Parameterizing the ellipse equation

We can take the equation found for the ellipse in the frame of the wires

$$\frac{(x' \operatorname{Cos}[6°]+y' \operatorname{Sin}[6°]+\Delta a)^2}{a^2}+\frac{(-x' \operatorname{Sin}[6°]+y' \operatorname{Cos}[6°])^2}{b^2}=1$$

$$\frac{x^2 \operatorname{Cos}^2[6°]+y^2 \operatorname{Sin}^2[6°]+\Delta a^2+2\,x\,y \operatorname{Cos}[6°] \operatorname{Sin}[6°]+2\,x\,\Delta a \operatorname{Cos}[6°]+2\,y\,\Delta a \operatorname{Sin}[6°]}{a^2}+\frac{x^2 \operatorname{Sin}^2[6°]+y^2 \operatorname{Cos}^2[6°]-2\,x\,y \operatorname{Sin}[6°] \operatorname{Cos}[6°]}{b^2}=1$$

We can use the parametrization for an ellipse

x = a cos t

y = b sin t

$$\frac{(x' \operatorname{Cos}[6°]+y' \operatorname{Sin}[6°]+\Delta a)^2}{a^2}+\frac{(-x' \operatorname{Sin}[6°]+y' \operatorname{Cos}[6°])^2}{b^2}=1$$

$$\frac{(a \operatorname{Cos}[t] \operatorname{Cos}[6°]+b \operatorname{Sin}[t] \operatorname{Sin}[6°]+\Delta a)^2}{a^2}+\frac{(-a \operatorname{Cos}[t] \operatorname{Sin}[6°]+b \operatorname{Sin}[t] \operatorname{Cos}[6°])^2}{b^2}=1$$

$$\frac{a^2 \operatorname{Cos}^2[t] \operatorname{Cos}^2[6°]+b^2 \operatorname{Sin}^2[t] \operatorname{Sin}^2[6°]+\Delta a^2+2\,a\,b \operatorname{Cos}[t] \operatorname{Cos}[6°] \operatorname{Sin}[t] \operatorname{Sin}[6°]+2\,a\,\Delta a \operatorname{Cos}[t] \operatorname{Cos}[6°]+2\,b\,\Delta a \operatorname{Sin}[t] \operatorname{Sin}[6°]}{a^2}+\frac{a^2 \operatorname{Cos}^2[t] \operatorname{Sin}^2[6°]+b^2 \operatorname{Sin}^2[t] \operatorname{Cos}^2[6°]-2\,a\,b \operatorname{Cos}[t] \operatorname{Sin}[6°] \operatorname{Cos}[6°] \operatorname{Sin}[t]}{b^2}=1$$

We have already stored the x and y values for the rotated wire frame, paired with the corresponding DC lab frame θ and ϕ values. If we set these two equations equal to each other, and solve for the ellipse parameter t, we can determine how the change θ effects the change in ϕ. The array constantθ stores the angle ϕ and wire mid-point for the intersections of the constant θ ellipse with the DC wires.

```
In[1355]:= constantθ[[2]]
Out[1355]= {{-13.4985, 3.5}, {2.08809, 3.5}, {14.0857, 2.5}, {21.26, 1.5}}
```

The array constantθxyzRotated contains the coordinates (x'', y'', z'') of the intersections in the frame of the wires. Setting

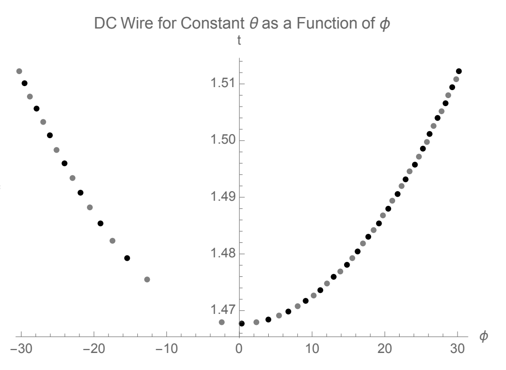
```
In[1356]:= ClearAll[θ, X, Y, t];

In[1479]:= FramePairs = constantθ;
constantθϕParameter = constantθ;
RowLengths = Table[{Nothing}, {i, 1, 36}];
For[rows = 1, rows < 37, rows ++,
  RowLengths[[rows]] = Length[constantθ[[rows]]];
  For[columns = 1, columns < RowLengths[[rows]] + 1, columns ++,
   X = constantθxyzRotated[[rows, columns, 1]];
   Y = constantθxyzRotated[[rows, columns, 2]];
   θ = rows + 4;
   t =
    t /. Solve[X^2 (Cos[6 °])^2 + Y^2 (Sin[6 °])^2 + Δa^2 + 2 X Y Cos[6 °] Sin[6 °] + 2 X Δa Cos[6 °] + 2 Y Δa Sin[6 °] + X^2 (Sin[6 °])^2 + Y^2 (Cos[6 °])^2 -
        2 X Y Sin[6 °] Cos[6 °] == a^2 (Cos[t])^2 (Cos[6 °])^2 + b^2 (Sin[t])^2 (Sin[6 °])^2 + Δa^2 + 2 a b Cos[t] Cos[6 °] Sin[t] Sin[6 °] +
        2 a Δa Cos[t] Cos[6 °] + 2 b Δa Sin[t] Sin[6 °] + a^2 (Cos[t])^2 (Sin[6 °])^2 + b^2 (Sin[t])^2 (Cos[6 °])^2 - 2 a b Cos[t] Sin[6 °] Cos[6 °] Sin[t], t];
   FramePairs[[rows, columns]] = t;
   For[element = 1, element < 5, element ++,
    If[Y < 0 && FramePairs[[rows, columns, element]] > 0,
     constantθϕParameter[[rows, columns]] = {constantθ[[rows, columns, 1]], FramePairs[[rows, columns, element]]};
    ];
    If[Y > 0 && FramePairs[[rows, columns, element]] > 0,
     constantθϕParameter[[rows, columns]] = {constantθ[[rows, columns, 1]], FramePairs[[rows, columns, element]]};
    ];
   ];
   ClearAll[X, Y, t, θ];

  ];
 ];

In[1483]:= DesiredFramePairs = Desiredconstantθ;
DesiredconstantθϕParameter = Desiredconstantθ;
DesiredRowLengths = Table[{Nothing}, {i, 1, 36}];
For[rows = 1, rows < 37, rows ++,
  DesiredRowLengths[[rows]] = Length[Desiredconstantθ[[rows]]];
  For[columns = 1, columns < DesiredRowLengths[[rows]] + 1, columns ++,
   X = DesiredconstantθxyzRotated[[rows, columns, 1]];
   Y = DesiredconstantθxyzRotated[[rows, columns, 2]];
   θ = rows + 4;
   t =
    t /. Solve[X^2 (Cos[6 °])^2 + Y^2 (Sin[6 °])^2 + Δa^2 + 2 X Y Cos[6 °] Sin[6 °] + 2 X Δa Cos[6 °] + 2 Y Δa Sin[6 °] + X^2 (Sin[6 °])^2 + Y^2 (Cos[6 °])^2 -
        2 X Y Sin[6 °] Cos[6 °] == a^2 (Cos[t])^2 (Cos[6 °])^2 + b^2 (Sin[t])^2 (Sin[6 °])^2 + Δa^2 + 2 a b Cos[t] Cos[6 °] Sin[t] Sin[6 °] +
        2 a Δa Cos[t] Cos[6 °] + 2 b Δa Sin[t] Sin[6 °] + a^2 (Cos[t])^2 (Sin[6 °])^2 + b^2 (Sin[t])^2 (Cos[6 °])^2 - 2 a b Cos[t] Sin[6 °] Cos[6 °] Sin[t], t];
   DesiredFramePairs[[rows, columns]] = t;
   For[element = 1, element < 5, element ++,
    If[Y < 0 && DesiredFramePairs[[rows, columns, element]] > 0,
     DesiredconstantθϕParameter[[rows, columns]] = {Desiredconstantθ[[rows, columns, 1]], DesiredFramePairs[[rows, columns, element]]};
    ];
    If[Y > 0 && DesiredFramePairs[[rows, columns, element]] > 0,
     DesiredconstantθϕParameter[[rows, columns]] = {Desiredconstantθ[[rows, columns, 1]], DesiredFramePairs[[rows, columns, element]]};
    ];
   ];
   ClearAll[X, Y, t, θ];

  ];
 ];
```

Cleaning up the left and right hand arrays

Creating plots

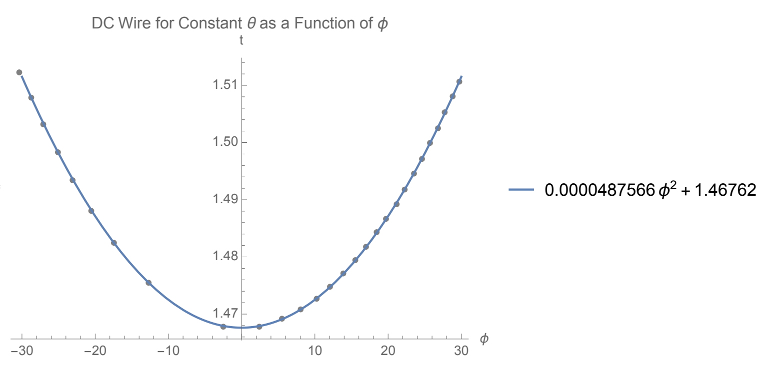
```
In[1487]:= tθ = Table[ListPlot[{constantθϕParameter[[i]]}, PlotStyle → Black, AxesLabel → {"ϕ", "t"}, PlotLabel → "DC Wire for Constant θ as a Function of ϕ"],
   {i, 1, 36}];
Desiredtθ = Table[ListPlot[{DesiredconstantθϕParameter[[i]]}, PlotStyle → Gray, AxesLabel → {"ϕ", "t"},
    PlotLabel → "DC Wire for Constant θ as a Function of ϕ"],
   {i, 1, 36}];

In[1489]:= Show[tθ[[36]], Desiredtθ[[36]]]
```

Out[1489]=

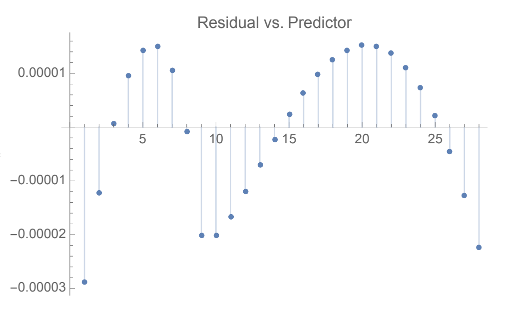



```
In[1490]:= ClearAll[FitForm, FitDegree, Fits];
FitForm = Table[0, {entry, 1, 36}];
For[rows = 1, rows < 37, rows ++,
  FitForm[[rows]] = A1 + A3 ϕ^2 ;
 ];
FitDegree = Table[NonlinearModelFit[DesiredconstantθϕParameter[[rows]], {FitForm[[rows]]}, {A1, A3}, ϕ], {rows, 1, 36}];
Fits = Table[Plot[FitForm[[i]] /. FitDegree[[i]]["BestFitParameters"], {ϕ, -30 , 30 }, PlotRange → Automatic,
    PlotLegends → {FitForm[[i]] /. FitDegree[[i]]["BestFitParameters"]}], {i, 1, 36}];
Show[Desiredtθ[[36]], Fits[[36]]]
ListPlot[FitDegree[[36]]["FitResiduals"], Filling -> Axis, PlotLabel → "Residual vs. Predictor"]
```

Out[1495]=

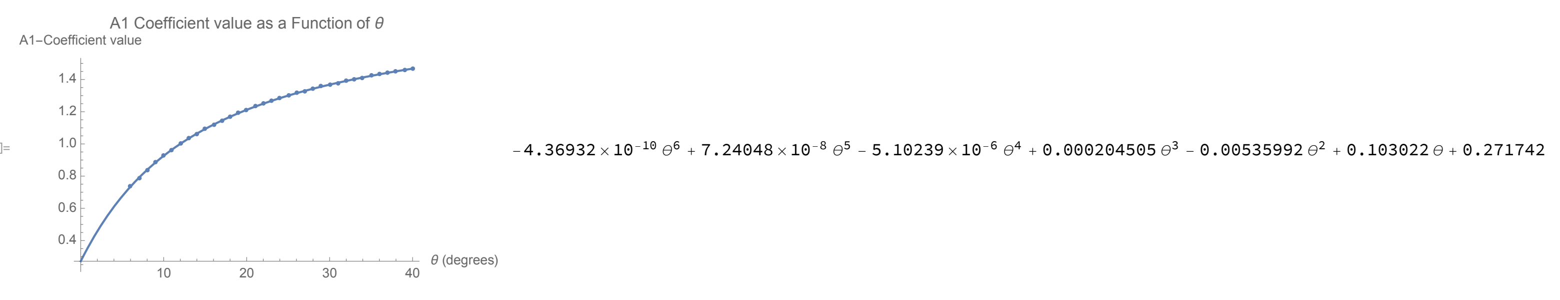


Out[1496]=

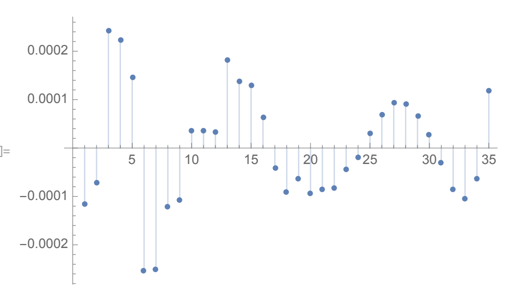



```
In[1497]:= A1coeff = Table[A1 /. FindFit[DesiredconstantθϕParameter[[i]], {FitForm[[i]]}, {A1, A2, A3}, ϕ], {i, 1, 36}];
A3coeff = Table[A3 /. FindFit[DesiredconstantθϕParameter[[i]], {FitForm[[i]]}, {A1, A2, A3}, ϕ], {i, 1, 36}];
For[element = 1, element < 37, element ++,
  A1coeff[[element]] = {element + 4, A1coeff[[element]]}
 ];
For[element = 1, element < 37, element ++,
  A3coeff[[element]] = {element + 4, A3coeff[[element]]}
 ];
A1coeff = Drop[A1coeff, 1];
A3coeff = Drop[A3coeff, 1];

In[1503]:= ClearAll[FitFormA1, FitDegreeA1, θ];
A1Plot = ListPlot[{A1coeff}, AxesLabel → {"θ (degrees)", "A1-Coefficient value"}, PlotLabel → "A1 Coefficient value as a Function of θ"];
FitFormA1 = B1 + B2 θ + B3 θ^2 + B4 θ^3 + B5 θ^4 + B6 θ^5 + B7 θ^6;
FitDegreeA1 = NonlinearModelFit[A1coeff, {FitFormA1}, {B1, B2, B3, B4, B5, B6, B7}, θ];
LegendA1 = FitFormA1 /. FitDegreeA1["BestFitParameters"];
A1fitPlot = Plot[FitFormA1 /. FitDegreeA1["BestFitParameters"], {θ, 0, 40}, AxesLabel → {"θ (degrees)", "A1-Coefficient value"},
   PlotLabel → "A1 Coefficient value as a Function of θ", PlotLegends → LegendA1];
Show[A1fitPlot, A1Plot]
ListPlot[FitDegreeA1["FitResiduals"], Filling -> Axis]
```

Out[1509]=

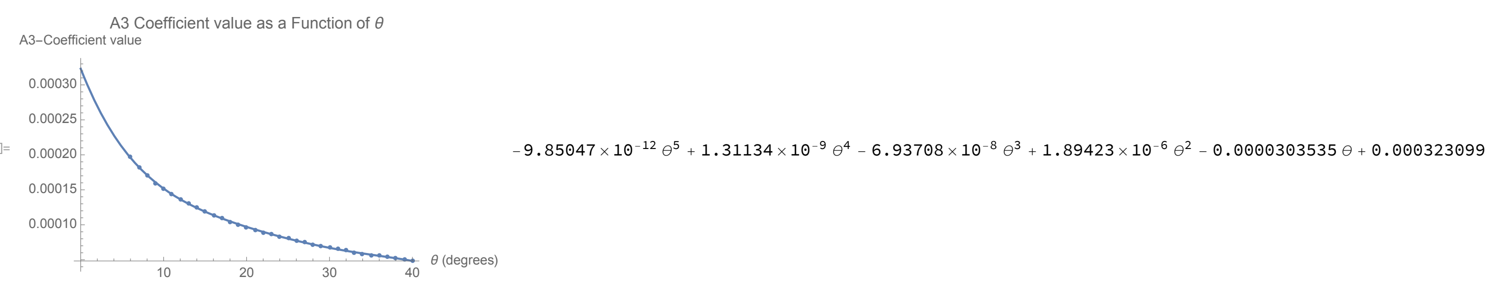


Out[1510]=

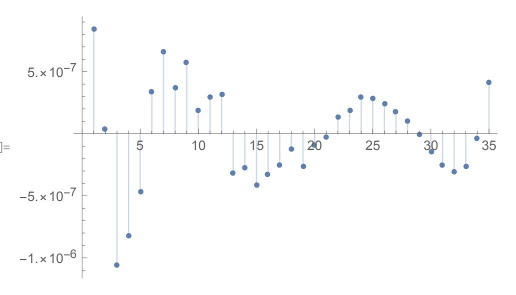


```
In[1344]:= a1[θ_] := -5.110673844469366`*^-10 θ^6 + 8.349113356981219`*^-8 θ^5 - 5.766102896274691`*^-6 θ^4 + 0.00022469104903106998` θ^3 -
   0.005684327766615618` θ^2 + 0.10556564278956385` θ + 0.2646112792693367`;

In[1511]:= A3Plot = ListPlot[{A3coeff}, AxesLabel → {"θ (degrees)", "A3-Coefficient value"}, PlotLabel → "A3 Coefficient value as a Function of θ"];
FitFormA3 = D1 + D2 θ + D3 θ^2 + D4 θ^3 + D5 θ^4 + D6 θ^5;
FitDegreeA3 = NonlinearModelFit[A3coeff, {FitFormA3}, {D1, D2, D3, D4, D5, D6}, θ];
LegendA3 = FitFormA3 /. FitDegreeA3["BestFitParameters"];
A3fitPlot = Plot[FitFormA3 /. FitDegreeA3["BestFitParameters"], {θ, 0, 40}, AxesLabel → {"θ (degrees)", "A3-Coefficient value"},
   PlotLabel → "A3 Coefficient value as a Function of θ", PlotLegends → LegendA3];
Show[A3fitPlot, A3Plot]
ListPlot[FitDegreeA3["FitResiduals"], Filling -> Axis]
```

Out[1516]=

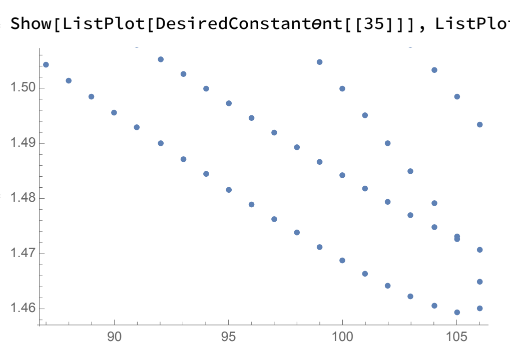


Out[1517]=



```
In[1518]:= a3[θ_] := -1.1732081633034528`*^-11 θ^5 + 1.5434310165672194`*^-9 θ^4 - 8.016600812026178`*^-8 θ^3 + 2.1259521653398487`*^-6 θ^2 -
   0.000032556006178456867` θ + 0.0003297559713932338`;

In[1519]:= a3[40] ϕ^2 + a1[40]
Out[1519]= 1.46748 + 0.0000482329 ϕ^2

In[1520]:= Desiredconstantθ[[36]]
Out[1520]= {{-30.3023, 102}, {-28.7241, 103}, {-27.0113, 104}, {-25.1242, 105}, {-22.9983, 106}, {-20.5159, 107}, {-17.4088, 108}, {-12.628, 109}, {-2.49093, 109},
 {2.33625, 108}, {5.48942, 107}, {8.01767, 106}, {10.189, 105}, {12.1214, 104}, {13.8792, 103}, {15.5021, 102}, {17.0167, 101}, {18.4418, 100},
 {19.7913, 99}, {21.0758, 98}, {22.3036, 97}, {23.4814, 96}, {24.6147, 95}, {25.708, 94}, {26.7651, 93}, {27.7892, 92}, {28.7831, 91}, {29.7491, 90}}

In[1521]:= Desiredconstantθϕ Parameter[[36]]
Out[1521]= {{-30.3023, 1.51236}, {-28.7241, 1.50784}, {-27.0113, 1.5032}, {-25.1242, 1.49841}, {-22.9983, 1.49343}, {-20.5159, 1.48816}, {-17.4088, 1.48241},
 {-12.628, 1.4754}, {-2.49093, 1.46791}, {2.33625, 1.46787}, {5.48942, 1.46908}, {8.01767, 1.47075}, {10.189, 1.47268}, {12.1214, 1.47478},
 {13.8792, 1.47702}, {15.5021, 1.47935}, {17.0167, 1.48175}, {18.4418, 1.48422}, {19.7913, 1.48674}, {21.0758, 1.4893}, {22.3036, 1.49189},
 {23.4814, 1.49452}, {24.6147, 1.49718}, {25.708, 1.49985}, {26.7651, 1.50255}, {27.7892, 1.50527}, {28.7831, 1.508}, {29.7491, 1.51075}}

In[1627]:= DesiredConstantθnt = Desiredconstantθ;
For[rows = 1, rows < 37, rows ++,
  DesiredRowLengths[[rows]] = Length[DesiredConstantθnt[[rows]]];
  For[columns = 1, columns < DesiredRowLengths[[rows]] + 1, columns ++,
   DesiredConstantθnt[[rows, columns]] = {DesiredConstantθnt[[rows, columns, 2]], Desiredconstantθϕ Parameter[[rows, columns, 2]]};
  ];
 ];

In[1629]:= DesiredConstantθnt[[36]]
Out[1629]= {{102, 1.51236}, {103, 1.50784}, {104, 1.5032}, {105, 1.49841}, {106, 1.49343}, {107, 1.48816}, {108, 1.48241}, {109, 1.4754}, {109, 1.46791},
 {108, 1.46787}, {107, 1.46908}, {106, 1.47075}, {105, 1.47268}, {104, 1.47478}, {103, 1.47702}, {102, 1.47935}, {101, 1.48175}, {100, 1.48422},
 {99, 1.48674}, {98, 1.4893}, {97, 1.49189}, {96, 1.49452}, {95, 1.49718}, {94, 1.49985}, {93, 1.50255}, {92, 1.50527}, {91, 1.508}, {90, 1.51075}}

In[1632]:= Show[ListPlot[DesiredConstantθnt[[35]]], ListPlot[DesiredConstantθnt[[36]]]]
```

Out[1632]=



The point that was the semi-major vertex, when rotated 6° to the right becomes

```
In[1603]:= rFromYtoX.{1.6831832367824053`, 0, 0} // MatrixForm
Out[1603]//MatrixForm=
 (1.67396
  0.175941
  0.)
```

Solving for the ellipse parameter given the angle and the corresponding X'' and Y'' components

```
In[1619]:= X = 1.6739625828969429`;
Y = 0.17594055713873974`;
θ = 40;
t =
 t /. Solve[X^2 (Cos[6 °])^2 + Y^2 (Sin[6 °])^2 + Δa^2 + 2 X Y Cos[6 °] Sin[6 °] + 2 X Δa Cos[6 °] + 2 Y Δa Sin[6 °] + X^2 (Sin[6 °])^2 + Y^2 (Cos[6 °])^2 - 2 X Y Sin[6 °] Cos[6 °] ==
    a^2 (Cos[t])^2 (Cos[6 °])^2 + b^2 (Sin[t])^2 (Sin[6 °])^2 + Δa^2 + 2 a b Cos[t] Cos[6 °] Sin[t] Sin[6 °] + 2 a Δa Cos[t] Cos[6 °] + 2 b Δa Sin[t] Sin[6 °] +
     a^2 (Cos[t])^2 (Sin[6 °])^2 + b^2 (Sin[t])^2 (Cos[6 °])^2 - 2 a b Cos[t] Sin[6 °] Cos[6 °] Sin[t], t]
ClearAll[X, Y, θ];
Out[1622]= {-1.31406, 1.4676, 3.06482 - 1.66742 i, 3.06482 + 1.66742 i}
```

We can see that at the vertex position, the parameter reaches it's minimum at 1.4676. This point reflects the right and left sides of the ellipse and the corresponding decreases in wire number as the parameter is increased.

```
In[1624]:= t
Out[1624]= {-1.31406, 1.4676, 3.06482 - 1.66742 i, 3.06482 + 1.66742 i}

In[87]:= Names["Global`*"]
Out[87]= {a, b, bottom, columns, constantθ, constantθl, constantθleft, constantθr, constantθright, constantθxyz, constantθxyzRotated, constantθyx,
 constantθyxRotated, D1P, D2P, DataXY, DataXYRotated, e, ellipse40, ellipse40Rotated, f, i, left, leftRotated, LeftSolutions, Lim,
 Limits, LineLeft, LineRight, n, number, number2, r, R, rD1, rD2, right, rightRotated, RightSolutions, row, RowLengths, rows, x, X, x0,
 x0forWireMiddles, x0forWires, x1, x2, xCenter, xD1, xD2, xP, xWire, y, Y, yD1, yD2, yP, yWire, yxPoints, zD1, zD2, zP, Δa, θ, θforϕat0, ϕ}

In[88]:= DumpSave["Part1.mx", "Global`"]
Out[88]= {Global`}
```