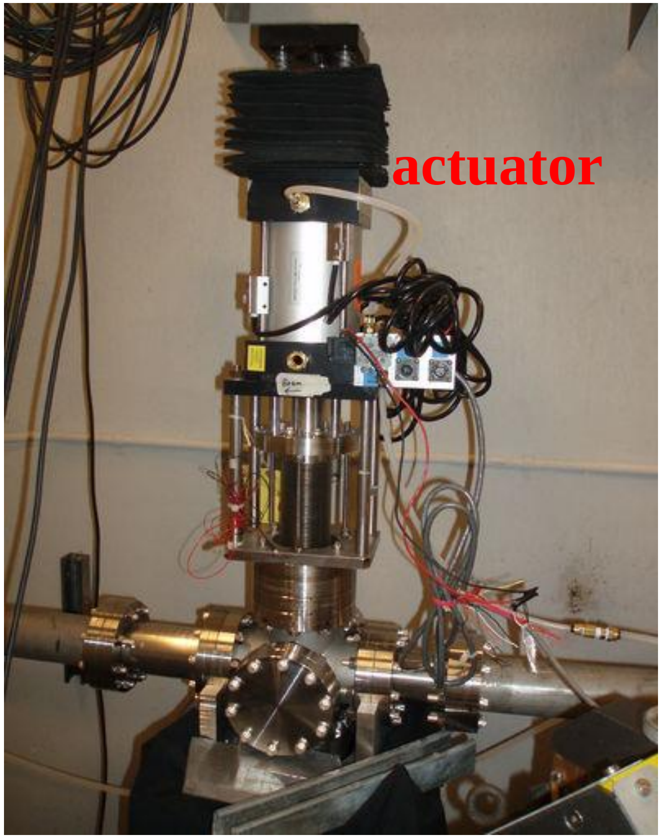
Parameter	Unit	Value
maximum Energy	MeV	16
peak current	mA	100
repetition rate	Hz	300
rms energy spread	MeV	2-4
macro pulse length	ns	>50

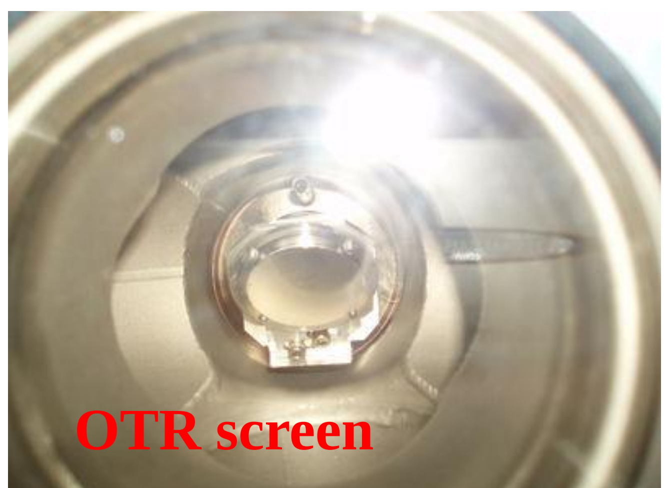


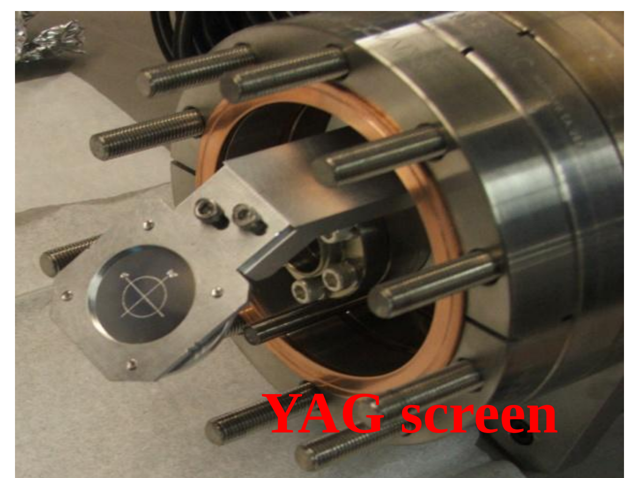
Experiment Setup Overview



Experiment Setup Overview





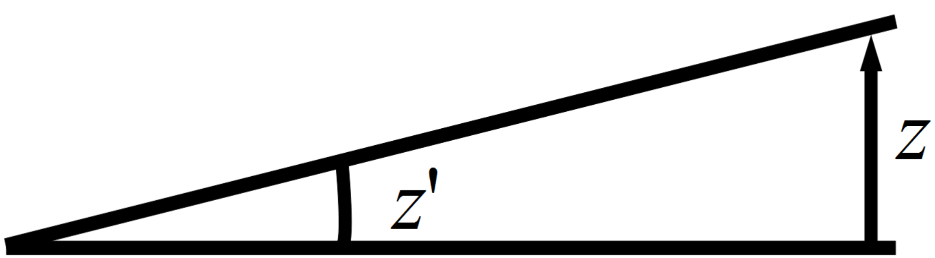


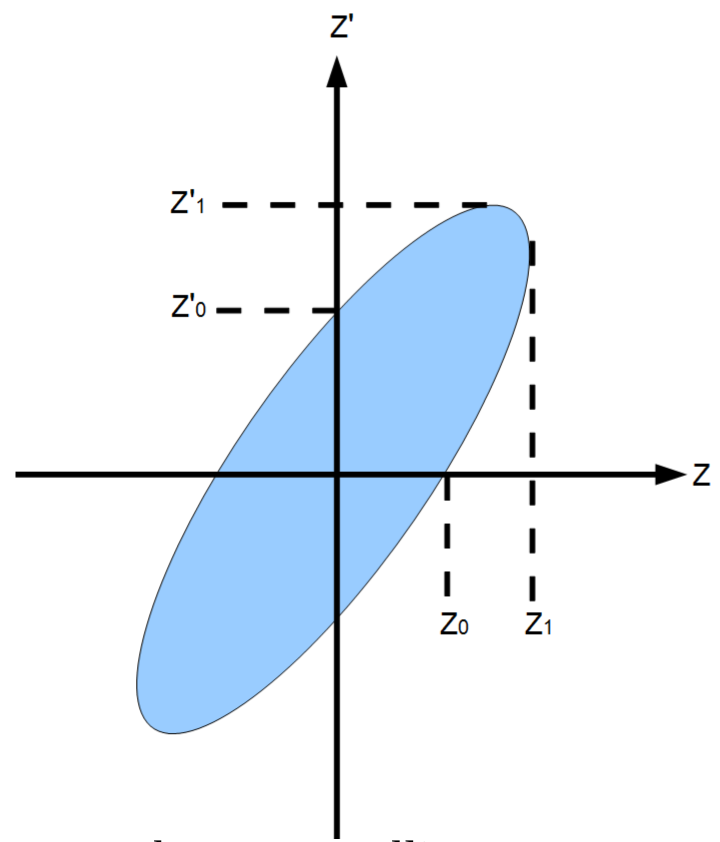
Emittance Measurement

- Why to measure?
- Emittance a key parameter of electron beam that determines the quality of positron beam.
- An Input parameter for simulation.

- What is emittance?
- The area of the ellipse is an invariant, which is called Courant-Snyder invariant.
- The transverse emittance of the beam is defined to be the area of the ellipse, which contains 90% of the particles.

$$z' = \frac{dz}{ds}$$



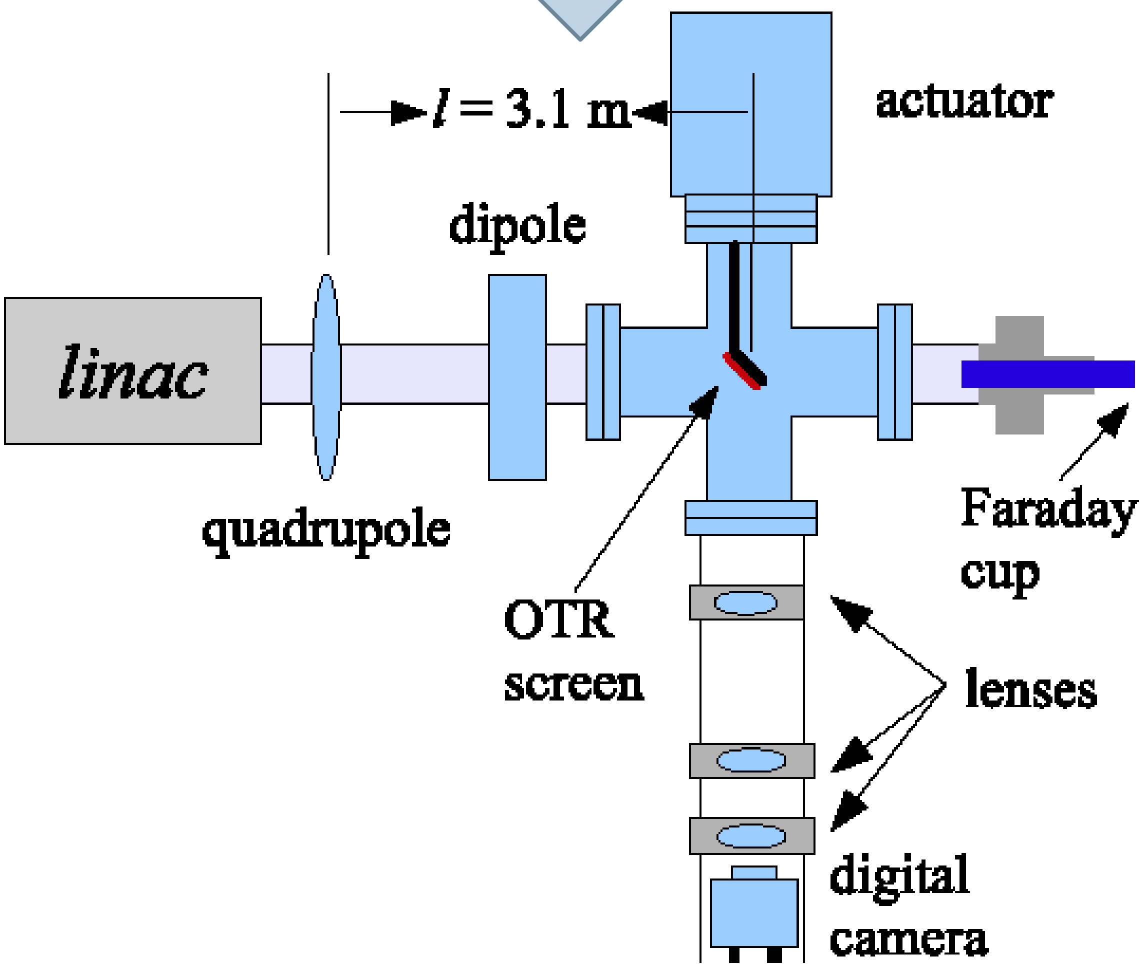


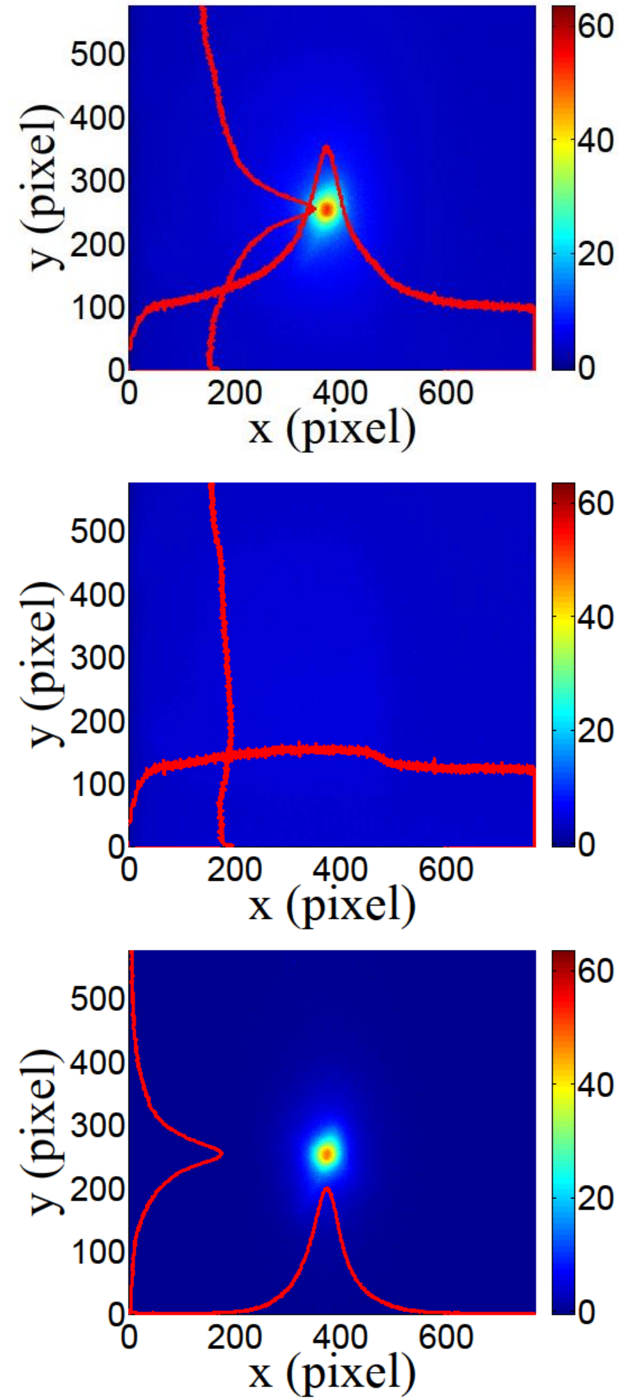
Phase space ellipse

Emittance Measurement

Background Subtraction.

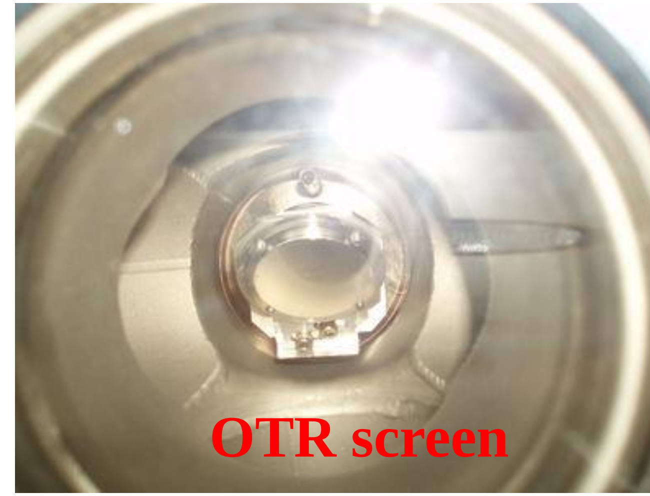
Quadrupole Scanning Method.





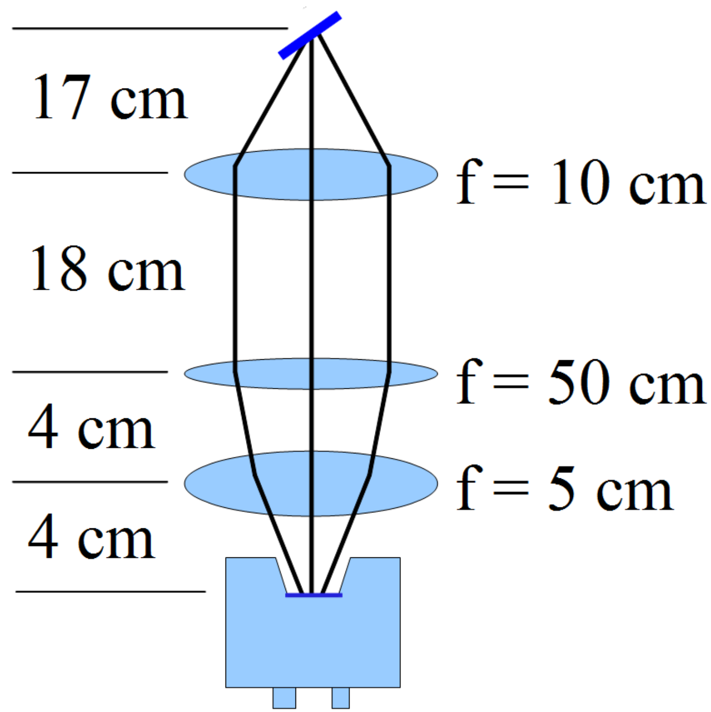
Emittance Measurement

Optical Transition Radiation: Electromagnetic radiation emitted when a charged particle passes through the boundary between two different media.



Cage system for camera:
3 lenses with different focal lengths.

- Collector: f = 10 cm.
- Fine tune: f = 50 cm.
- Course tune : f = 5cm.



Emittance Measurement

Quadrupole scanning of electron beam on OTR Screen.

Scan Q1 from -5 Amp to 5 Amp in 51 steps, at 0.2 Amp increments.

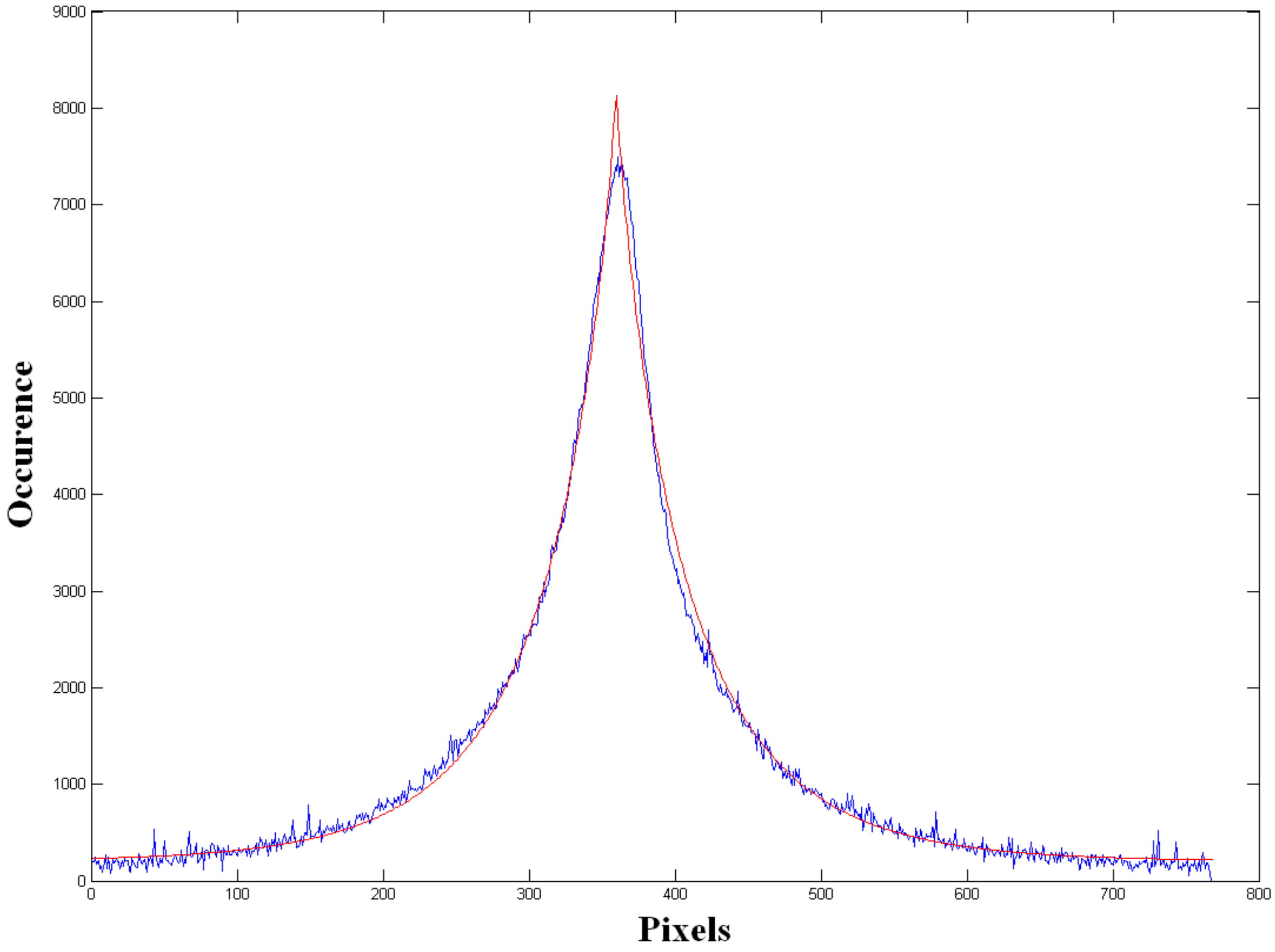
Emittance Measurement

Electron beam projected profile is not Gaussian

Super-Gaussian Fitting:

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(\frac{-(\mathrm{abs}(x))^N}{2\sigma_0^N} \right) \quad \text{with} \quad \sigma = \sigma_0 \cdot \left(\frac{\pi}{2}\right)^{2/N-1}.$$

- **N = 2 → Normal Gaussian**
- **N < 2 → Super Gaussian**
- **N > 2 → Flat-top shape**
- **fitted the Super-Gaussian.**
- **σ = 39 px**

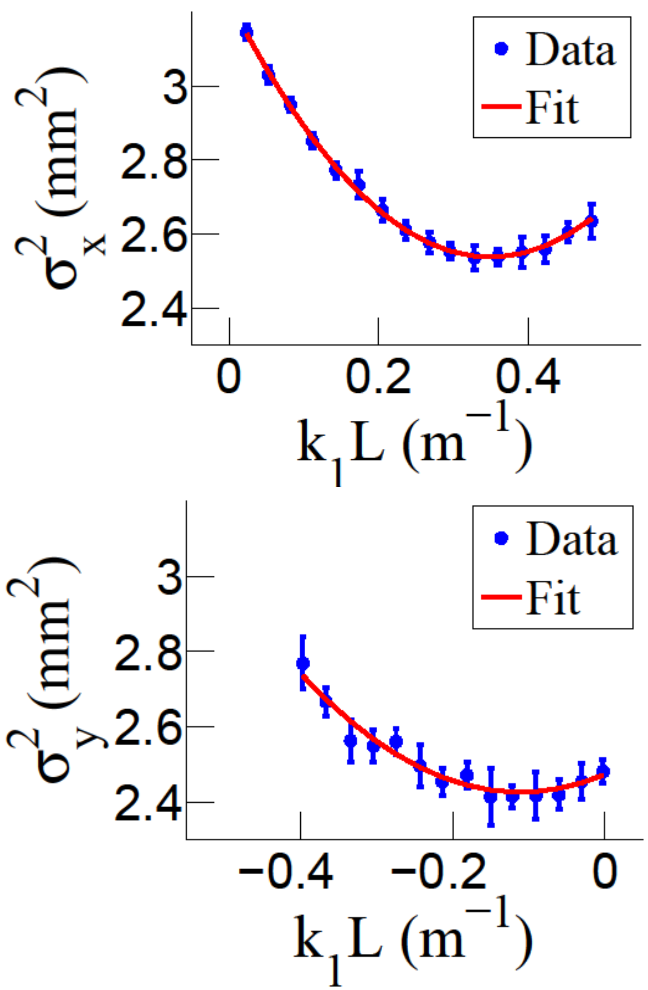


Emittance Measurement

Fit is parabolic function, emittance and Twiss parameters are extracted from constants A, B, and C.

$$\sigma_x^2 = A(kL-B)^2 + C = A(kL)^2 - 2AB(kL) + (C + AB^2)$$

The projected emittance of the HRRL was measured to be less than 0.4 μm using the OTR, at an peak energy of 15 MeV.

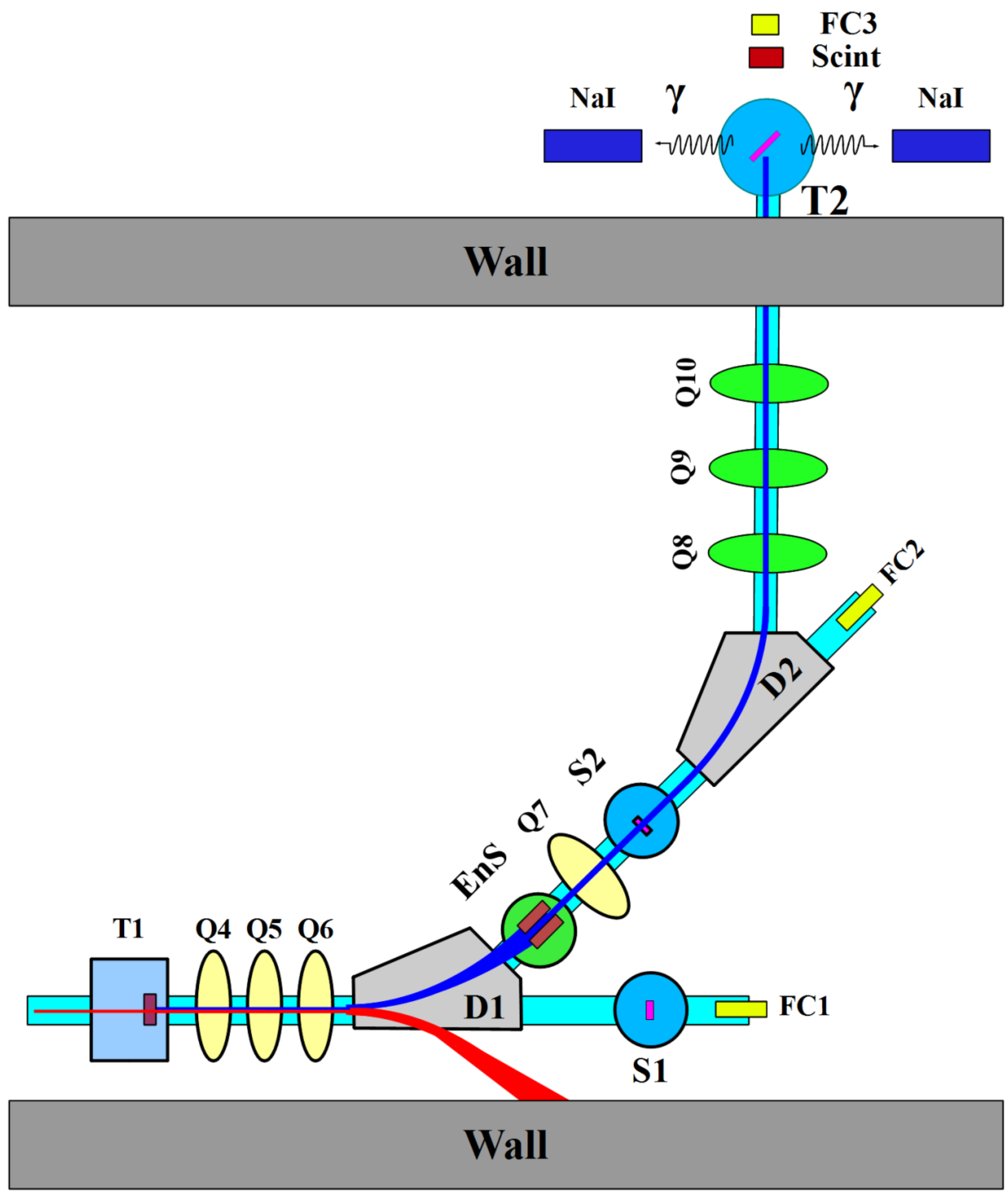


Emittance Measurement Results.

Parameter	Unit	Value
projected emittance ε_x	μm	0.37 ± 0.02
projected emittance ε_y	μm	0.30 ± 0.04
β_x	m	1.40 ± 0.06
β_y	m	1.17 ± 0.13
α_x	rad	0.97 ± 0.06
α_y	rad	0.24 ± 0.07
micro-pulse charge	pC	11
micro-pulse length	ps	35
energy of the beam E	MeV	15 ± 1.6
relative energy spread ΔE/E	%	10.4

Positron Transportation

- Quadrupole triplet system collects positrons.
- First dipole separates positrons and electrons.
- Magnets in beamline can be optimized for transporting electrons. Then we reverse polarity on magnets to have beamline optimized for positrons.



Positron Detection

T2: tungsten foil with 0.04 inch (1.016 mm) thickness, 1.25 inch (31.75 mm) diameter. Placed with 45 degree angle regarding to the beam direction.

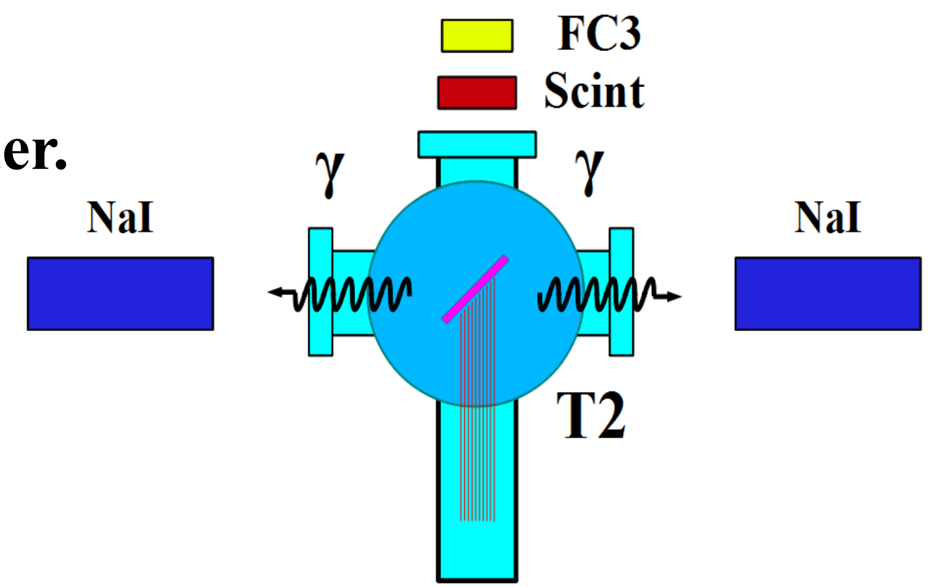
Positrons annihilate, produce two 511 keV photons back to back.

Scintilator: Detect photons and electrons. Used to tune electron beam.

FC3: Faraday Cup to measure electron beam.

2 NaI detectors measure photons.

Crystal: 3" diameter, 3" thick cylinder.

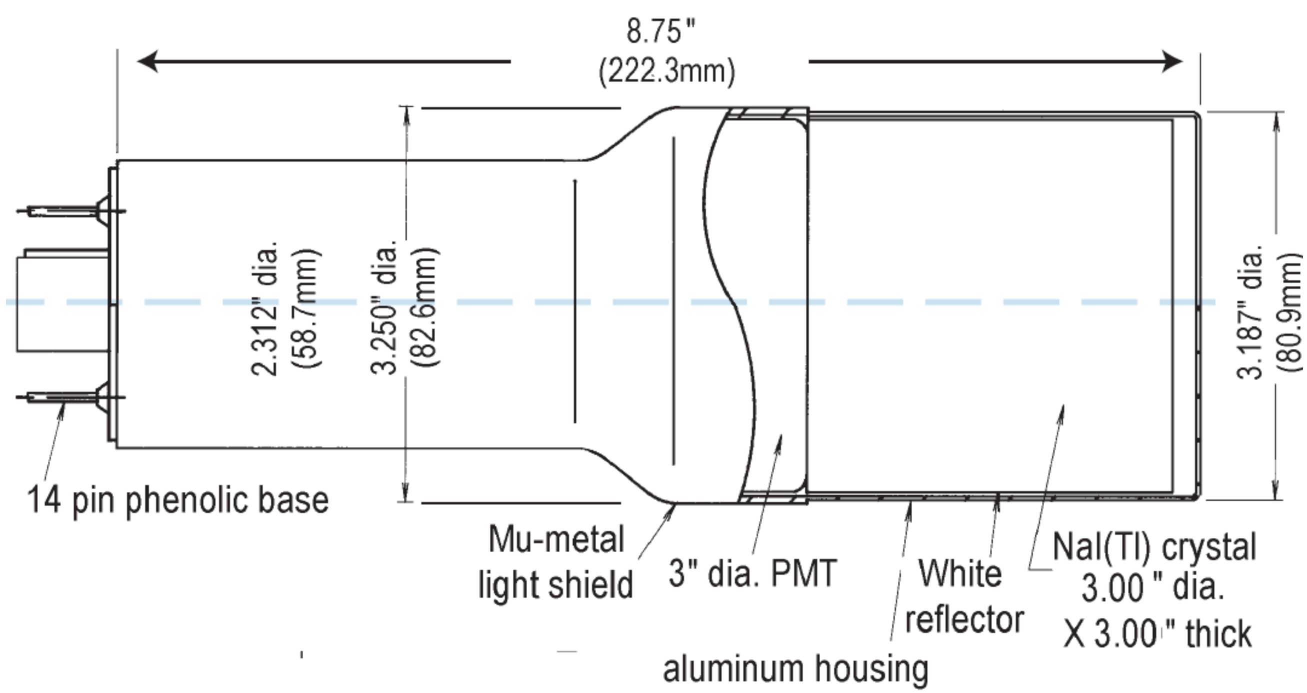


Positron Detection

2 NaI detectors measure photons.
Crystal: 3” diameter, 3” thick.

Rebuilt PMT bases:

old base pulse length ~ 400 μs.
new base pulse length ~ 1 μs.



Model 3M3/3

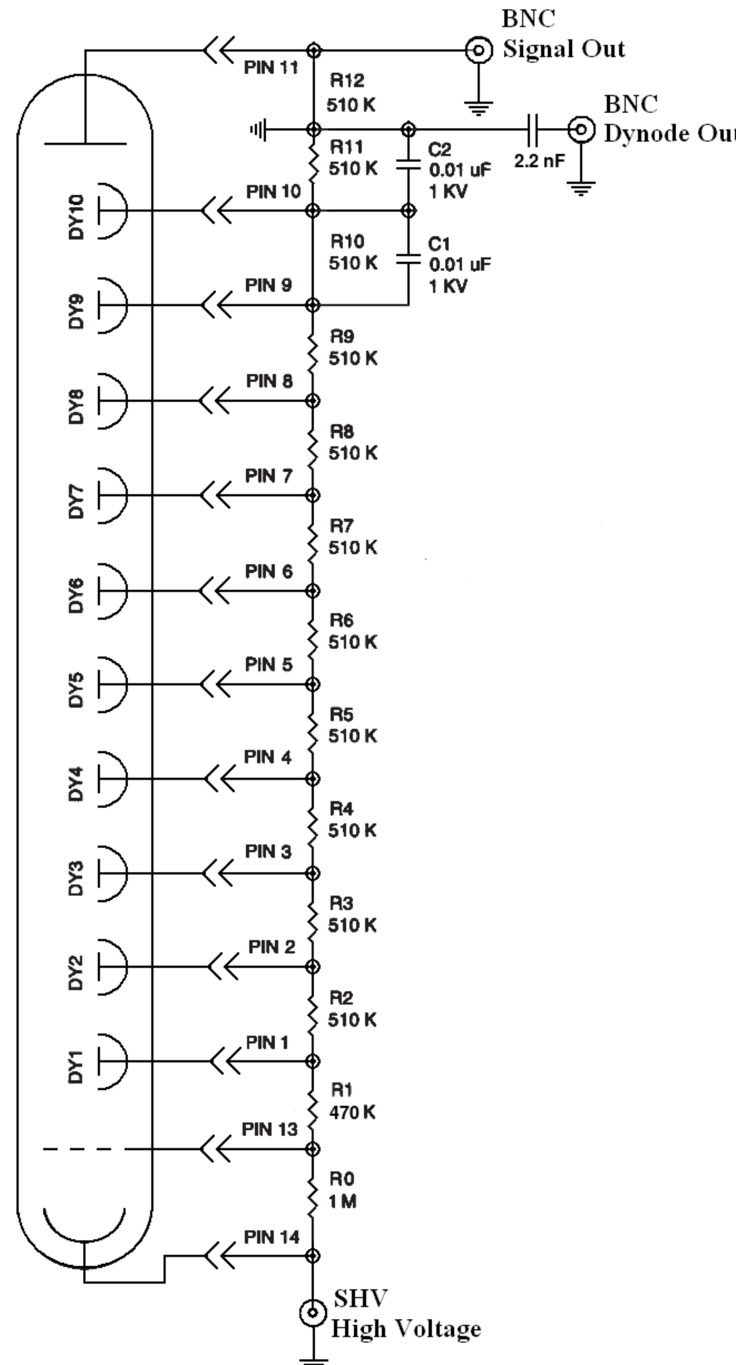


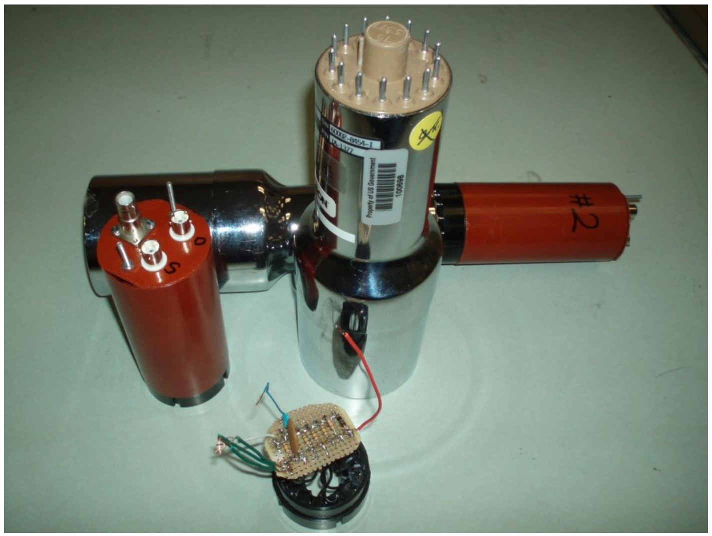
Figure from SAINT-GOBAIN CRYSTAL & DETECTORS

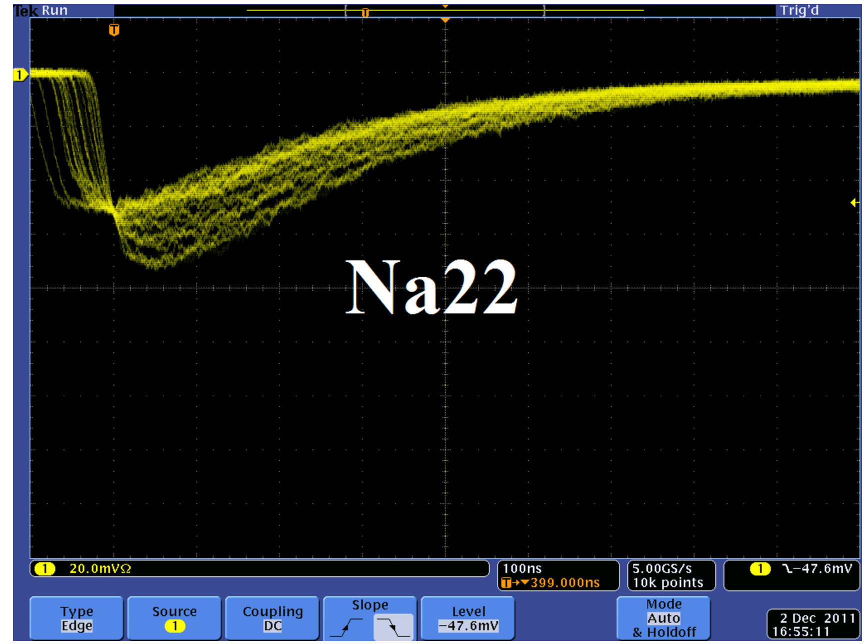
Positron Detection

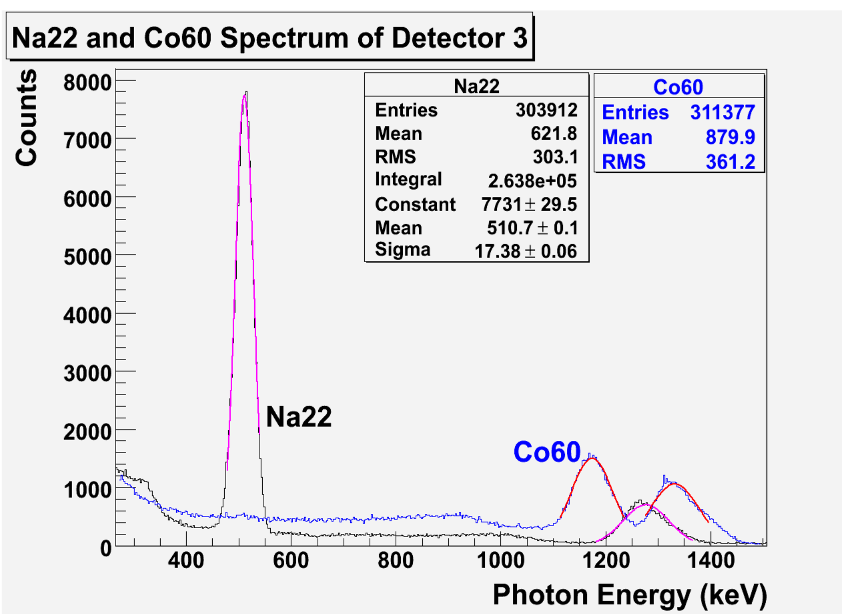
PMT base HV = -1150 V

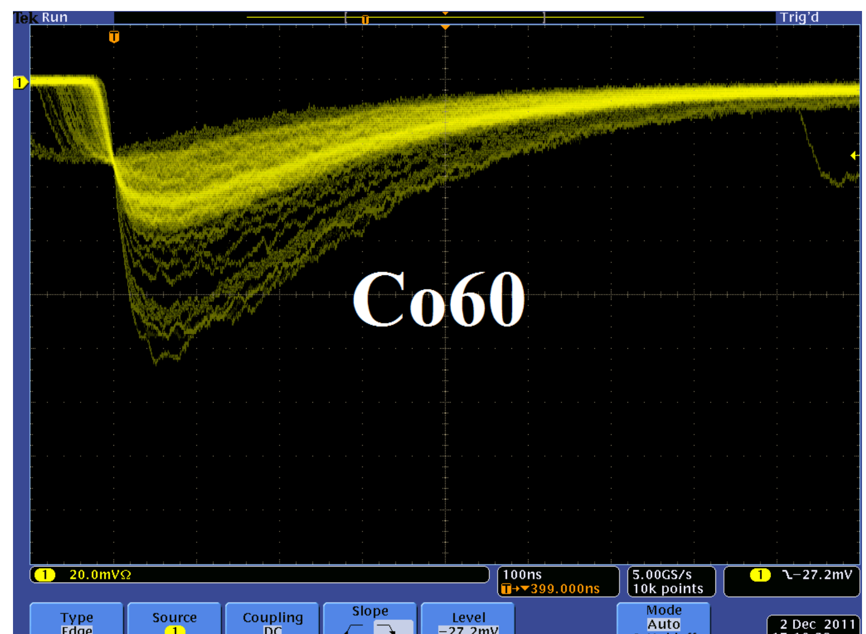
Co60: 1173 keV and 1332 keV

Na22: 1275 keV and 511 keV

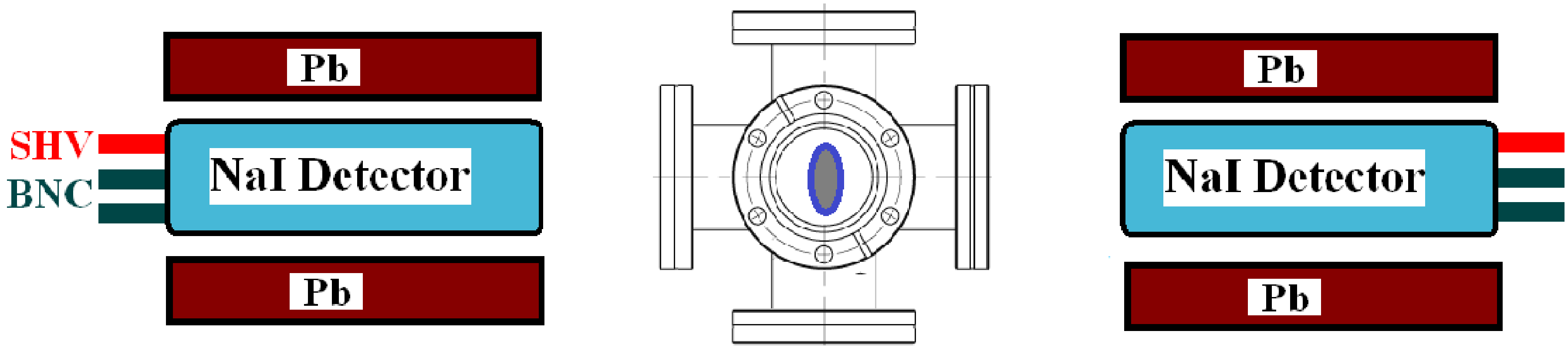








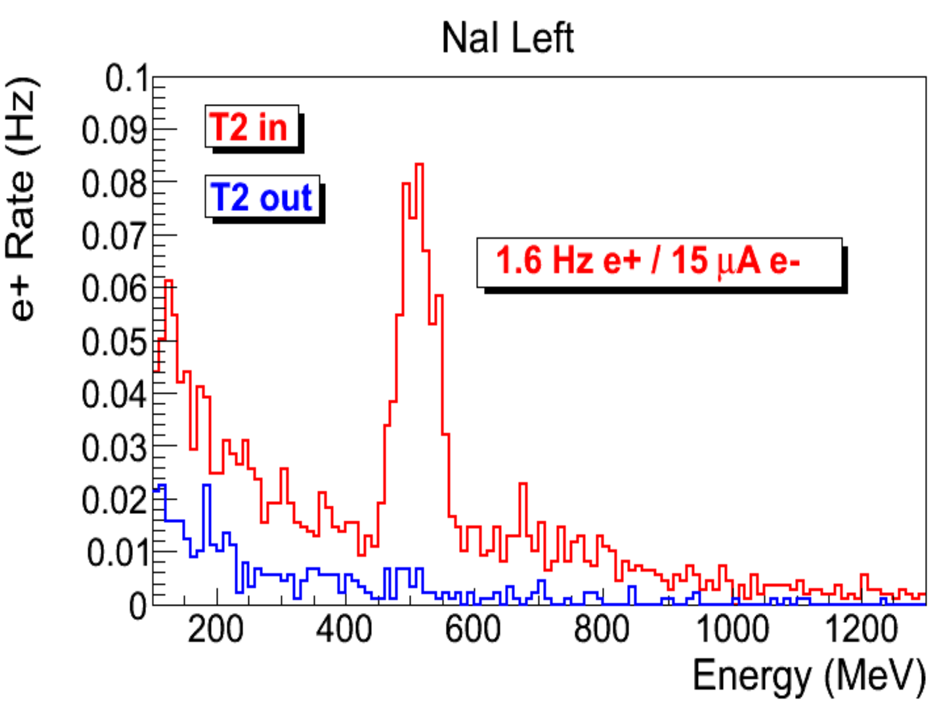
Positron Detection

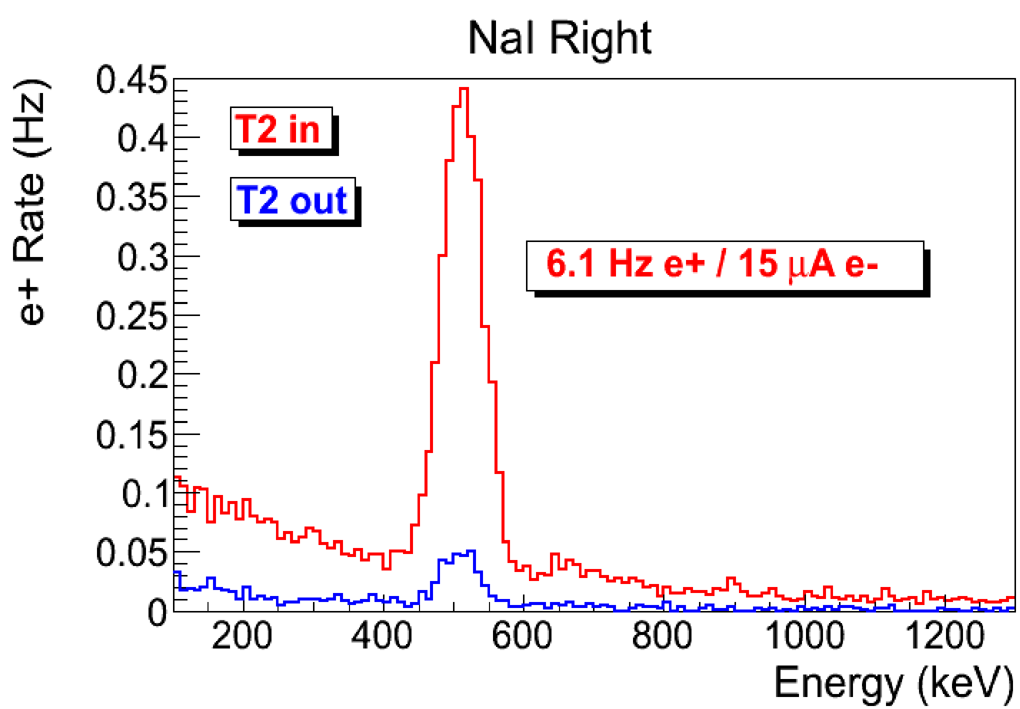


T2 in: annihilate target in the beamline.

T2 out: annihilate target out of the beamline (background run).

Coincidence of detectors and gun trigger: (DL & GunTrig) & (DR & GunTrig)



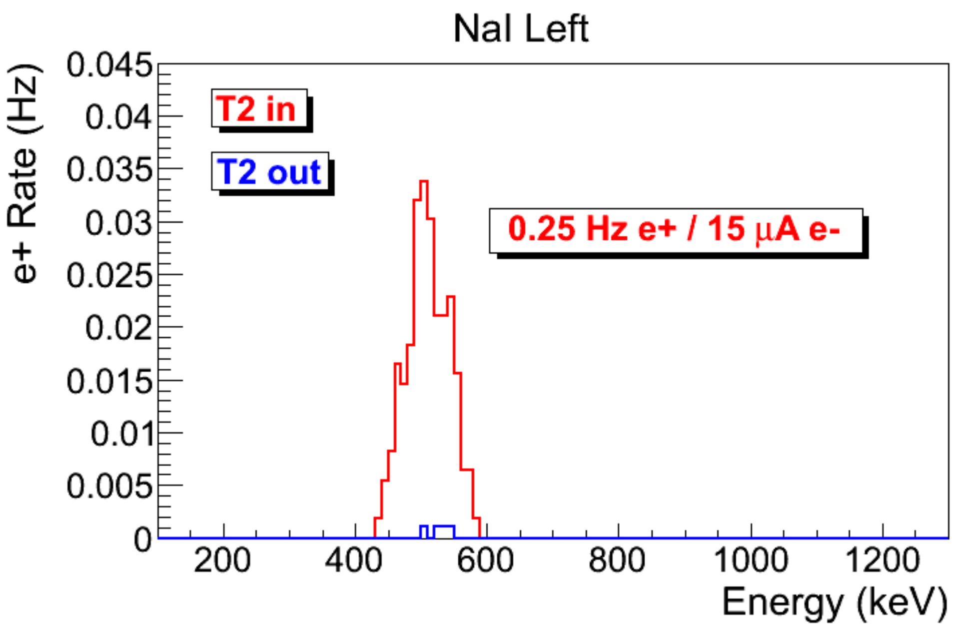


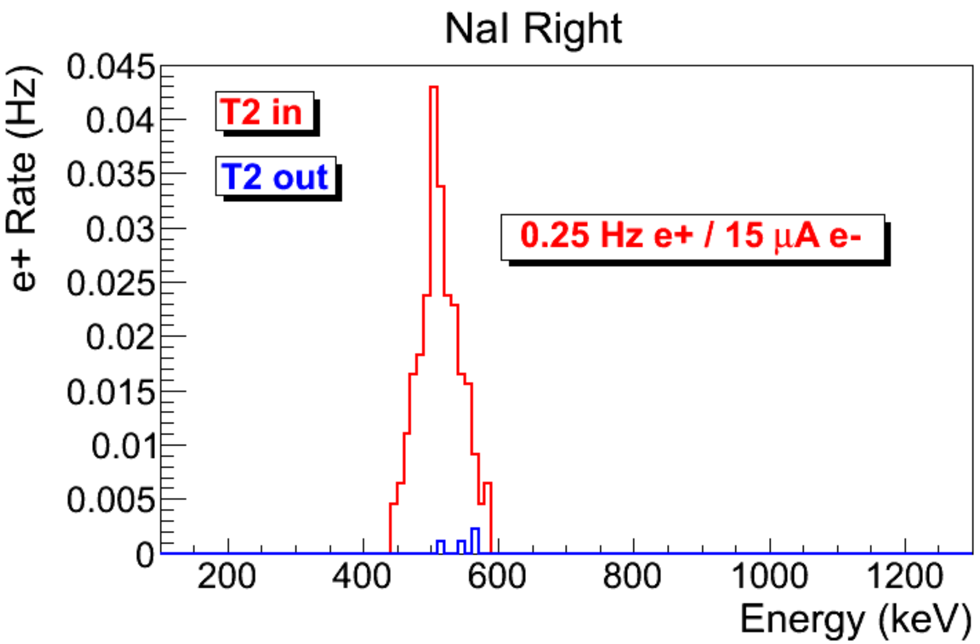
Positron Detection

T2 in run conditions.

Parameter	Unit	Value
repetition rate	Hz	300
run time	s	1094
macro pulse number		329368
event number		15361
e^- rate	Hz	$(1.37 \pm 0.02) \times 10^{14}$
e^- current	μA	15 ± 0.4

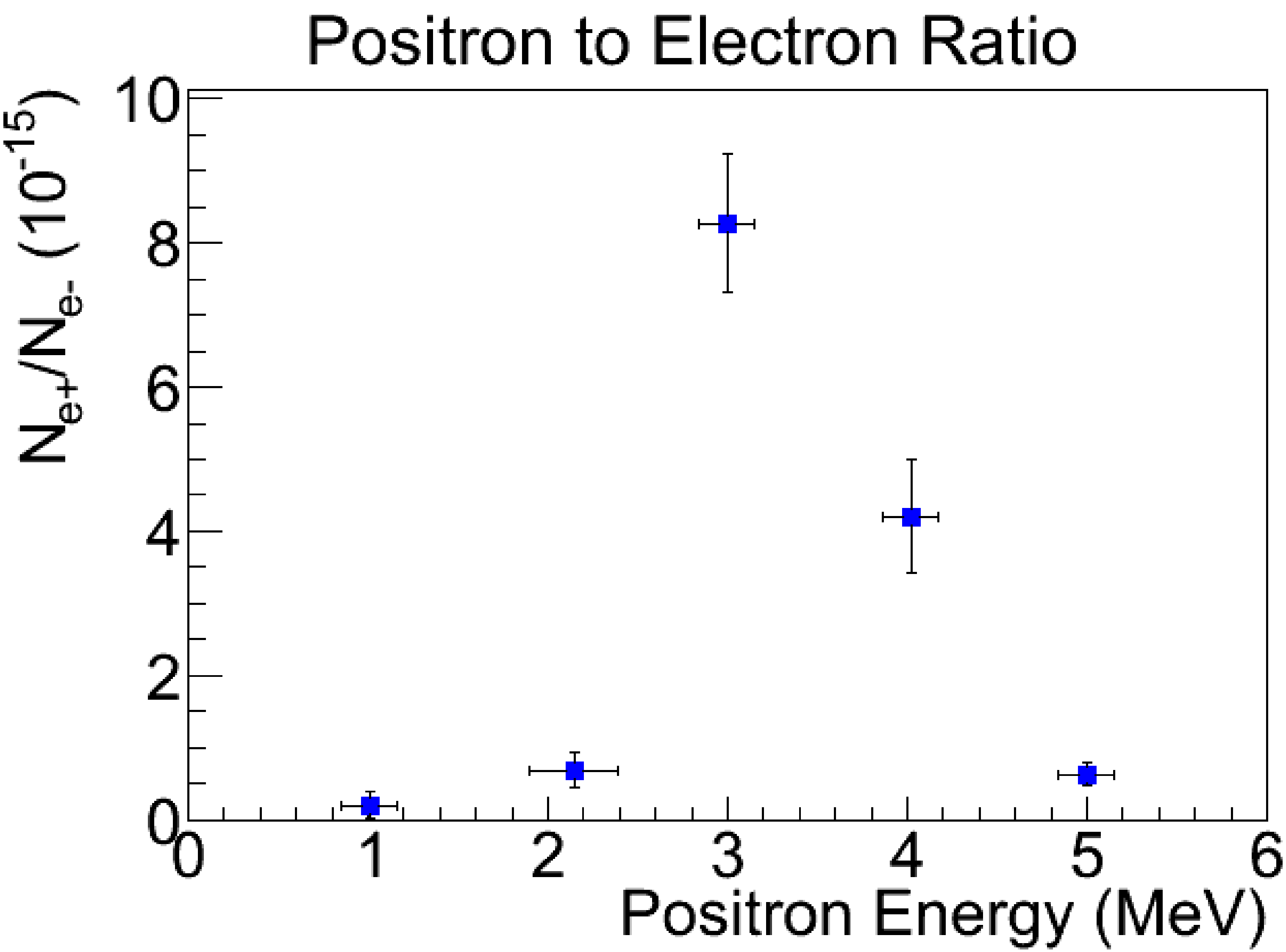
Cut: events around 511 keV regions on both detectors.





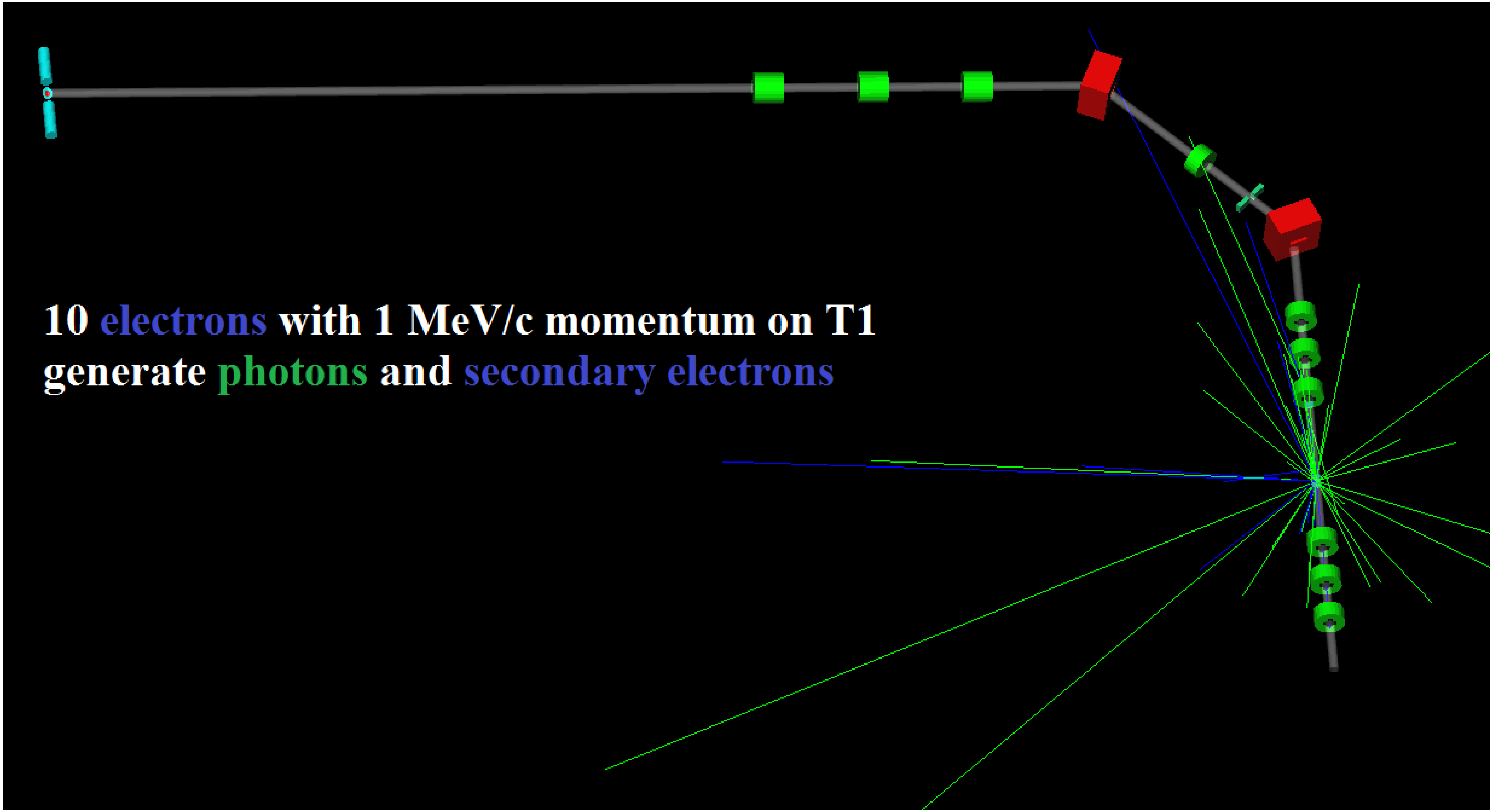
Positron Detection

$$Ratio = \frac{\text{number of positrons (with certain energy) detected at NaI}}{\text{total number of electrons on the target}}$$



Simulation: Positron Beam Loss

G4beamline is a particle tracking and simulation program based on the Geant4 toolkit that is specifically designed to easily simulate beamlines and other systems using single-particle tracking.

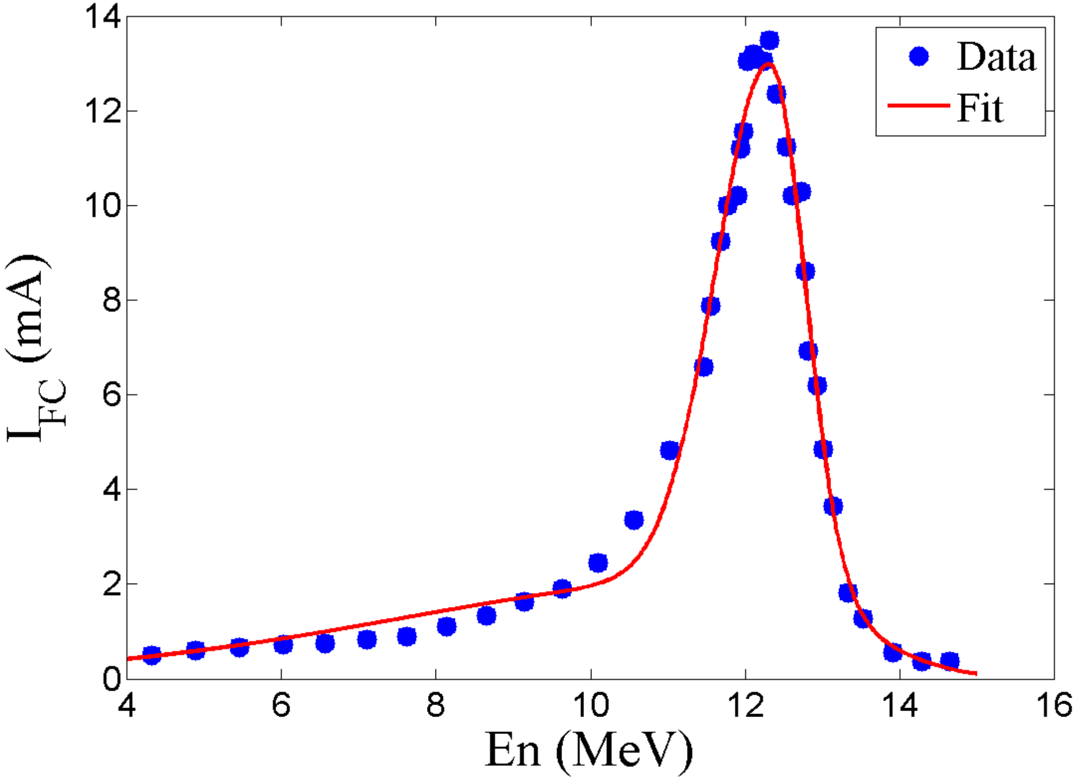


Simulation: Positron Beam Loss

Simulation toll: G4beamline

Electron Beam Generation:
The beam energy spread does not follow Gaussian distribution, but the overlapping of two skewed Gaussian found to be the best description of beam energy profile [2].

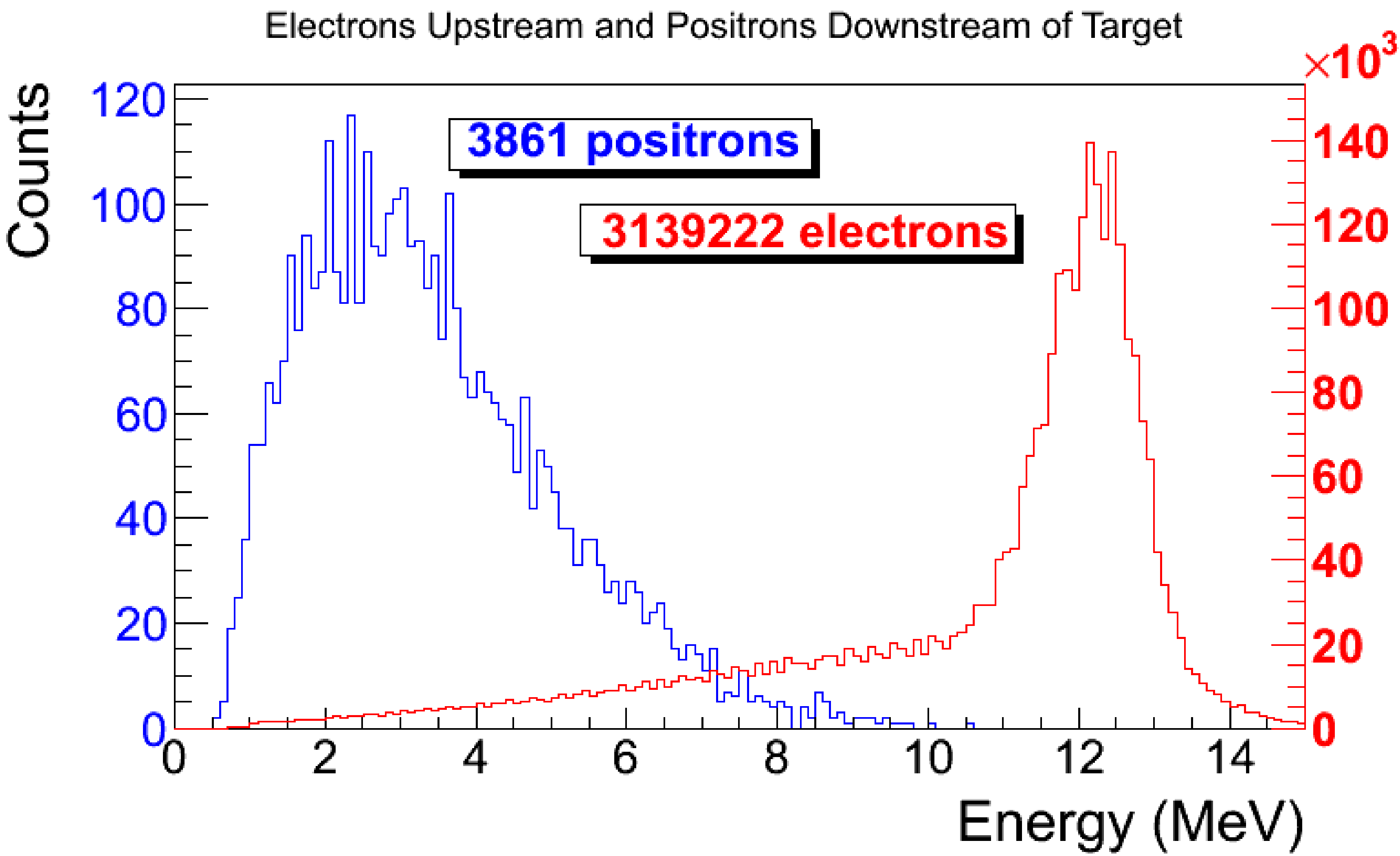
Electron Beam Energy Spread at 12 MeV



Parameter	Notation	First Gaussian	Second Gaussian
amplitude	A	2.13894	10.88318
mean	μ	12.07181	12.32332
sigma left	σ_L	4.46986	0.69709
sigma right	σ_R	1.20046	0.45170

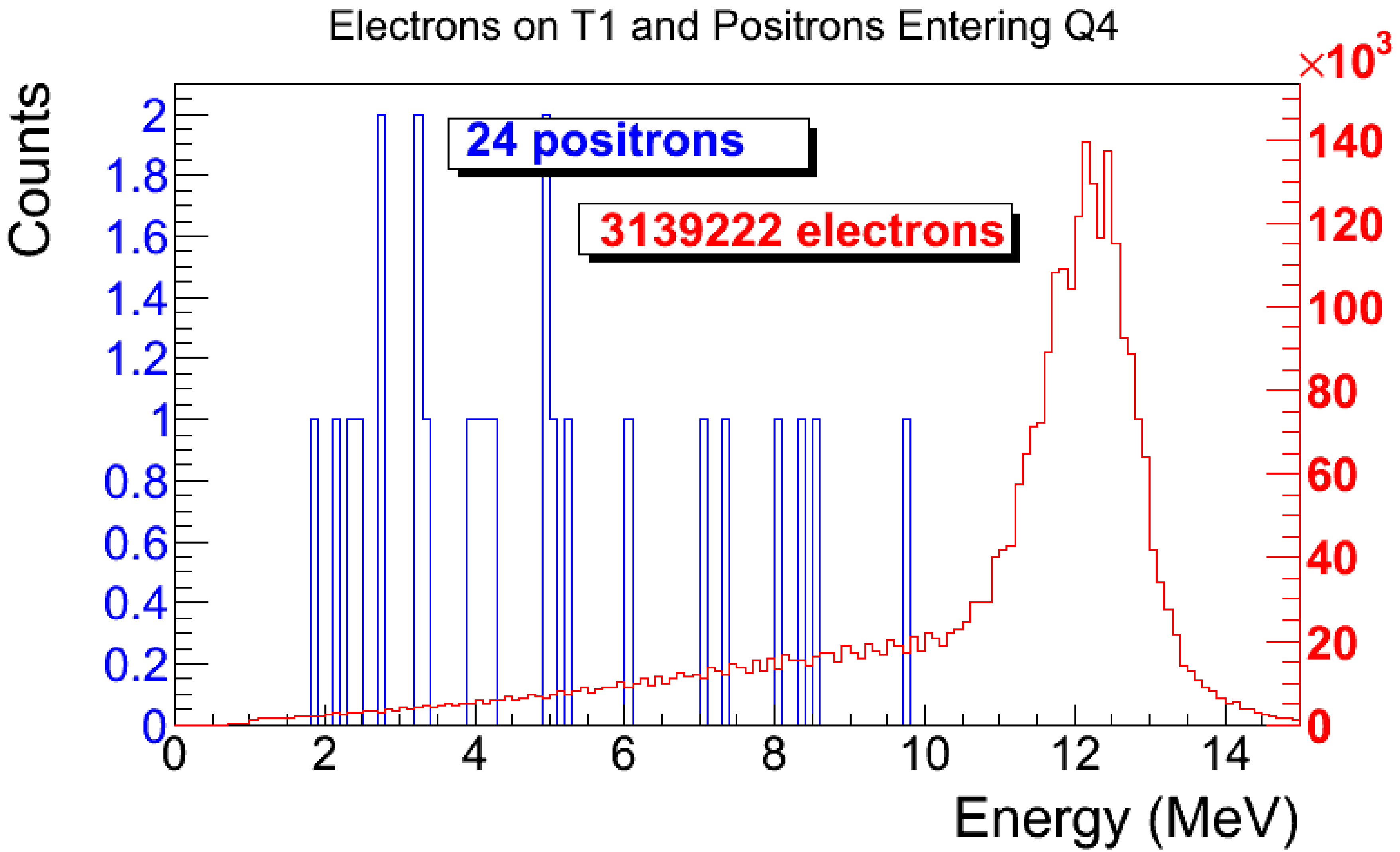
Simulation: Positron Beam Loss

3, 139, 222 electrons on T1 generated 3861 positrons.



Simulation: Positron Beam Loss

3, 139, 222 electrons on T1 generated 3861 positrons.
Only 24 positrons enters Q4 (0.6%).
Positrons come out with π angle.



Summary

1. Reconfigured HRRL for positron production.

2. Constructed a positron detection system using two NaI detectors.

3. Experimentally measured positrons at different energies (1-5 MeV). Peak is around 3 MeV.

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Also Thanks to my friends Chris, Jack, Tony, and many others.

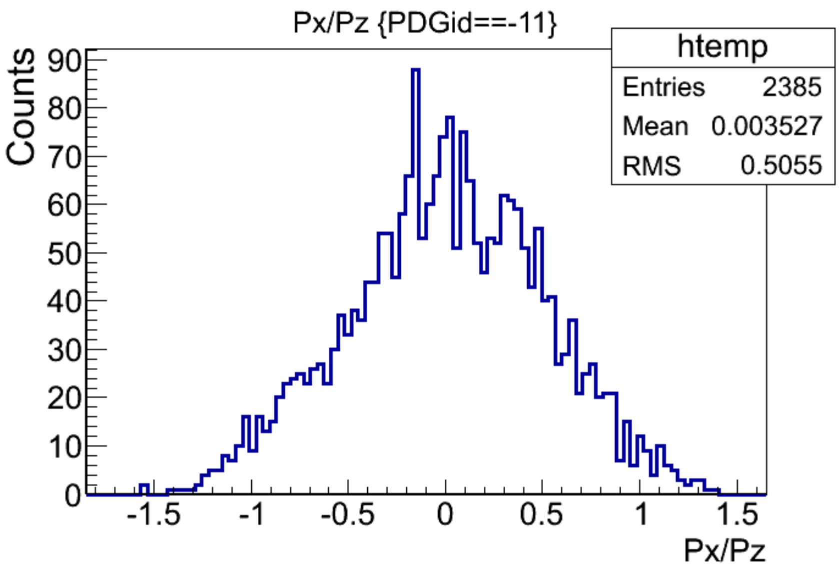
Thanks for listening !

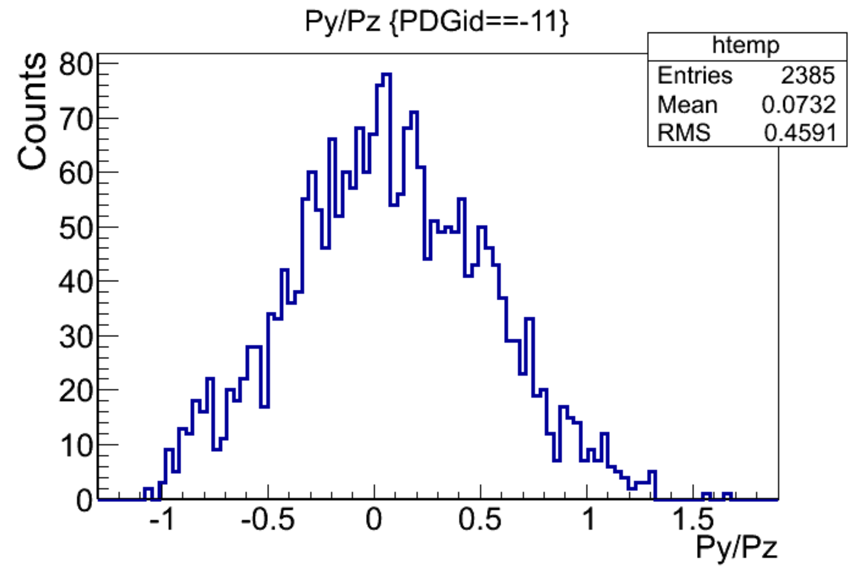
Questions ?

Simulation: Positron Beam Loss

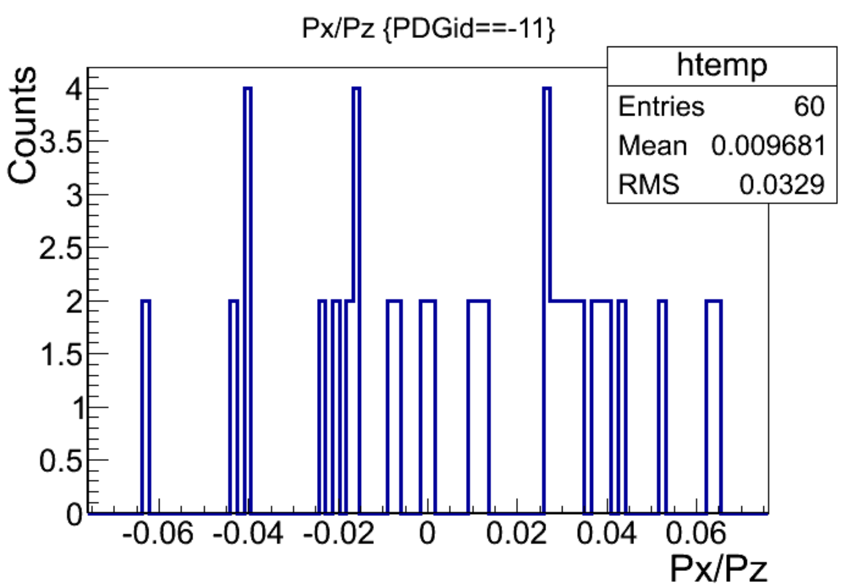
Positrons come out with π angle. Only 2.5% (60/2385) makes to the Q4.

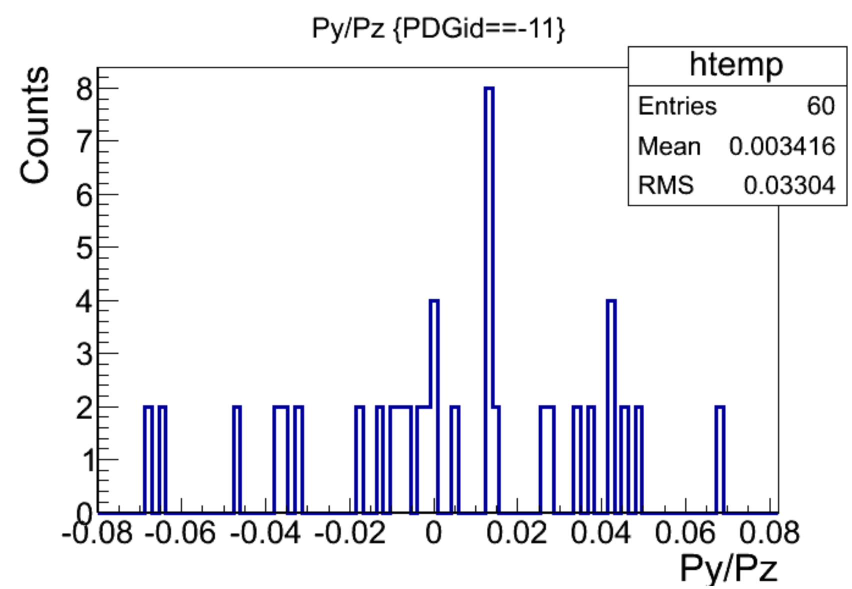
Positrons come right after T1.





Positrons come right after T1.





Motivation

The nucleon electromagnetic form factors are fundamental quantities that related to the charge and magnetization distribution in the nucleon.

$G_E(Q^2)$: Electric form factors.

$G_M(Q^2)$: Magnetic form factors

- **Rosenbluth Technique: A method to measure electric and magnetic form factors [1] .**

- **Recoil Polarization Technique measures the ratio: $G_E(Q^2)/G_M(Q^2)$ [2].**

Note: Both are electron scattering off nuclei.

Motivation

Rosenbluth Technique:

The unpolarized differential cross section for elastic scattering (in one-photon approximation):

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

where σ_{Mott} is the Mott cross section, θ is electron scattering angle, Q is transferred four momentum, and $\tau = Q^2/4M_p^2$.

longitudinal virtual photon polarization: $\varepsilon = (1 + 2(1 + \tau)\tan^2(\theta/2))^{-1}$:

$$\frac{d\sigma}{d\Omega} = \frac{\tau\sigma_{Mott}}{\epsilon(1+\tau)}\left[G_M^2 + \frac{\epsilon}{\tau}G_E^2\right]$$

Measure ***dσ/dΩ*** measured at different initial electron energies (in σ_{Mott}) and scattering angles, θ, while keeping momentum transfer (***Q***) the same [1].

Motivation

Recoil Polarization Technique:

A ratio of **transverse (P_t)** and **longitudinal (P_l)** **polarization of recoil protons** from elastic scattering of **longitudinally polarized electrons** on an **unpolarized hydrogen target** was measured.

the ratio of proton form factors can be extracted directly from the ratio of P_t and P_l

$$\frac{G_E}{G_M} = -\frac{P_t}{P_l}\frac{(E+E')}{2M_p}\tan\frac{\theta}{2}$$

where E, E′ are the electron energy before and after scattering.
M_p is the proton mass.

Motivation

What is the effect of Two Photon Exchange (TPE)?

In the standard one-photon exchange (Born) approximation, the reduced Bron Cross section can be written as

$$\sigma_R = G_M(Q^2) + (\varepsilon / \tau) G_M(Q^2),$$

TPE arises as radiative correction δ of order α:

$$\sigma_R \rightarrow \sigma_R(1 + \delta)$$

Dente:

The amplitude of one photon exchange $\mathscr{M}_\gamma$

The amplitude of two photon exchange $\mathscr{M}_{\gamma\gamma}$

Then to leading order in α, δ arises from the interference term

$$\delta = \frac{2\mathrm{R}\{\mathscr{M}_\gamma^{+} \mathscr{M}_{\gamma\gamma}\}}{\left|\mathscr{M}_\gamma\right|^2}.$$

Direct experimental evidence for the contribution of TPE can be obtained by comparing the ratio of e^+p to e^-p cross sections. Because $\mathscr{M}_\gamma$ changes sign for positron scattering, where as $\mathscr{M}_{\gamma\gamma}$ does not, δ has the opposite sign for the electron and positron scattering [1] .

HRRL Energy Spread:

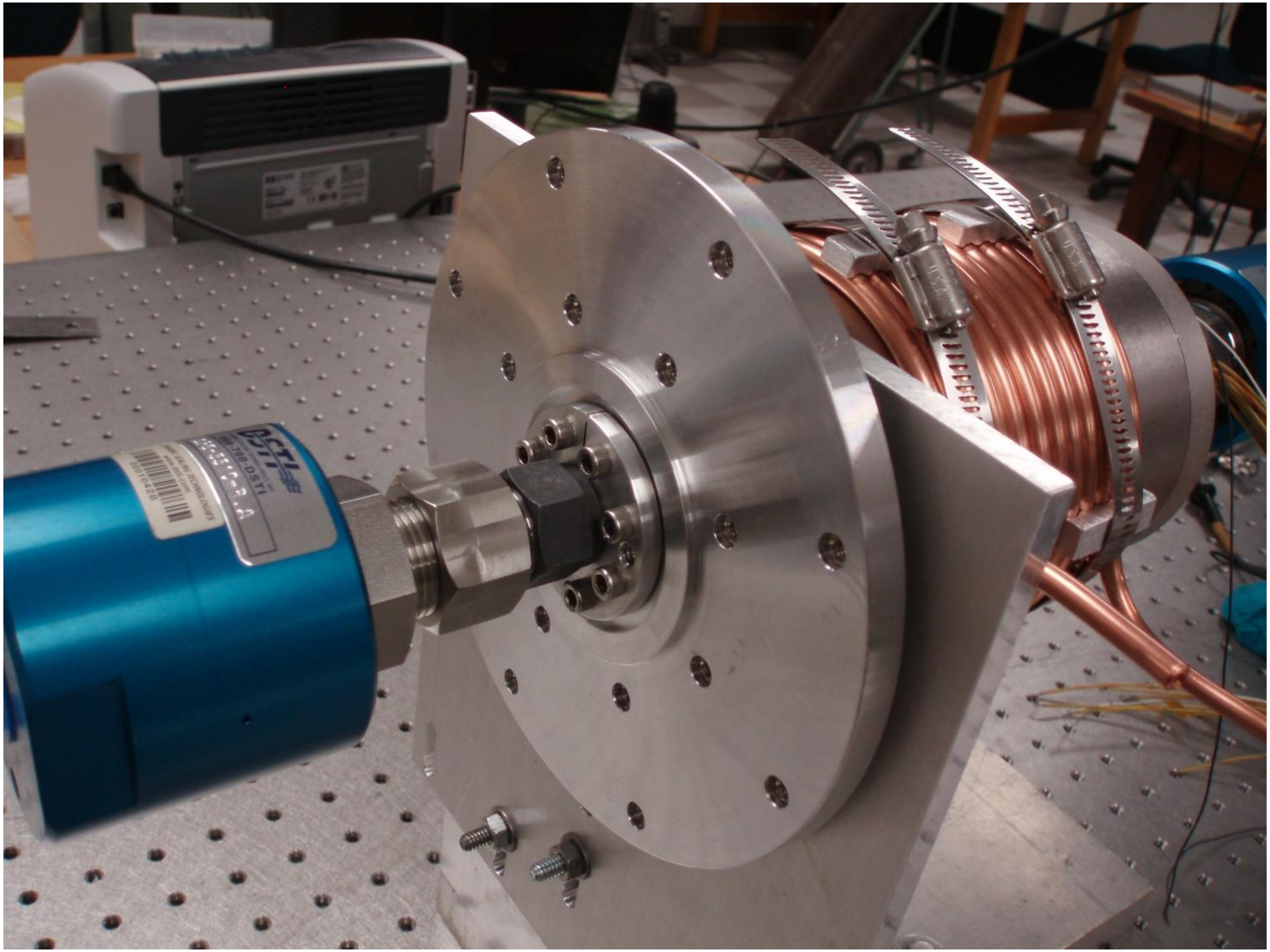
When En = 4 MeV: $\Delta E = 2$ MeV

When En = 8 MeV: $\Delta E = 3$ MeV

When En = 12 MeV: $\Delta E = 4$ MeV

Future work: Rotating tungsten target

8 rotating targets cooled by hub heat sink.



Possible use of HRRL positron beam:

- More intense positron source for Positron Annihilation Spectroscopy a technique to study defects in the materials.

- Experiments on positrons intensity explore possible techniques for increasing currents in positron accelerators (colliders).

- Positrons can be accelerated and used as a tool just like electron beam.

Concept of “low energy” positron source:

•10-mA 10-MeV CW electron beam (JLab FEL injector)

• tungsten radiator target

• collection and energy selection with quadrupole triplets

•maximize yield in CEBAF admittance 200 π mm mrad (rms, normalized) transverse ± 2% longitudinal

Advantages:

- below neutron activation threshold
- energy spread of positron limited by primary electron energy
- compact in size
- unique continuous source

Disadvantages:

- lower pair-production cross section
- large divergence of positron beam
- heat load on target

Summary

Goal: Measure the positron production efficiency.

1. Reconfigured HRRL for positron production.

2. Constructed a positron detection system using two NaI detectors.

3. Experimentally measured positrons at different energies (1-5 MeV).

4. Construct high power positron production target.

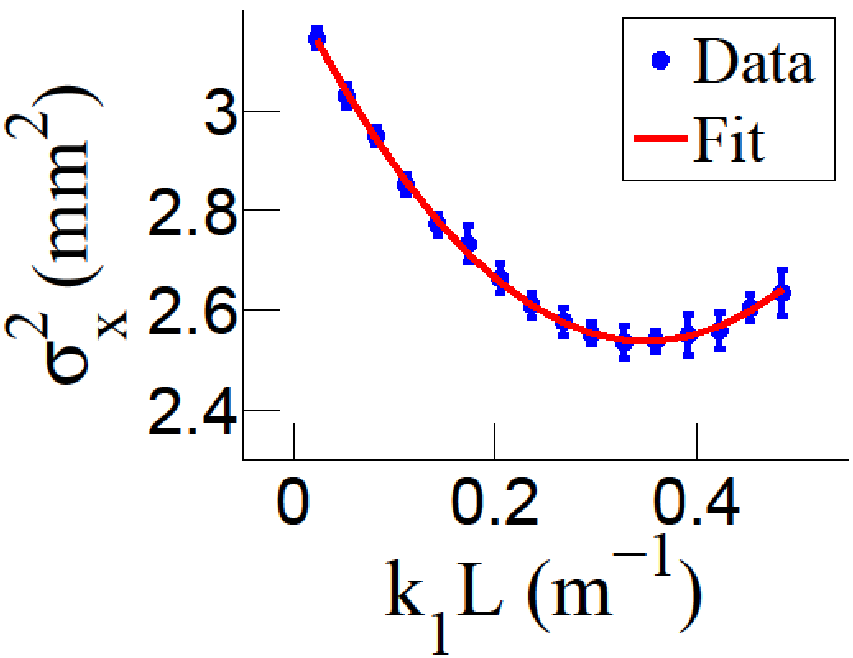
Data Analysis

Q = 14.7 pC, E = 14 MeV, macro pulse length = 200 ns FWHM pulse

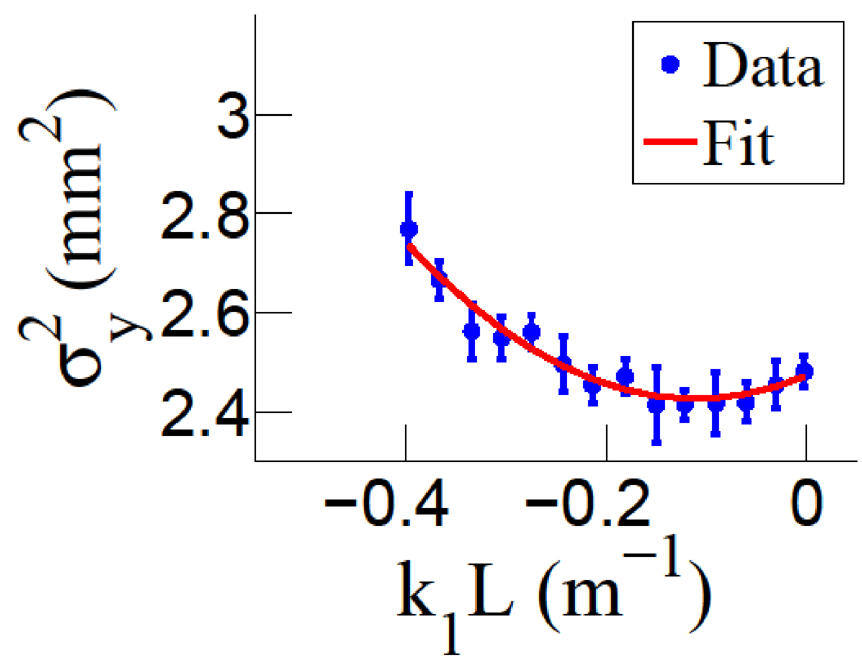
$$\sigma_x^{\ 2} = (3.678 \pm 0.022) + (-4.17 \pm 0.22) k_1 L + (5.55 \pm 0.42)(k_1 L)^2$$

$$\sigma_y^{\ 2} = (2.843 \pm 0.044) + (1.02 \pm 0.52) k_1 L + (3.8 \pm 1.2)(k_1 L)^2$$

Positive scan, X-projection



Positive scan, Y-projection



Thickness of the Tungsten target: 2 mm

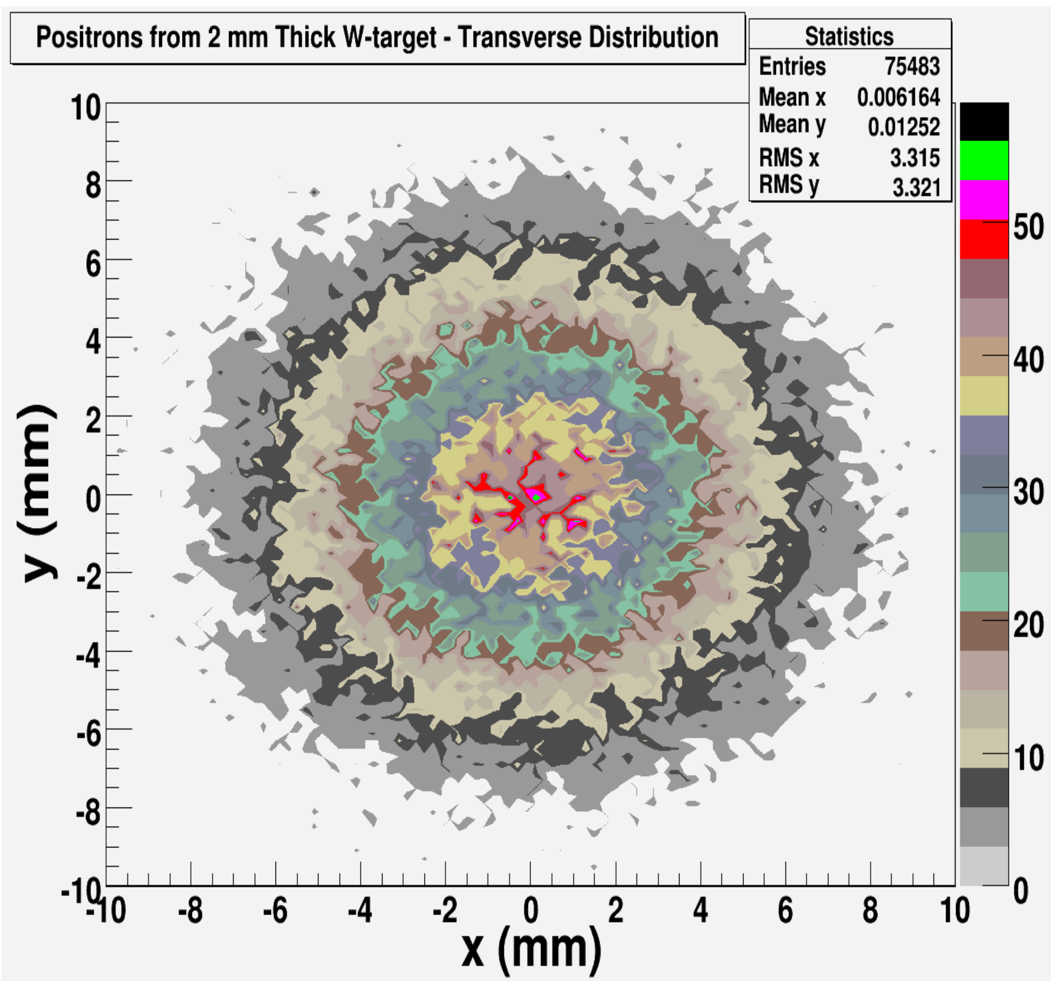
Electrons: 100 million

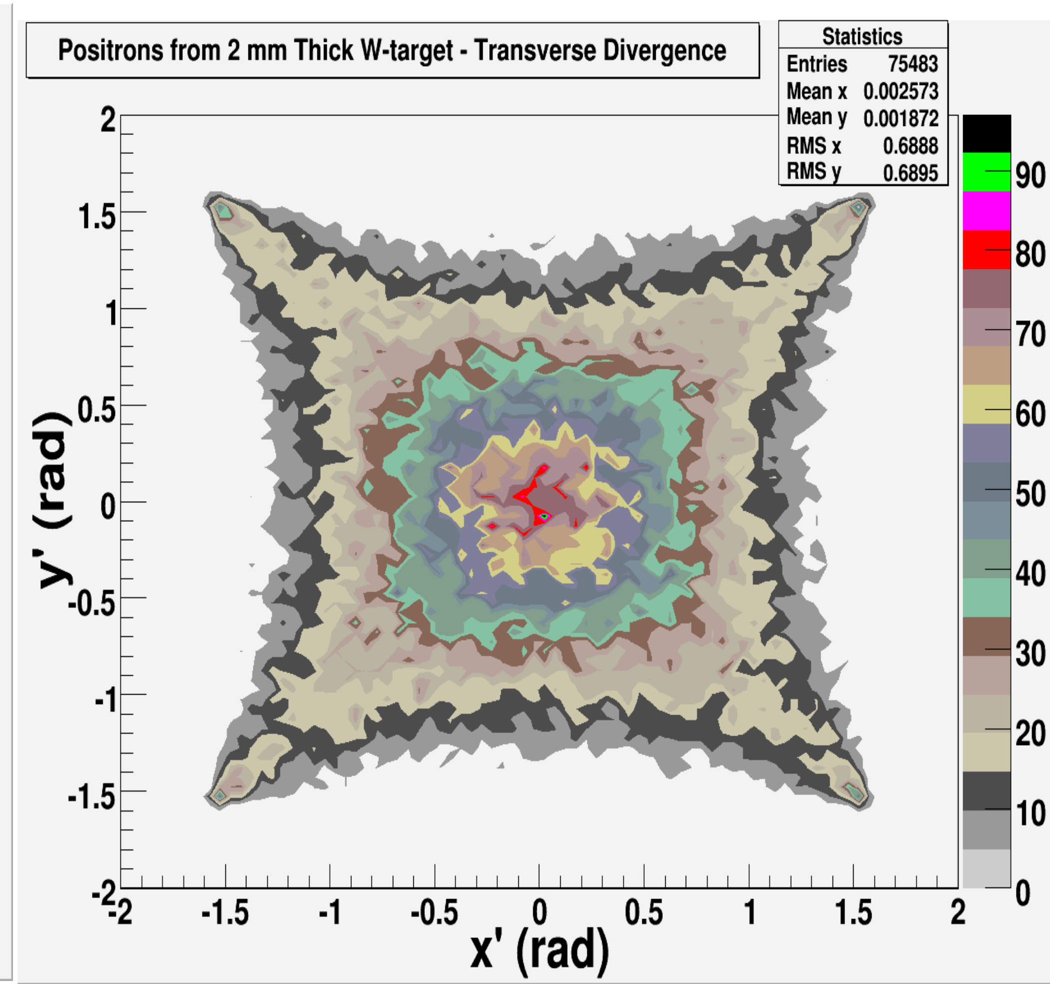
Beam size at Tungsten target: Gaussian, $\sigma_{x,y}$ = 3 mm

Beam divergence at target: 0

Beam Energy: 10 MeV

Energy Spread: $\sigma\delta$ = 4.23%





[Bremsstrahlung]: http://www.physics.hku.hk/~phys0607/lectures/chap06.html

[Pair production]http://resources.yesican-science.ca/trek/radiation/final/index.html

What?

I propose to measure the positron production efficiency for a positron source that uses a quadrupole triplet system to collect positrons from a tungsten target that are produced when the target is impinged by electrons from the HRRL.

Why ?

- A positron beam, as a new probe to explore nuclear and particle physics at Jefferson Lab. It is a complement to electron beam.
- HRRL positron beam line will be production efficiency testing beamline for the Jlab injector.
- A positron beam at ISU is also potential tool for more nuclear physics studies.
- More intense positron source for Positron Annihilation Spectroscopy.
- Possible tests to increase currents in positron accelerators (colliders).

Motivation

A large number of experiments have measured elastic electron-proton scattering cross sections in order to extract the electric and magnetic form factors, GEp (Q2) and GMp (Q2) (where −Q2 is the fourmomentum transfer squared), using the Rosenbluth technique [1].

A low Q2 measurement at MITBates [4, 5] obtained values of GEp/GMp consistent with previous Rosenbluth separations. Later experiments at Jefferson Lab (JLab) extended these measurements up to Q2 = 5.6 GeV2 [6, 7, 8], and show significant deviations from form factor scaling. They show a roughly linear decrease of the value of μpGEp/GMp from unity at low Q2 to approximately 0.3 at Q2 = 5.6 GeV2. Figure 1 shows the JLab polarization transfer measurements from refs. [6, 8], along with a global Rosenbluth analysis of the cross section measurements [2].

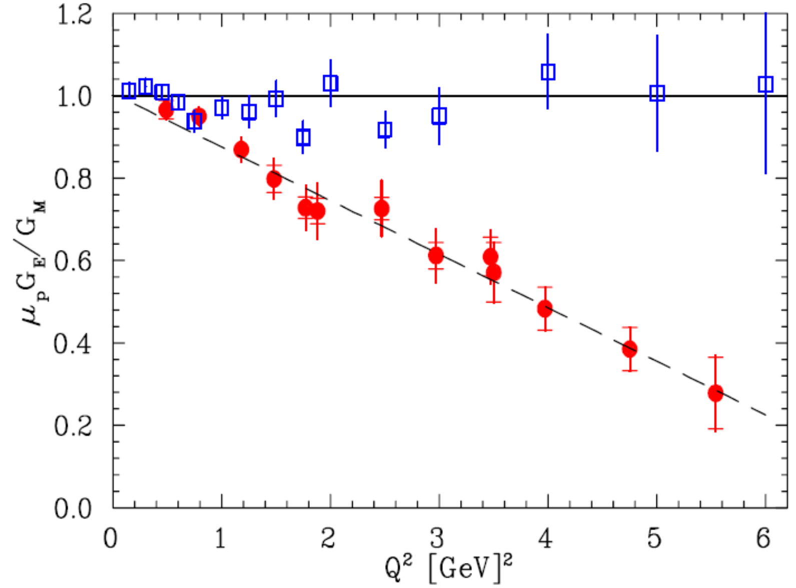


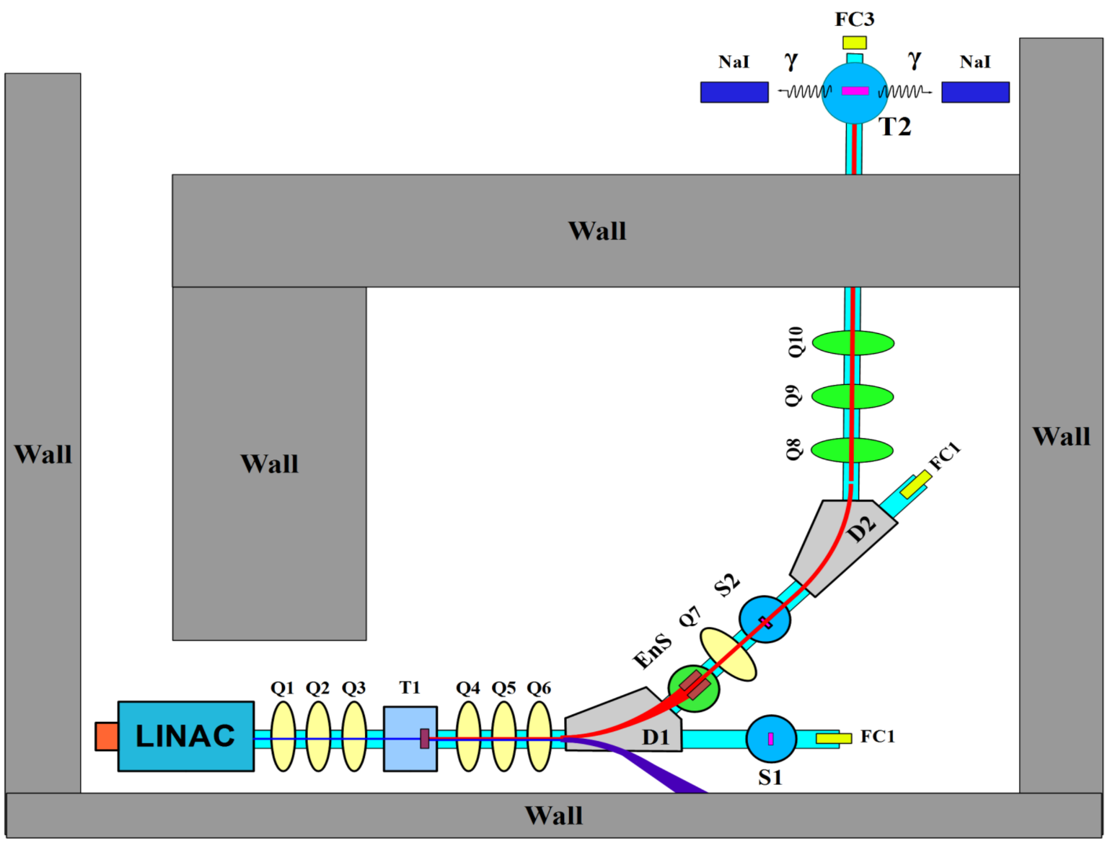
FIG. 1: Ratio of electric to magnetic form factor as extracted by Rosenbluth measurements (hollow squares) and from the JLab measurements of recoil polarization (solid circles). The dashed line is the fit to the polarization transfer data.

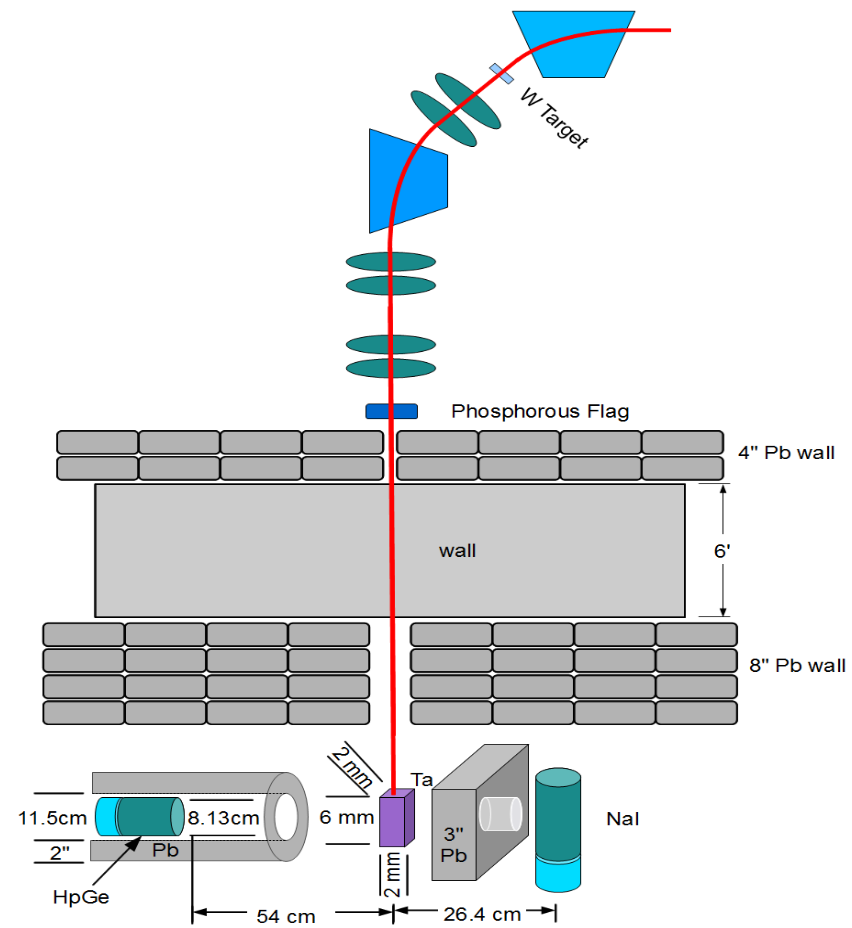
[1] M. N. Rosenbluth, Phys. Rev. 79, 615(1950).
[2] R. C. Walker et al., Phys. Rev. D 49, 5671(1994).
[3] R. C. Walker, Ph.D. thesis, California Institute of Technology(1989).
[4] B. D. Milbrath et al., Phys. Rev. Lett. 82, 2221(E) (1999).
[5] B. D. Milbrath et al., Phys. Rev. Lett. 80, 452(1998).
[6] M. K. Jones et al., Phys. Rev. Lett. 84, 1398(2000).
[7] O. Gayou et al., Phys. Rev. C 64, 038292(2001).
[8] O. Gayou et al., Phys. Rev. Lett. 88, 092301(2002).

Future Plan and Improvements

We want to produce positrons using the HRRL beam line. We can improve positron collection efficiency by applying following methods:

1. **By using a quadrupole triplet before tungsten a target, electron beam can be optimized of maximum positron beam current.**
2. **Positrons will be collected by the quadrupole triplet system, which will improve collection efficiency.**

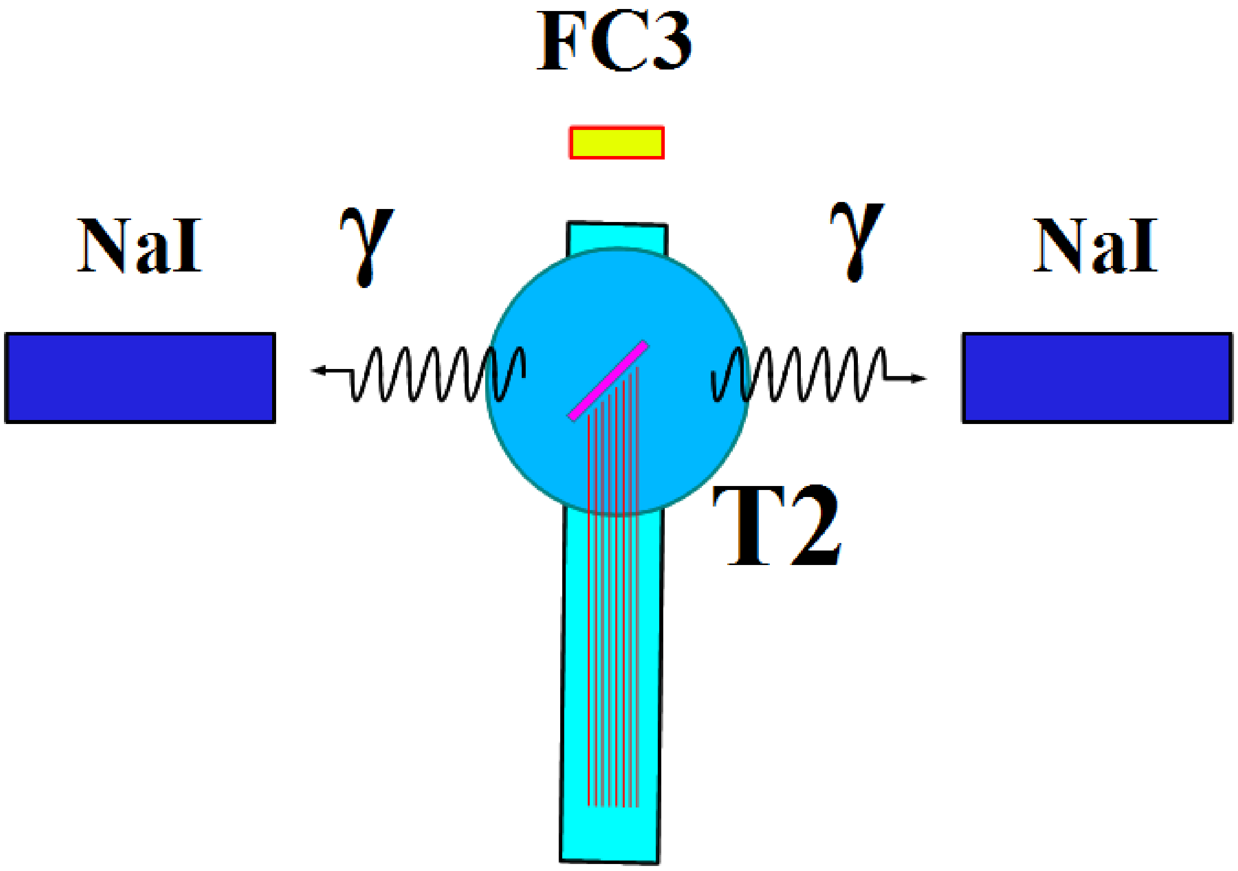




Future Plan and Improvements

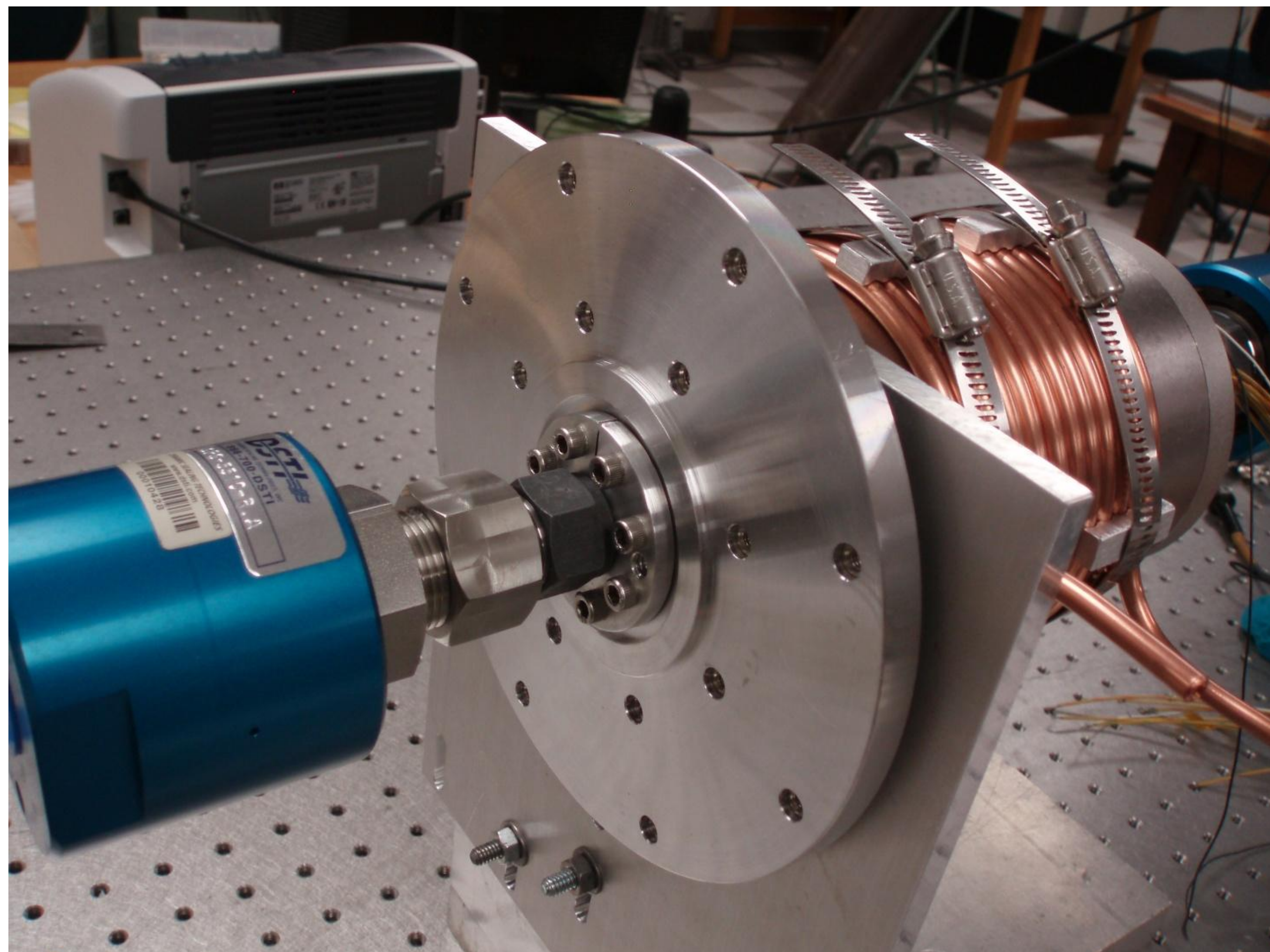
We want to produce positrons using the HRRL beam line. We can improve positron collection efficiency by applying following methods:

3. By doing coincidence between 2 NaI detectors, we can improve NaI spectrums.



Future Runs

4. Rotating motor system to cryogenically cool 8 tungsten targets which can take more beam current without overheating target.

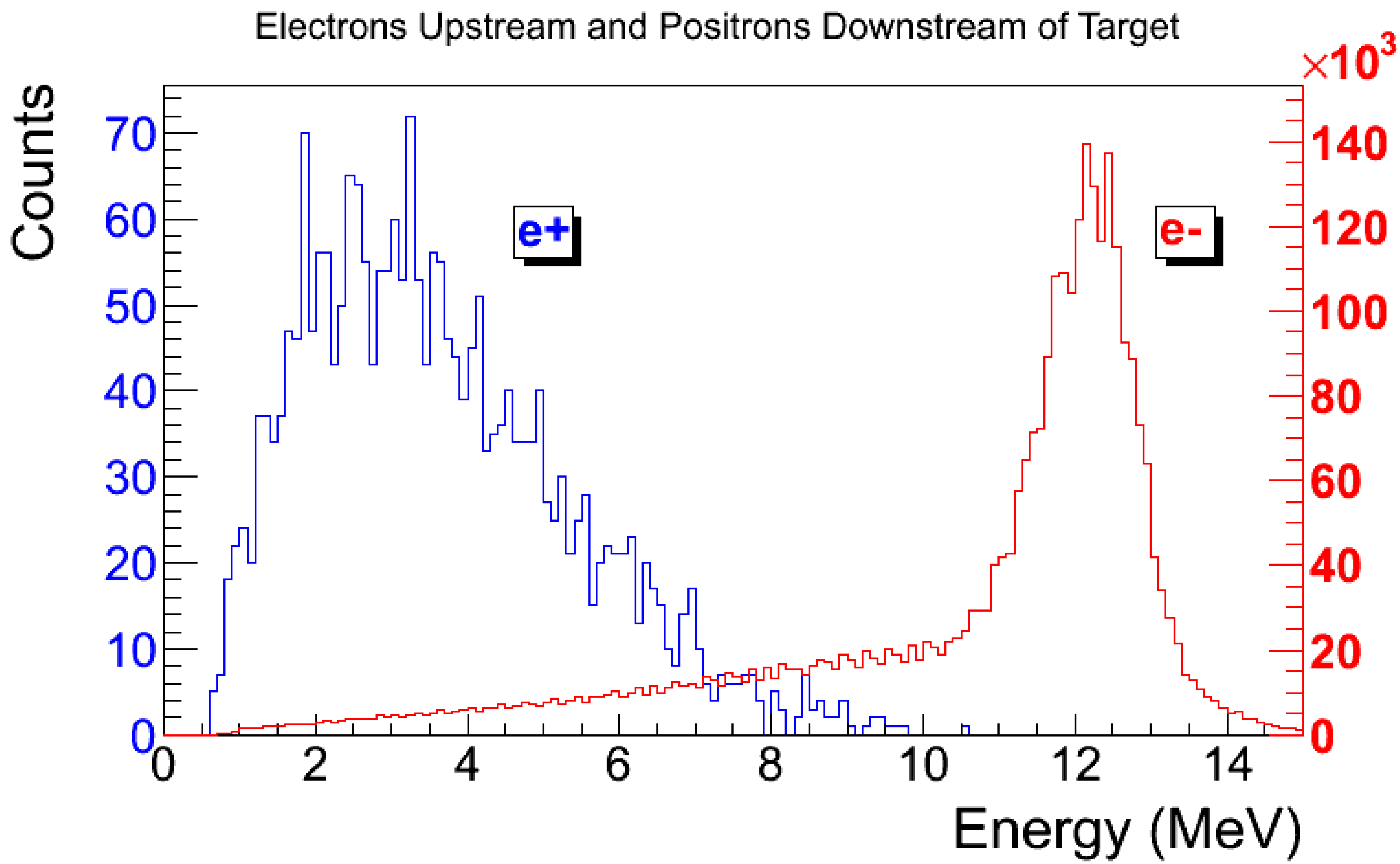


References:

Positron Production: Simulation

G4beamline simulation of the experiment using electron beam:

- Incident electron beam on T1 given in red.
- Positrons escaped downstream of T1 given in blue.



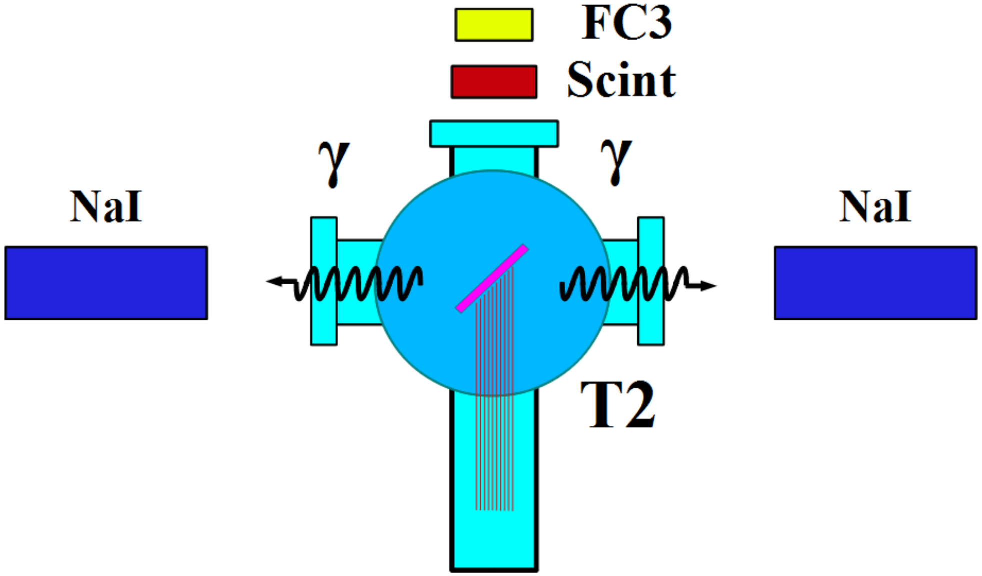
Positron Detection

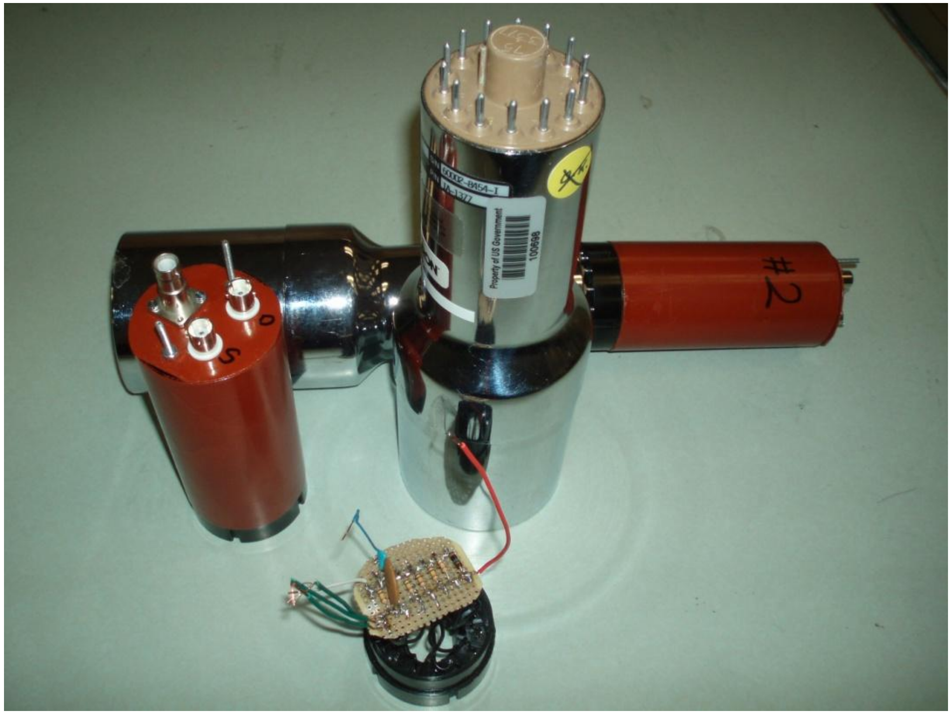
Positrons annihilate, produce two 511 keV photons back to back.

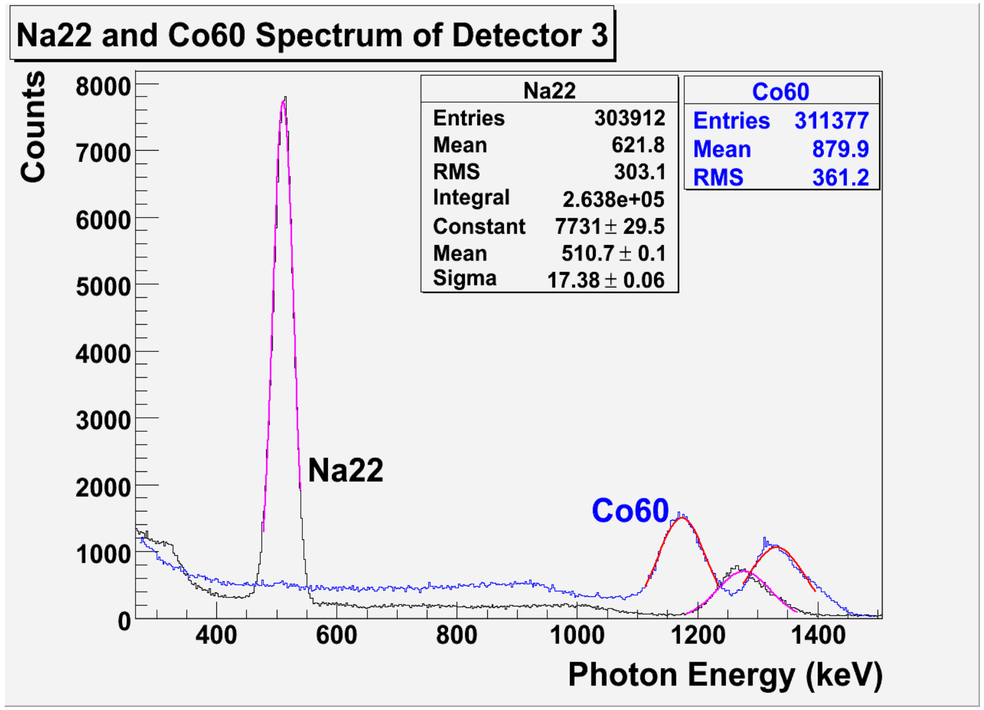
2 NaI detectors measure photons.

Rebuilt PMT bases:

old base rise time = 400 μs.
new base rise time = 1 μ s.







Motivation

Why do we want a polarized Positron source (see Grames talk Sept 2012 in europe)

Two Photon Exchange explain Formfactor disagreement. Blunden, Arrington

Deeply Virtual Compton Scattering (DVCS) , removes BH contributions to measurement

Motivation

Rosenbluth technique:

$G_E(Q^2)$ and $G_M(Q^2)$ determined by the separation of longitudinal and transversal contributions to the electron–proton scattering cross section.

Differential cross section of the elastic scattering in one–photon approximation, assuming P– and T–invariance, can be written [1] as:

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

where σ_{Mott} is the Mott cross section, θ is electron scattering angle, Q is transferred fourmomentum, and $\tau = Q^2/4M_p^2$. Introducing the longitudinal virtual photon polarization, $\varrho = (1 + 2(1 + \tau)\tan^2(\theta/2))^{-1}$, one can re-write the above formula as:

$$\frac{d\sigma}{d\Omega} = \frac{\tau\sigma_{Mott}}{\epsilon(1+\tau)}\left[G_M^2 + \frac{\epsilon}{\tau}G_E^2\right]$$

The two form factors can be disentangled by measuring scattering cross sections at different initial electron energies and scattering angles while keeping momentum transfer (Q) the same.

As is seen from the last formula, the contribution of the electric form factor to the cross section drops down with increasing Q^2. Therefore it becomes difficult to measure G_E using the Rosenbluth method at high Q^2.

[1] M. N. Rosenbluth, Phys. Rev. 79 (1950) 615.

Mott scattering, also referred to as spin-coupling inelastic Coulomb scattering, is the separation of the two spin states of an electron beam by scattering the beam off the Coulomb field of heavy atoms. It is mostly used to measure the spin polarization of an electron beam.

In lay terms, Mott Scattering is similar to Rutherford Scattering but electrons are used instead of Alpha particles as they do not interact via the strong force (only weak and electromagnetic). This enables them to penetrate the atomic nucleus, giving valuable insight into the nuclear structure.

Mott scattering is quantum form off Rutherford scattering

When an electron passes near an atom it is defected by the Coulomb field of the atom. Classically this scattering is usually called Rutherford scattering; the quantum form is Mott scattering. During the encounter the electron is accelerated and so emits radiation. This is called bremsstrahlung, which is literally "braking radiation".

Mott cross section

$$\frac{d\sigma}{d^2\boldsymbol{\Omega}} = \frac{Z^2 e^4 (1 - v^2 \sin^2 \frac{1}{2}\theta)}{4(4\pi\varepsilon_0)^2 |\boldsymbol{p}|^2 v^2 \sin^4 \frac{1}{2}\theta}$$

In the field I am (experimental particle physics) the term **Born cross section** usually refers to the **calculated cross section at lowest order in perturbation theory.** The observed cross section should in principle be compared to the cross section calculated including all order in perturbation theory. In practice, it is compared to the cross section calculated to the highest order which is available.

$$\psi(\mathbf{r}) = \phi(\mathbf{r}) - \frac{m}{2\pi\,\hbar^2}\frac{\exp(\mathrm{i}\,k\,r)}{r}\int \exp(-\mathrm{i}\,\mathbf{k}'\!\cdot\mathbf{r}')\,V(\mathbf{r}')\,\psi(\mathbf{r}')\,d^3\mathbf{r}',$$

The *Born approximation*: $$\psi(\mathbf{r}) \rightarrow \phi(\mathbf{r}) = \frac{\exp(\mathrm{i}\,\mathbf{k}\cdot\mathbf{r})}{(2\pi)^{3/2}}.$$

Thus, in the Born approximation, the differential cross-section for scattering by a Yukawa potential is

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{2\,m\,V_0}{\hbar^2\,\mu}\right)^2 \frac{1}{[2\,k^2\,(1-\cos\theta)+\mu^2]^2},$$

$$q^2 = 4\,k^2\,\sin^2(\theta/2) = 2\,k^2\,(1-\cos\theta).$$

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{2\,m\,Z\,Z'\,e^2}{4\pi\epsilon_0\,\hbar^2}\right)^2 \frac{1}{16\,k^4\,\sin^4(\theta/2)}.$$

$$\frac{d\sigma}{d\Omega} \simeq \left(\frac{Z\,Z'\,e^2}{16\pi\epsilon_0\,E}\right)^2 \frac{1}{\sin^4(\theta/2)},$$

The Born approximation is valid provided that $\psi(\mathbf{r})$ is not too different from $\varphi(\mathbf{r})$ in the scattering region.

[1]:http://farside.ph.utexas.edu/teaching/qm/lectures/node69.html

$$\mathbf{A}^* = (\overline{\mathbf{A}})^{\mathrm{T}} = \overline{\mathbf{A}^{\mathrm{T}}}$$

Other names for the conjugate transpose of a matrix are **Hermitian conjugate**, or **transjugate**. The conjugate transpose of a matrix **A** can be denoted by any of these symbols:

- $\mathbf{A}^*$ or $\mathbf{A}^{\mathbf{H}}$, commonly used in linear algebra

$\mathbf{A}^\dagger$ (sometimes pronounced "**A** dagger"), universally used in quantum mechanics

$\mathbf{A}^+$ although this symbol is more commonly used for the Moore-Penrose pseudoinverse

http://en.wikipedia.org/wiki/Conjugate_transpose