In[1]:=
```
ClearAll["Global`*"]
```

Defining the conditions for transfering from the cone cooridinates of θ and ϕ to the plane, we start at θ=20°

In[2]:=
```
θ = 20;
```

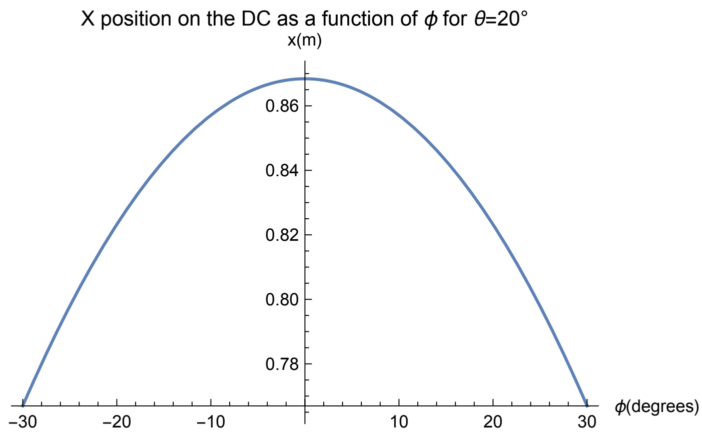
All of the conditions dependent on θ and ϕ, where ϕ is left as a variable

In[3]:=
```
Δa_1 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] - Csc[115 ° - θ °]);
e_1 = Sin[25 °] / Cos[θ °];
a_1 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] + Csc[115 ° - θ °]);
rD1_1 = (a_1 e_1 - Δa_1) Tan[65 °] Cos[θ °];
rD2_1 = (a_1 e_1 + Δa_1) Tan[65 °] Cos[θ °];
xD1_1 = rD1_1 Cos[ϕ °];
yD1_1 = rD1_1 Sin[ϕ °];
zD1_1 = rD1_1 Cot[θ °];
xD2_1 = rD2_1 Cos[ϕ °];
yD2_1 = rD2_1 Sin[ϕ °];
zD2_1 = rD2_1 Cot[θ °];
xP_1 = (2.52934271645 Cos[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
yP_1 = (2.52934271645 Sin[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
zP_1 = (2.52934271645 Cot[θ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
x1_1 = (rD2_1^2 - rD1_1^2 + Cot[θ °]^2 (rD2_1^2 - rD1_1^2) - 2 xP_1 (xD2_1 - xD1_1) -
        2 yP_1 (yD2_1 - yD1_1) - 2 zP_1 (zD2_1 - zD1_1)) / (4 a_1 e_1) - a_1 e_1;
x_1 = x1_1 - Δa_1 + a_1 e_1 // N;
n_1 = -957.412 / (Tan[θ °] + 2.14437) + 430.626;
D2P_1 = Sqrt[(xD2_1 - xP_1)^2 + (yD2_1 - yP_1)^2 + (zD2_1 - zP_1)^2] // N;
D1P_1 = Sqrt[(xP_1 - xD1_1)^2 + (yP_1 - yD1_1)^2 + (zP_1 - zD1_1)^2] // N;
y_1 = Sqrt[D1P_1^2 - x1_1^2] // N;
b_1 = a_1 (1 - e_1);
```

We can examine the x and y position as a function of ϕ

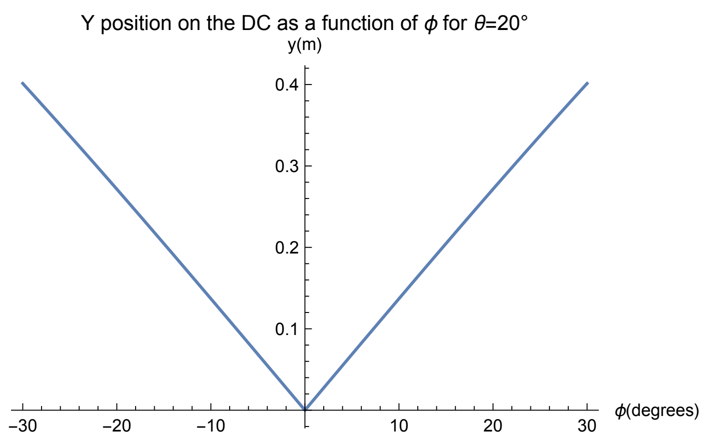
In[24]:=
```
Plot[x_1, {ϕ, -30, 30}, AxesLabel → {"ϕ(degrees)", "x(m)"},
 PlotLabel → "X position on the DC as a function of ϕ for θ=20°"]
```

Out[24]=



In[25]:=
```
Plot[y_1, {ϕ, -30, 30}, AxesLabel → {"ϕ(degrees)", "y(m)"},
 PlotLabel → "Y position on the DC as a function of ϕ for θ=20°"]
```

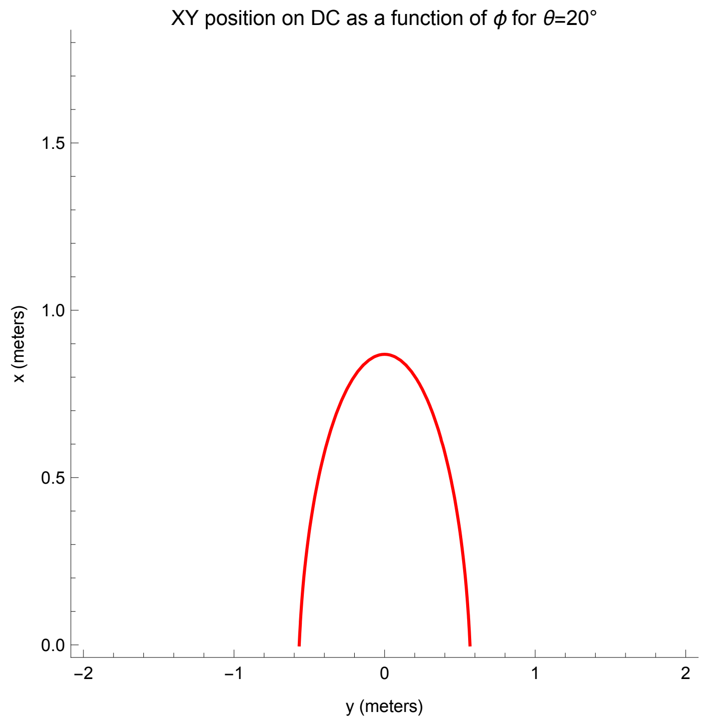
Out[25]=



We can use the x and y coordinates to plot the ellipse they mark out on the conic section

In[26]:=
```
ellipse20 = ContourPlot[(x_1 + Δa_1)^2/a_1^2 + y_1^2/b_1^2 == 1,
  {y_1, -2, 2}, {x_1, 0, 1.8}, Frame → {True, True, False, False},
  PlotLabel → "XY position on DC as a function of ϕ for θ=20°",
  FrameLabel → {"y (meters)", "x (meters)"},
  ContourStyle → Red, PlotLegends → Automatic]
```

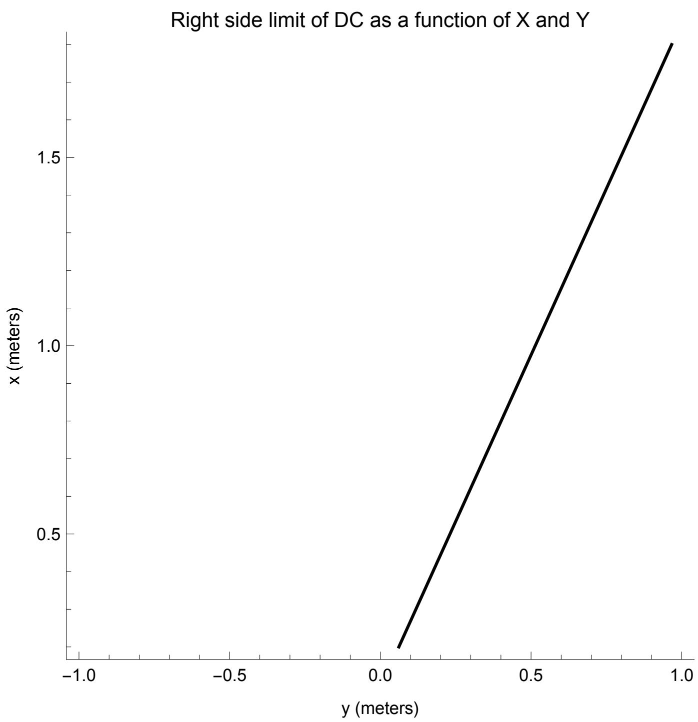
Out[26]=



We can define the constraints of the plane the DC is in

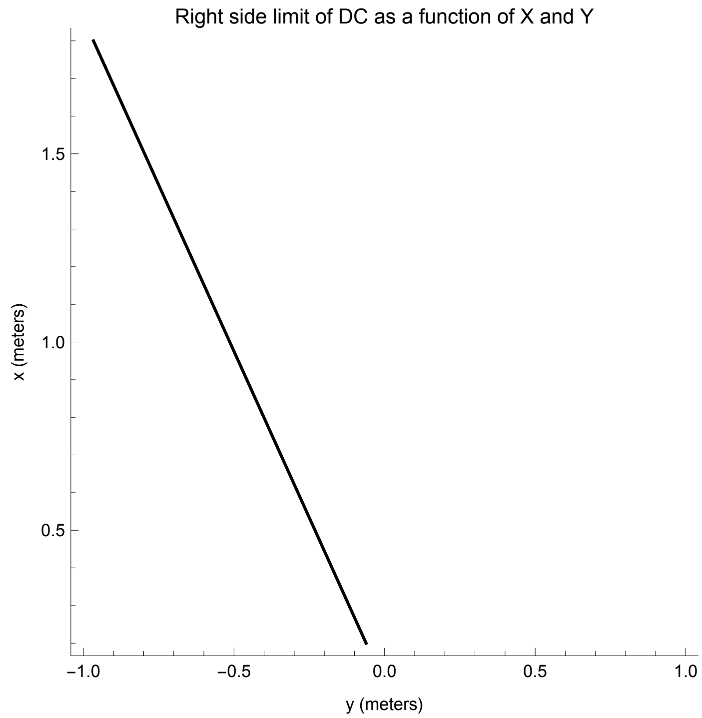
In[27]:=
```
right = ContourPlot[x2 == Cot[29.5 °] y + .09156,
  {y, -1, 1}, {x2, .2, 1.8}, Frame → {True, True, False, False},
  PlotLabel → "Right side limit of DC as a function of X and Y",
  FrameLabel → {"y (meters)", "x (meters)"},
  ContourStyle → Black, PlotLegends → Automatic]
```

Out[27]=



In[28]:=
```
left = ContourPlot[x2 == -Cot[29.5 °] y + .09156,
  {y, -1, 1}, {x2, .2, 1.8}, Frame → {True, True, False, False},
  PlotLabel → "Right side limit of DC as a function of X and Y",
  FrameLabel → {"y (meters)", "x (meters)"},
  ContourStyle → Black, PlotLegends → Automatic]
```

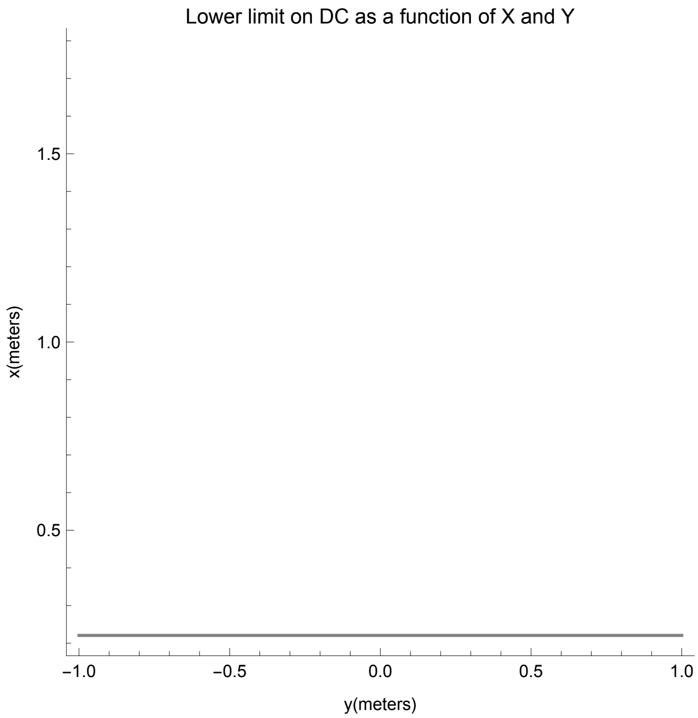
Out[28]=



Similarly, the bottom limit of the DC can be modeled

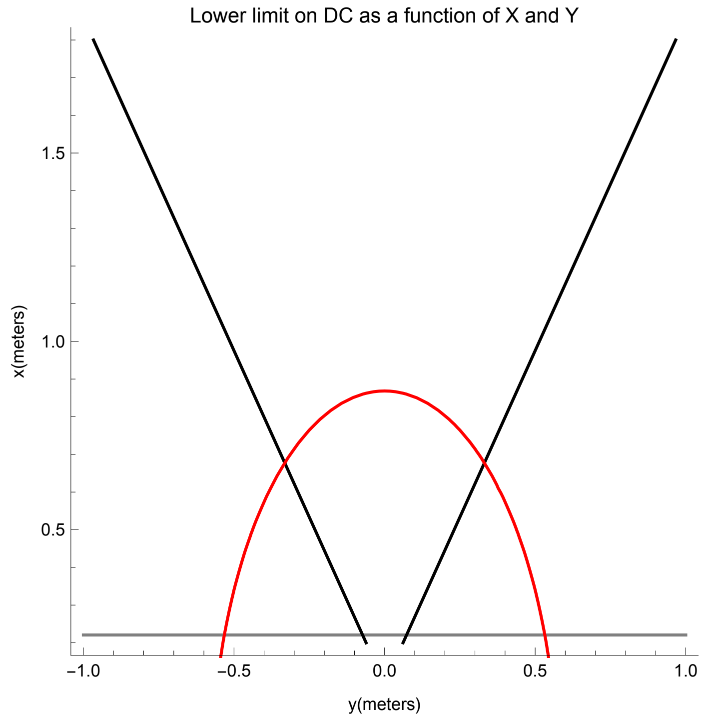
In[29]:=
```
bottom = ContourPlot[x0 == .22070, {y, -1, 1},
  {x0, .2, 1.8}, Frame → {True, True, False, False},
  PlotLabel → "Lower limit on DC as a function of X and Y",
  FrameLabel → {"y(meters)", "x(meters)"},
  ContourStyle → Gray, PlotLegends → Automatic]
```

Out[29]=



In[30]:=
```
Show[bottom, right, left, ellipse20]
```

Out[30]=



The cone of constant θ=20° cross the midpoint plane of the DC at ϕ=0. At this position, we know that the wire number for this area is

In[31]:= $n = \frac{-957.412}{\text{Tan}[\theta\,°] + 2.14437} + 430.626$

Out[31]= 48.9346

This corresponds to lower and upper limits on θ which agree with θ=20°

In[32]:=
```
θlower = 4.49876 + 0.293001 ( n1) + 0.000679074 (n1)^2 - 3.57132*^-6 (n1)^3 /. n1 → 48
```

Out[32]= 19.7324

In[33]:=
```
θupper = 4.49876 + 0.293001 (n1) + 0.000679074 (n1)^2 - 3.57132*^-6 (n1)^3 /. n1 → 49
```

Out[33]= 20.0661

We can define the x coordinate of the wires as they cross the midpoint plane as shown earlier

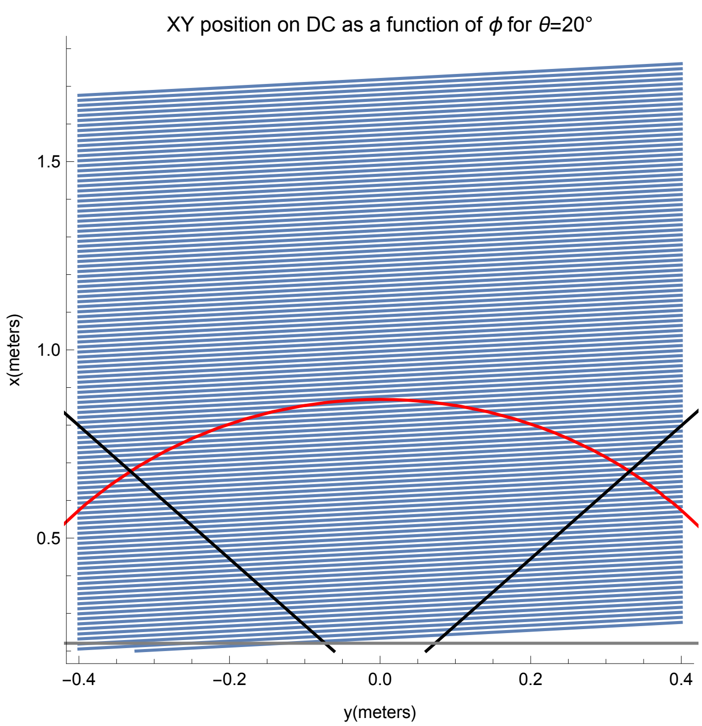
In[34]:=
```
x0forWires[number_] := .2207 + .01337 * (number);
```

Combining the graphics, we can see how the DC constraints assemble

In[35]:=

```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.4, 0.4}, {xWire[number], .2, 1.8},
    Frame → {True, True, False, False},
    PlotLabel → "XY position on DC as a function of ϕ for θ=20°",
    FrameLabel → {"y(meters)", "x(meters)"}],
  {number, 1, 112}], ellipse20, right, left, bottom]
```

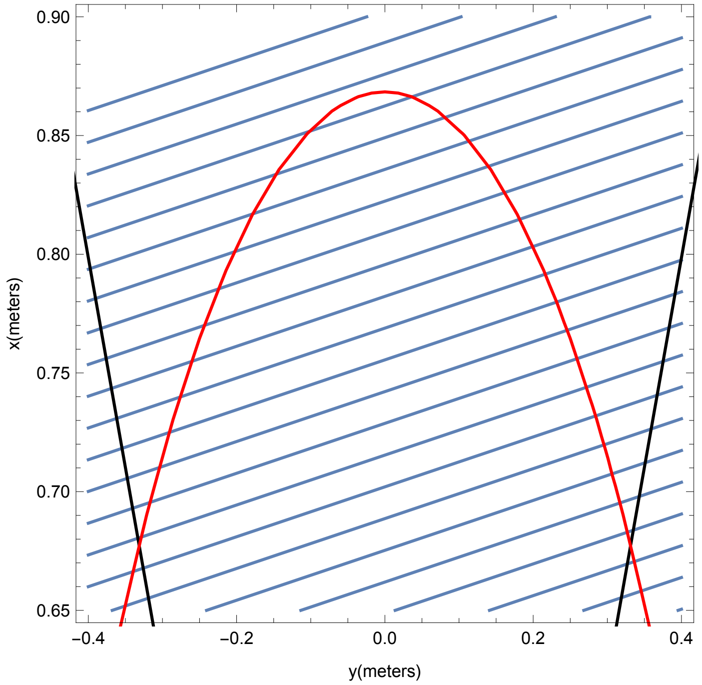
Out[35]=



Zooming in

In[36]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.4, 0.4}, {xWire[number], .65, .9},
    FrameLabel → {"y(meters)", "x(meters)"}],
  {number, 22, 51}], bottom, right, left, ellipse20]
```

Out[36]=



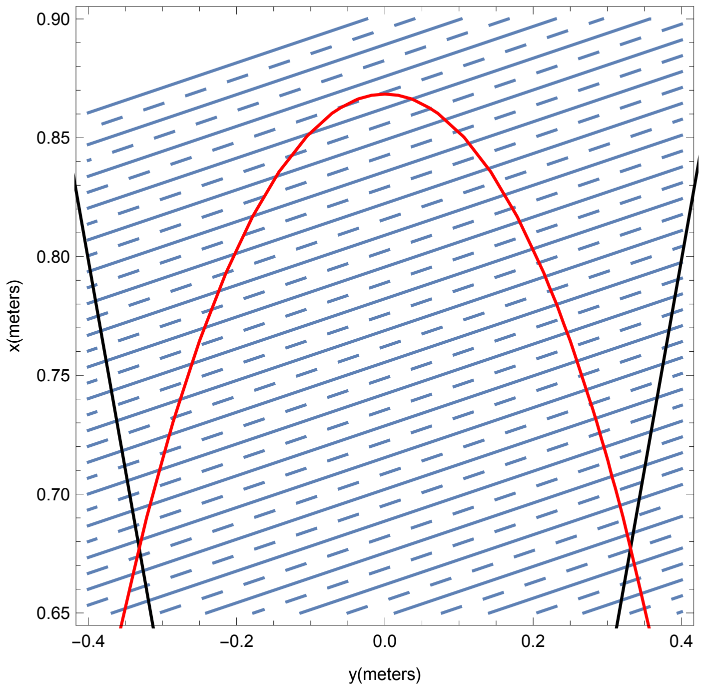
We can define the point midway between two parallel lines as the point where one wire is recorded versus its neighbor

In[37]:=
```
x0forWireMiddles[number_] :=
  ((.2207 + .01337 * (number)) + (.2207 + .01337 * (number - 1))) / 2;
```

Viewing this

In[38]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.4, 0.4}, {xWire[number], .65, .9},
    FrameLabel → {"y(meters)", "x(meters)"}], {number, 22, 51}], Table[
   ContourPlot[xWire[number2] == Tan[6 °] yWire[number2] + x0forWireMiddles[number2],
    {yWire[number2], -0.4, 0.4}, {xWire[number2], .65, .9},
    ContourStyle → {Dashing[Large]}], {number2, 22, 51}],
  bottom, right, left, ellipse20]
```

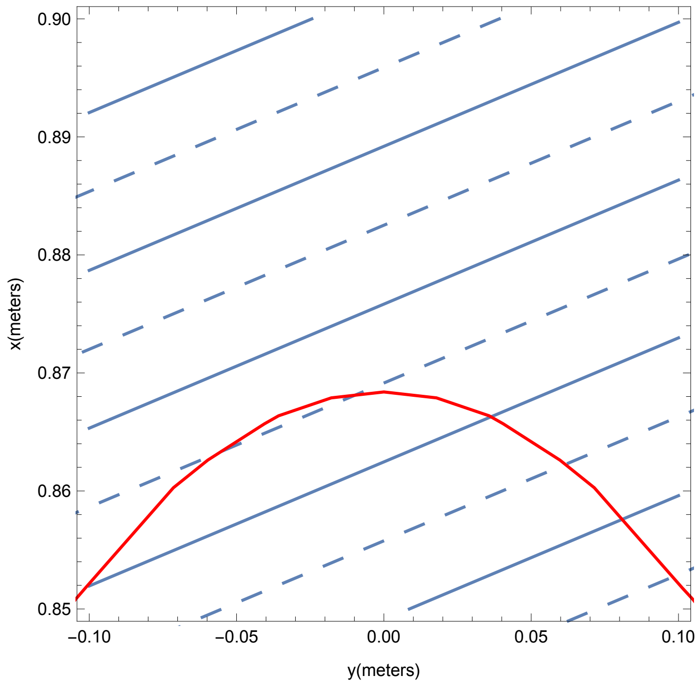
Out[38]=



The ellipse for θ=20° does cross over the wire midpoint twice

In[636]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.1, 0.1}, {xWire[number], .85, .9},
    FrameLabel → {"y(meters)", "x(meters)"}], {number, 22, 51}], Table[
  ContourPlot[xWire[number2] == Tan[6 °] yWire[number2] + x0forWireMiddles[number2],
    {yWire[number2], -0.4, 0.4}, {xWire[number2], .65, .9},
    ContourStyle → {Dashing[Large]}], {number2, 22, 51}],
 bottom, right, left, ellipse20]
```

Out[636]=



Doing the same as above, but now for θ=40°

In[39]:=
```
ClearAll[θ]
```

In[40]:=
```
θ = 40;
```

In[41]:=
```
Δa_2 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] - Csc[115 ° - θ °]);
e_2 = Sin[25 °] / Cos[θ °];
a_2 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] + Csc[115 ° - θ °]);
rD1_2 = (a_2 e_2 - Δa_2) Tan[65 °] Cos[θ °];
rD2_2 = (a_2 e_2 + Δa_2) Tan[65 °] Cos[θ °];
xD1_2 = rD1_2 Cos[ϕ °];
yD1_2 = rD1_2 Sin[ϕ °];
zD1_2 = rD1_2 Cot[θ °];
xD2_2 = rD2_2 Cos[ϕ °];
yD2_2 = rD2_2 Sin[ϕ °];
zD2_2 = rD2_2 Cot[θ °];
xP_2 = (2.52934271645 Cos[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
yP_2 = (2.52934271645 Sin[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
zP_2 = (2.52934271645 Cot[θ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
x1_2 = (rD2_2^2 - rD1_2^2 + Cot[θ °]^2 (rD2_2^2 - rD1_2^2) - 2 xP_2 (xD2_2 - xD1_2) -
      2 yP_2 (yD2_2 - yD1_2) - 2 zP_2 (zD2_2 - zD1_2)) / (4 a_2 e_2) - a_2 e_2;
x_2 = x1_2 - Δa_2 + a_2 e_2 // N;
n_2 = -957.412 / (Tan[θ °] + 2.14437) + 430.626;
```

$$D2P\_2 = \sqrt{(xD2\_2 - xP\_2)^2 + (yD2\_2 - yP\_2)^2 + (zD2\_2 - zP\_2)^2} \;\; // \; N;$$

$$D1P\_2 = \sqrt{(xP\_2 - xD1\_2)^2 + (yP\_2 - yD1\_2)^2 + (zP\_2 - zD1\_2)^2} \;\; // \; N;$$

```
y_2 = Sqrt[D1P_2^2 - x1_2^2] // N;
b_2 = a_2 (1 - e_2);
```

In[57]:= $n = \frac{-957.412}{Tan[\theta °] + 2.14437} + 430.626$

Out[57]= 109.72

In[58]:= $\theta lower = 4.49876 + 0.293001\ (n1) + 0.000679074\ (n1)^2 - 3.57132*\hat{}-6\ (n1)^3$ /. n1 → 109

Out[58]= 39.879

In[59]:= $\theta upper = 4.49876 + 0.293001\ (n1) + 0.000679074\ (n1)^2 - 3.57132*\hat{}-6\ (n1)^3$ /. n1 → 110

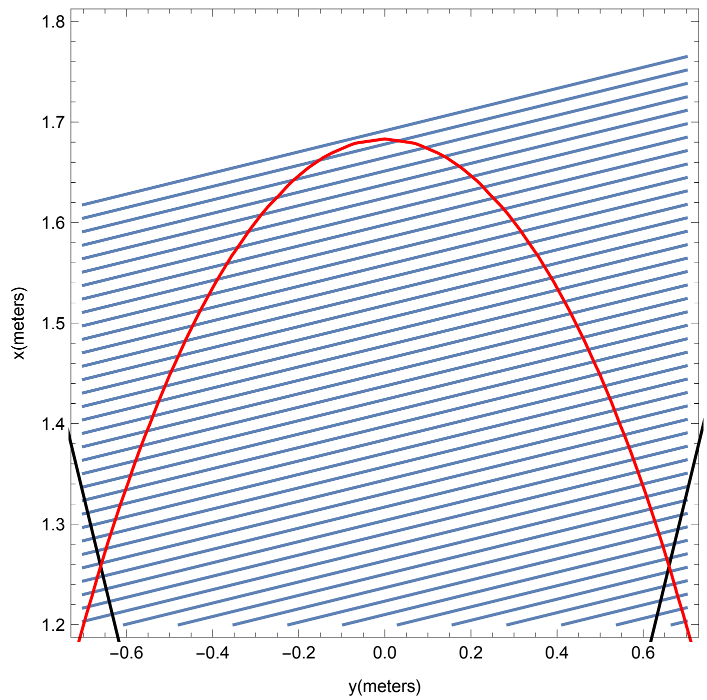
Out[59]= 40.1922

We can define the xy position on the DC plane as a function of ϕ for θ=40°

In[60]:= ellipse40 = ContourPlot[$\frac{(x\_2 + \Delta a\_2)^2}{a\_2^2} + \frac{y\_2^2}{b\_2^2}$ == 1, {y_2, -1, 1}, {x_2, .2, 1.8},
FrameLabel → {"y(meters)", "x(meters)"}, ContourStyle → Red];

In[61]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.7, 0.7}, {xWire[number], 1.2, 1.8},
    FrameLabel → {"y(meters)", "x(meters)"}],
  {number, 55, 110}], bottom, right, left, ellipse40]
```

Out[61]=



Doing the same as above, but now for θ=10°

In[330]:=
```
ClearAll[θ]
```

In[331]:=
```
θ = 10;
```

In[470]:=
```
Δa_3 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] - Csc[115 ° - θ °]);
e_3 = Sin[25 °] / Cos[θ °];
a_3 = (2.52934271645 Sin[θ °]) / 2 (Csc[65 ° - θ °] + Csc[115 ° - θ °]);
rD1_3 = (a_3 e_3 - Δa_3) Tan[65 °] Cos[θ °];
rD2_3 = (a_3 e_3 + Δa_3) Tan[65 °] Cos[θ °];
xD1_3 = rD1_3 Cos[ϕ °];
yD1_3 = rD1_3 Sin[ϕ °];
zD1_3 = rD1_3 Cot[θ °];
xD2_3 = rD2_3 Cos[ϕ °];
yD2_3 = rD2_3 Sin[ϕ °];
zD2_3 = rD2_3 Cot[θ °];
xP_3 = (2.52934271645 Cos[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
yP_3 = (2.52934271645 Sin[ϕ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
zP_3 = (2.52934271645 Cot[θ °]) / (Cot[θ °] + Cos[ϕ °] Cot[65 °]);
x1_3 = (rD2_3^2 - rD1_3^2 + Cot[θ °]^2 (rD2_3^2 - rD1_3^2) - 2 xP_3 (xD2_3 - xD1_3) -
      2 yP_3 (yD2_3 - yD1_3) - 2 zP_3 (zD2_3 - zD1_3)) / (4 a_3 e_3) - a_3 e_3 ;
x_3 = x1_3 - Δa_3 + a_3 e_3 //
   N;
n_3 = -957.412 / (Tan[θ °] + 2.14437) + 430.626;
D2P_3 = Sqrt[(xD2_2 - xP_3)^2 + (yD2_3 - yP_3)^2 + (zD2_3 - zP_3)^2] // N;
D1P_3 = Sqrt[(xP_3 - xD1_3)^2 + (yP_3 - yD1_3)^2 + (zP_3 - zD1_3)^2] // N;
y_3 = Sqrt[D1P_3^2 - x1_3^2] // N;
b_3 = a_3 (1 - e_3);
```

In[348]:= $n = \frac{-957.412}{\mathrm{Tan}[\theta °] + 2.14437} + 430.626$

Out[348]= 18.0724

In[349]:=
```
θlower = 4.49876 + 0.293001 ( n1) + 0.000679074 (n1)^2 - 3.57132*^-6 (n1)^3 /. n1 → 18
```

Out[349]= 9.97197

In[350]:=
```
θupper = 4.49876 + 0.293001 (n1) + 0.000679074 (n1)^2 - 3.57132*^-6 (n1)^3 /. n1 → 19
```

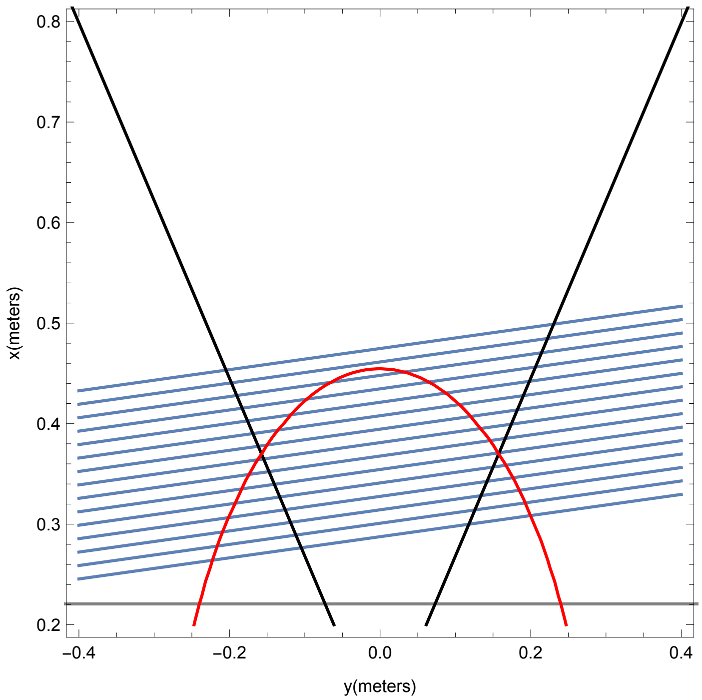
Out[350]= 10.2864

We can define the xy position on the DC plane as a function of ϕ for θ=10°

In[351]:=
```
ellipse10 = ContourPlot[(x_3 + Δa_3)^2 / a_3^2 + y_3^2 / b_3^2 == 1, {y_3, -1, 1}, {x_3, .2, 1.8},
   FrameLabel → {"y(meters)", "x(meters)"}, ContourStyle → Red];
```

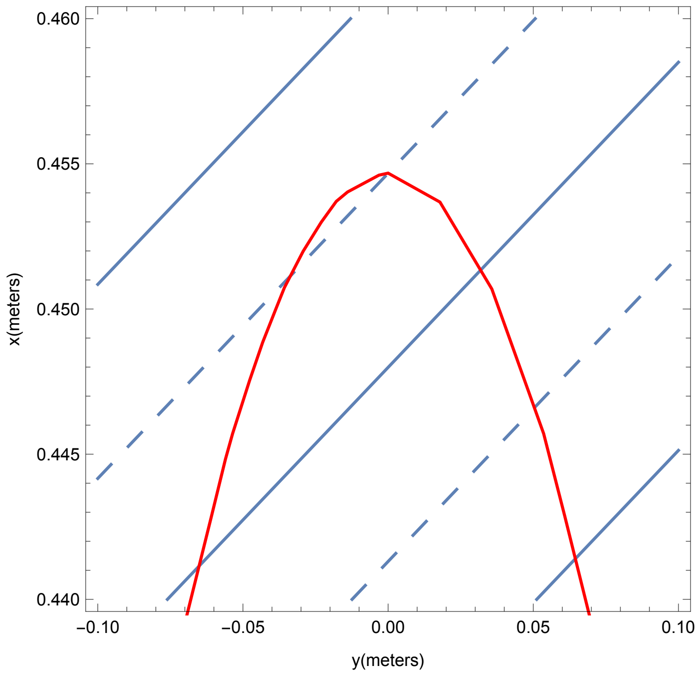
In[352]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.4, 0.4}, {xWire[number], .2, .8},
    FrameLabel → {"y(meters)", "x(meters)"}],
  {number, 5, 19}], bottom, right, left, ellipse10]
```

Out[352]=



In[642]:=
```
Show[Table[ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWires[number],
    {yWire[number], -0.1, 0.1}, {xWire[number], .44, .46},
    FrameLabel → {"y(meters)", "x(meters)"}], {number, 5, 19}], Table[
  ContourPlot[xWire[number2] == Tan[6 °] yWire[number2] + x0forWireMiddles[number2],
    {yWire[number2], -0.1, 0.1}, {xWire[number2], .44, .46},
    ContourStyle → {Dashing[Large]}], {number2, 5, 18}],
 bottom, right, left, ellipse10]
```

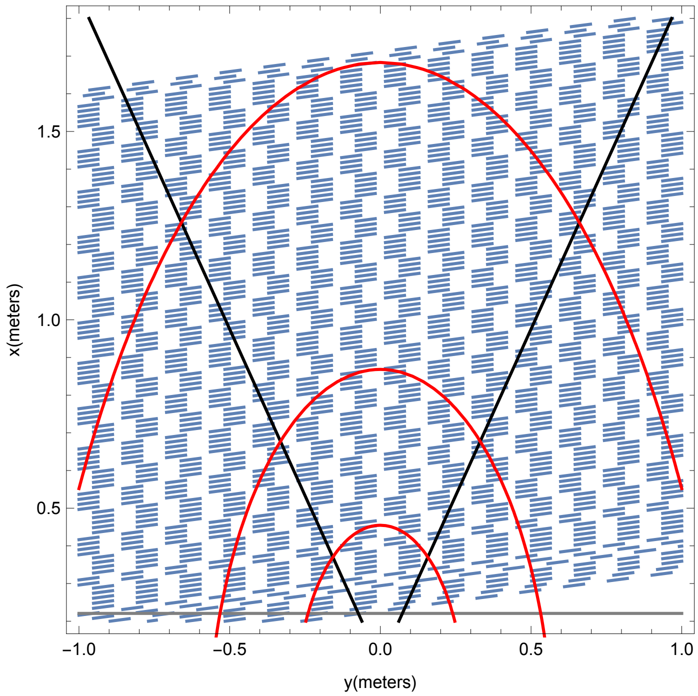
Out[642]=



We can examine the ellipses for θ=20°, 30°, and 40° in the DC plane as they cross the wires

In[353]:=
```
Show[Table[
  ContourPlot[xWire[number2] == Tan[6 °] yWire[number2] + x0forWireMiddles[number2],
   {yWire[number2], -1, 1}, {xWire[number2], .2, 1.8},
   FrameLabel → {"y(meters)", "x(meters)"},
   ContourStyle → {Dashing[Large]}], {number2, 1, 112}]
 , bottom, right, left, ellipse10, ellipse20, ellipse40]
```

Out[353]=



We can find the angle ϕ where the elliptical xy path for θ=40° crosses the midway point in between wires. This corresponds to hits at a constant θ occurring on different wires. The elliptical path hits the right and left wall at ϕ = ±24.9252 and will not cross additional wires lower than 78 in the 1st quadrant. On the left hand side, the limit is lower since the slope of this plane is positive.

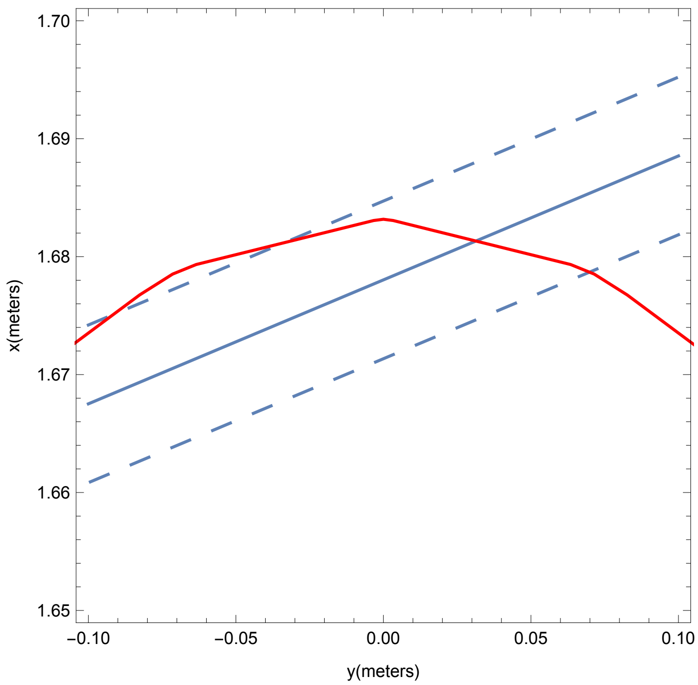
In[354]:=
$$\text{Solve}\left[\sqrt{a\_2^2\left(1-\frac{y\_2^2}{b\_2^2}\right)} - \Delta a\_2 == \text{Cot}[29.5\,°]\; y\_2 + .09156 \;\&\&\; \phi > .1 \;\&\&\; \phi < 30,\ \phi\right]$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[354]= {{ϕ → 24.9252}}

In[355]:=
```
Show[Table[
   ContourPlot[xWire[number] == Tan[6 °] yWire[number] + x0forWireMiddles[number],
    {yWire[number], -0.1, 0.1}, {xWire[number], 1.65, 1.7},
    FrameLabel → {"y(meters)", "x(meters)"},
    ContourStyle → {Dashing[Large]}], {number, 109, 110}],
  Table[ContourPlot[xWire[number2] == Tan[6 °] yWire[number2] + x0forWires[number2],
    {yWire[number2], -0.1, 0.1}, {xWire[number2], 1.65, 1.7},
    FrameLabel → {"y(meters)", "x(meters)"}],
   {number2, 109, 109}], bottom, right, left, ellipse40]
```

Out[355]=



In[356]:=
```
Degree40LineRight = Sort[Table[
   {ϕ /. Solve[Sqrt[a_2^2 (1 - y_2^2/b_2^2)] - Δa_2 == Tan[6 °] y_2 + x0forWireMiddles[number] &&
      ϕ > .01 && ϕ < 30, ϕ]}, {number, 73, 109}]]
```

$$\left\{\phi \;/.\; \text{Solve}\left[\sqrt{a\_2^2\left(1-\frac{y\_2^2}{b\_2^2}\right)} - \Delta a\_2 == \text{Tan}[6\,°]\,y\_2 + \text{x0forWireMiddles}[\text{number}] \;\&\&\; \phi > .01 \;\&\&\; \phi < 30,\ \phi\right]\right\}$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

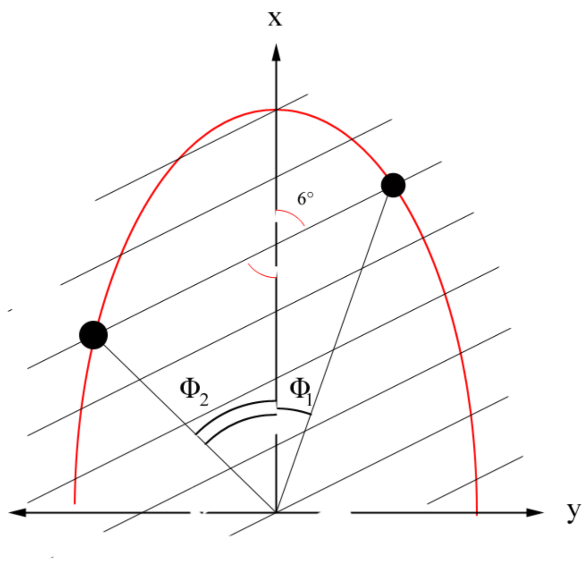
Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General: Further output of Solve::ratnz will be suppressed during this calculation.

Out[356]=
```
{{{2.64046}}, {{4.46211}}, {{5.88012}}, {{7.08111}}, {{8.14077}},
 {{9.09882}}, {{9.97917}}, {{10.7975}}, {{11.5649}}, {{12.2894}},
 {{12.9771}}, {{13.6327}}, {{14.2601}}, {{14.8624}}, {{15.442}},
 {{16.0012}}, {{16.5418}}, {{17.0652}}, {{17.573}}, {{18.0661}},
 {{18.5457}}, {{19.0127}}, {{19.4678}}, {{19.9118}}, {{20.3454}},
 {{20.769}}, {{21.1833}}, {{21.5888}}, {{21.9858}}, {{22.3748}}, {{22.7561}},
 {{23.1301}}, {{23.497}}, {{23.8573}}, {{24.2111}}, {{24.5587}}, {{24.9003}}}
```

The fact the equation for the y component in the plane of the detector uses a square root function, we know there must be two solutions. Due to the use of inverse trigonometric functions within Mathematica, only the positive values of y will be used.



By symmetry, we know that the positions within the 2nd quadrant can be reflected into the 1st quadrant by taking the opposite slope of the wire function

In[357]:= Solve[$\sqrt{a\_2^2 \left(1 - \frac{y^2}{b\_2^2}\right)}$ - Δa_2 == Tan[6 °] y + x0forWireMiddles[109] , y]

Out[357]= {{y → -0.186361}, {y → 0.0702975}}

In[358]:= Solve[$\sqrt{a\_2^2 \left(1 - \frac{y^2}{b\_2^2}\right)}$ - Δa_2 == -Tan[6 °] y + x0forWireMiddles[109] , y]

Out[358]= {{y → -0.0702975}, {y → 0.186361}}

In[359]:= Degree40LineLeft = Sort[Table[

$$\left\{\phi\ /.\ \text{Solve}\left[\sqrt{a\_2^2\left(1-\frac{y\_2^2}{b\_2^2}\right)}-\Delta a\_2 == -\text{Tan}[6\,°]\ y\_2+\text{x0forWireMiddles}[\text{number}]\ \&\&\ \phi>.01\ \&\&\ \phi<30,\ \phi\right]\right\},\ \{\text{number},\ 84,\ 109\}\Big]\Big]$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General: Further output of Solve::ratnz will be suppressed during this calculation.

```
Out[359]= {{{7.00229}}, {{8.80492}}, {{10.2039}}, {{11.3859}}, {{12.4265}},
 {{13.3654}}, {{14.2267}}, {{15.0259}}, {{15.7742}}, {{16.4795}},
 {{17.1481}}, {{17.7845}}, {{18.3927}}, {{18.9757}}, {{19.5361}},
 {{20.0761}}, {{20.5974}}, {{21.1015}}, {{21.59}}, {{22.0638}}, {{22.524}},
 {{22.9716}}, {{23.4074}}, {{23.832}}, {{24.2461}}, {{24.6503}}}
```

```
In[360]:= Degree40LineLeft = -1 Degree40LineLeft
```

```
Out[360]= {{{-7.00229}}, {{-8.80492}}, {{-10.2039}}, {{-11.3859}}, {{-12.4265}},
 {{-13.3654}}, {{-14.2267}}, {{-15.0259}}, {{-15.7742}}, {{-16.4795}},
 {{-17.1481}}, {{-17.7845}}, {{-18.3927}}, {{-18.9757}}, {{-19.5361}},
 {{-20.0761}}, {{-20.5974}}, {{-21.1015}}, {{-21.59}}, {{-22.0638}}, {{-22.524}},
 {{-22.9716}}, {{-23.4074}}, {{-23.832}}, {{-24.2461}}, {{-24.6503}}}
```

```
In[361]:= Degree40LineLeft
```

```
Out[361]= {{{-7.00229}}, {{-8.80492}}, {{-10.2039}}, {{-11.3859}}, {{-12.4265}},
 {{-13.3654}}, {{-14.2267}}, {{-15.0259}}, {{-15.7742}}, {{-16.4795}},
 {{-17.1481}}, {{-17.7845}}, {{-18.3927}}, {{-18.9757}}, {{-19.5361}},
 {{-20.0761}}, {{-20.5974}}, {{-21.1015}}, {{-21.59}}, {{-22.0638}}, {{-22.524}},
 {{-22.9716}}, {{-23.4074}}, {{-23.832}}, {{-24.2461}}, {{-24.6503}}}
```

In[362]:=
```
Degree40Line = Union[Degree40LineLeft, Degree40LineRight]
```

Out[362]=
```
{{{-24.6503}}, {{-24.2461}}, {{-23.832}}, {{-23.4074}}, {{-22.9716}}, {{-22.524}},
 {{-22.0638}}, {{-21.59}}, {{-21.1015}}, {{-20.5974}}, {{-20.0761}}, {{-19.5361}},
 {{-18.9757}}, {{-18.3927}}, {{-17.7845}}, {{-17.1481}}, {{-16.4795}},
 {{-15.7742}}, {{-15.0259}}, {{-14.2267}}, {{-13.3654}}, {{-12.4265}},
 {{-11.3859}}, {{-10.2039}}, {{-8.80492}}, {{-7.00229}}, {{2.64046}},
 {{4.46211}}, {{5.88012}}, {{7.08111}}, {{8.14077}}, {{9.09882}}, {{9.97917}},
 {{10.7975}}, {{11.5649}}, {{12.2894}}, {{12.9771}}, {{13.6327}}, {{14.2601}},
 {{14.8624}}, {{15.442}}, {{16.0012}}, {{16.5418}}, {{17.0652}}, {{17.573}},
 {{18.0661}}, {{18.5457}}, {{19.0127}}, {{19.4678}}, {{19.9118}}, {{20.3454}},
 {{20.769}}, {{21.1833}}, {{21.5888}}, {{21.9858}}, {{22.3748}}, {{22.7561}},
 {{23.1301}}, {{23.497}}, {{23.8573}}, {{24.2111}}, {{24.5587}}, {{24.9003}}}
```

In[363]:=
```
TableOfValues40a =
  Prepend[Replace[Degree40Line, {x_List} :→ x, {0, -1}], {"ϕ(degrees)"}];
```

We can add the number of successive crossings

In[499]:=
```
TableOfValues40b =
  MapThread[Prepend, {TableOfValues40a, {"Crossing", -26, -25, -24, -23, -22, -21,
     -20, -19, -18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5,
     -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37}}];
```

In[623]:=
```
TableOfValues40c =
  MapThread[Append, {TableOfValues40a, {"Wire Number", 84, 85, 86, 87, 88, 89, 90, 91,
     92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109,
     109, 108, 107, 106, 105, 104, 103, 102, 101, 100, 99, 98, 97, 96, 95, 94, 93, 92,
     91, 90, 89, 88, 87, 86, 85, 84, 83, 82, 81, 80, 79, 78, 77, 76, 75, 74, 73}}];
```

In[677]:=
```
Table40degree = Labeled[Grid[TableOfValues40b, Frame → All],
  "Crossing Number as a function of ϕ", Top]
```

Crossing Number as a function of ϕ

Crossing	ϕ(degrees)
-26	-24.6503
-25	-24.2461
-24	-23.832
-23	-23.4074
-22	-22.9716
-21	-22.524
-20	-22.0638
-19	-21.59
-18	-21.1015
-17	-20.5974
-16	-20.0761
-15	-19.5361
-14	-18.9757
-13	-18.3927
-12	-17.7845
-11	-17.1481

Out[677]=

-10	-16.4795
-9	-15.7742
-8	-15.0259
-7	-14.2267
-6	-13.3654
-5	-12.4265
-4	-11.3859
-3	-10.2039
-2	-8.80492
-1	-7.00229
1	2.64046
2	4.46211
3	5.88012
4	7.08111
5	8.14077
6	9.09882
7	9.97917
8	10.7975
9	11.5649
10	12.2894
11	12.9771
12	13.6327
13	14.2601
14	14.8624
15	15.442
16	16.0012
17	16.5418
18	17.0652
19	17.573
20	18.0661
21	18.5457
22	19.0127
23	19.4678
24	19.9118
25	20.3454
26	20.769
27	21.1833
28	21.5888
29	21.9858
30	22.3748
31	22.7561
32	23.1301
33	23.497
34	23.8573
35	24.2111
36	24.5587
37	24.9003

In[678]:=
```
Table40degreeWire = Labeled[Grid[TableOfValues40c, Frame → All],
  "Midpoint Wire Number as a function of ϕ", Top]
```

Midpoint Wire Number as a function of ϕ

ϕ (degrees)	Wire Number
-24.6503	84
-24.2461	85
-23.832	86

Out[678]=

−23.4074	87
−22.9716	88
−22.524	89
−22.0638	90
−21.59	91
−21.1015	92
−20.5974	93
−20.0761	94
−19.5361	95
−18.9757	96
−18.3927	97
−17.7845	98
−17.1481	99
−16.4795	100
−15.7742	101
−15.0259	102
−14.2267	103
−13.3654	104
−12.4265	105
−11.3859	106
−10.2039	107
−8.80492	108
−7.00229	109
2.64046	109
4.46211	108
5.88012	107
7.08111	106
8.14077	105
9.09882	104
9.97917	103
10.7975	102
11.5649	101
12.2894	100
12.9771	99
13.6327	98
14.2601	97
14.8624	96
15.442	95
16.0012	94
16.5418	93
17.0652	92
17.573	91
18.0661	90
18.5457	89
19.0127	88
19.4678	87
19.9118	86
20.3454	85
20.769	84
21.1833	83
21.5888	82
21.9858	81
22.3748	80
22.7561	79
23.1301	78
23.497	77
23.8573	76

23.8573	76
24.2111	75
24.5587	74
24.9003	73

As was done for the θ=40° line, we can do the same for θ=20°

The θ = 20 ° elliptical path hits the right wall at ϕ=24.5611°, hence no further wires will be crossed after this angle

In[366]:= $\text{Solve}\left[\sqrt{a\_1^2\left(1-\frac{y\_1^2}{b\_1^2}\right)} - \Delta a\_1 == \text{Cot}[29.5\,°]\; y\_1 + .09156\;\&\&\;\phi > .1\;\&\&\;\phi < 30,\; \phi\right]$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[366]= {{ϕ → 24.5611}}

In[560]:= $\text{Degree20LineRight} = \text{Sort}\Big[\text{Table}\Big[\Big\{\phi\;/.\;\text{NSolve}\Big[\sqrt{a\_1^2\left(1-\frac{y\_1^2}{b\_1^2}\right)} - \Delta a\_1 == \text{Tan}[6\,°]\; y\_1 + \text{x0forWireMiddles}[\text{number}]\;\&\&\;\phi > .01\;\&\&\;\phi < 30\;,\;\phi\Big]\Big\},\;\{\text{number}, 33, 48\}\Big]\Big]$

Out[560]= {{{4.51922}}, {{7.21621}}, {{9.30026}}, {{11.0575}}, {{12.6025}}, {{13.9946}}, {{15.2697}}, {{16.4512}}, {{17.5554}}, {{18.5943}}, {{19.577}}, {{20.5105}}, {{21.4003}}, {{22.2512}}, {{23.0667}}, {{23.8501}}}

In[586]:= $\phi\;/.\;\text{Solve}\left[\sqrt{a\_1^2\left(1-\frac{y\_1^2}{b\_1^2}\right)} - \Delta a\_1 == -\text{Tan}[6\,°]\; y\_1 + \text{x0forWireMiddles}[49]\;\&\&\;\phi > .1\;\&\&\;\phi < 30\;,\;\phi\right]$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[586]= {0.595741, 4.24005}

In[593]:=
```
Degree20LineLeft = Sort[Table[
  {ϕ /. Solve[Sqrt[a_1^2 (1 - y_1^2/b_1^2)] - Δa_1 == -Tan[6 °] y_1 + x0forWireMiddles[number] &&
      ϕ > .1 && ϕ < 30 , ϕ]}, {number, 38, 48}]]
```

$$\left\{\phi\ /.\ \text{Solve}\left[\sqrt{a\_1^2\left(1-\frac{y\_1^2}{b\_1^2}\right)}-\Delta a\_1 == -\text{Tan}[6\,°]\,y\_1+\text{x0forWireMiddles}[\text{number}]\ \&\&\ \phi>.1\ \&\&\ \phi<30,\ \phi\right]\right\},\ \{\text{number},38,48\}$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General: Further output of Solve::ratnz will be suppressed during this calculation.

Out[593]=
```
{{{9.31161}}, {{11.9648}}, {{14.0045}}, {{15.7171}}, {{17.2168}},
 {{18.5633}}, {{19.7922}}, {{20.9271}}, {{21.9843}}, {{22.9756}}, {{23.9103}}}
```

In[594]:=
```
Degree20LineLeft = Prepend[Degree20LineLeft, {{4.240053353652899`}}]
```

Out[594]=
```
{{{4.24005}}, {{9.31161}}, {{11.9648}}, {{14.0045}}, {{15.7171}}, {{17.2168}},
 {{18.5633}}, {{19.7922}}, {{20.9271}}, {{21.9843}}, {{22.9756}}, {{23.9103}}}
```

In[595]:=
```
Degree20LineLeft = Prepend[Degree20LineLeft, {{0.5957412339502922`}}]
```

Out[595]=
```
{{{0.595741}}, {{4.24005}}, {{9.31161}}, {{11.9648}},
 {{14.0045}}, {{15.7171}}, {{17.2168}}, {{18.5633}},
 {{19.7922}}, {{20.9271}}, {{21.9843}}, {{22.9756}}, {{23.9103}}}
```

In[596]:=
```
Degree20LineLeft = -1 * Degree20LineLeft;
```

In[597]:=
```
Degree20Line = Union[Degree20LineLeft, Degree20LineRight];
```

In[598]:=
```
TableOfValues20a =
 Prepend[Replace[Degree20Line, {x_List} :> x, {0, -1}], {"ϕ(degrees)"}]
```

Out[598]=
```
{{ϕ(degrees)}, {-23.9103}, {-22.9756}, {-21.9843}, {-20.9271},
 {-19.7922}, {-18.5633}, {-17.2168}, {-15.7171}, {-14.0045}, {-11.9648},
 {-9.31161}, {-4.24005}, {-0.595741}, {4.51922}, {7.21621}, {9.30026},
 {11.0575}, {12.6025}, {13.9946}, {15.2697}, {16.4512}, {17.5554},
 {18.5943}, {19.577}, {20.5105}, {21.4003}, {22.2512}, {23.0667}, {23.8501}}
```

```
In[609]:= TableOfValues20b =
  MapThread[Prepend, {TableOfValues20a, {"Crossing", -13, -12, -11, -10, -9, -8, -7,
     -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}}]

Out[609]= {{Crossing, ϕ(degrees)}, {-13, -23.9103}, {-12, -22.9756}, {-11, -21.9843},
  {-10, -20.9271}, {-9, -19.7922}, {-8, -18.5633}, {-7, -17.2168}, {-6, -15.7171},
  {-5, -14.0045}, {-4, -11.9648}, {-3, -9.31161}, {-2, -4.24005}, {-1, -0.595741},
  {1, 4.51922}, {2, 7.21621}, {3, 9.30026}, {4, 11.0575}, {5, 12.6025}, {6, 13.9946},
  {7, 15.2697}, {8, 16.4512}, {9, 17.5554}, {10, 18.5943}, {11, 19.577},
  {12, 20.5105}, {13, 21.4003}, {14, 22.2512}, {15, 23.0667}, {16, 23.8501}}

In[618]:= TableOfValues20c =
  MapThread[Append, {TableOfValues20a, {"Wire Number", 38, 39, 40, 41, 42, 43, 44, 45,
     46, 47, 48, 49, 49, 48, 47, 46, 45, 44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33}}]

Out[618]= {{ϕ(degrees), Wire Number}, {-23.9103, 38}, {-22.9756, 39},
  {-21.9843, 40}, {-20.9271, 41}, {-19.7922, 42}, {-18.5633, 43},
  {-17.2168, 44}, {-15.7171, 45}, {-14.0045, 46}, {-11.9648, 47},
  {-9.31161, 48}, {-4.24005, 49}, {-0.595741, 49}, {4.51922, 48},
  {7.21621, 47}, {9.30026, 46}, {11.0575, 45}, {12.6025, 44}, {13.9946, 43},
  {15.2697, 42}, {16.4512, 41}, {17.5554, 40}, {18.5943, 39}, {19.577, 38},
  {20.5105, 37}, {21.4003, 36}, {22.2512, 35}, {23.0667, 34}, {23.8501, 33}}
```

In[679]:=
```
Table20degree = Labeled[Grid[TableOfValues20b, Frame → All],
  "Crossing Number as a function of ϕ", Top]
```

Out[679]=

Crossing Number as a function of ϕ

Crossing	ϕ (degrees)
-13	-23.9103
-12	-22.9756
-11	-21.9843
-10	-20.9271
-9	-19.7922
-8	-18.5633
-7	-17.2168
-6	-15.7171
-5	-14.0045
-4	-11.9648
-3	-9.31161
-2	-4.24005
-1	-0.595741
1	4.51922
2	7.21621
3	9.30026
4	11.0575
5	12.6025
6	13.9946
7	15.2697
8	16.4512
9	17.5554
10	18.5943
11	19.577
12	20.5105
13	21.4003
14	22.2512
15	23.0667
16	23.8501

In[680]:=
```
Table20degreeWires =
 Labeled[Grid[TableOfValues20c, Frame → All], "Wire Number as a function of ϕ", Top]
```

Out[680]=

Wire Number as a function of ϕ

ϕ (degrees)	Wire Number
-23.9103	38
-22.9756	39
-21.9843	40
-20.9271	41
-19.7922	42
-18.5633	43
-17.2168	44
-15.7171	45
-14.0045	46
-11.9648	47
-9.31161	48
-4.24005	49
-0.595741	49
4.51922	48
7.21621	47
9.30026	46
11.0575	45
12.6025	44
13.9946	43
15.2697	42
16.4512	41
17.5554	40
18.5943	39
19.577	38
20.5105	37
21.4003	36
22.2512	35
23.0667	34
23.8501	33

As was done for the θ=20° line, we can do the same for θ=10°

The θ = 10 ° elliptical path hits the right wall at ϕ=22.3762°, hence no further wires will be crossed after this angle

In[487]:=

$$\text{Solve}\left[\sqrt{a\_3^2\left(1-\frac{y\_3^2}{b\_3^2}\right)} - \Delta a\_3 == \text{Cot}[29.5\,°]\; y\_3 + .09156\ \&\&\ \phi > .1\ \&\&\ \phi < 30,\ \phi\right]$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[487]= {{ϕ → 22.3762}}

In[536]:=
```
Degree10LineRight = Sort[Table[
   {ϕ /. Solve[Sqrt[a_3^2 (1 - y_3^2/b_3^2)] - Δa_3 == Tan[6 °] y_3 + x0forWireMiddles[number] &&
       ϕ > .01 && ϕ < 30 , ϕ]}, {number, 11, 18}]]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General: Further output of Solve::ratnz will be suppressed during this calculation.

Out[536]=
```
{{{0.045343}}, {{7.07936}}, {{10.7465}}, {{13.5653}},
 {{15.9292}}, {{17.9949}}, {{19.8442}}, {{21.5257}}}
```

In[644]:=
```
Degree10LineLeft = Sort[Table[
   {ϕ /. Solve[Sqrt[a_3^2 (1 - y_3^2/b_3^2)] - Δa_3 == -Tan[6 °] y_3 + x0forWireMiddles[number] &&
       ϕ > .001 && ϕ < 30 , ϕ]}, {number, 13, 18}]]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General: Further output of Solve::ratnz will be suppressed during this calculation.

Out[644]=
```
{{{4.75099}}, {{11.7028}}, {{15.2855}}, {{18.0175}}, {{20.2921}}, {{22.2662}}}
```

In[537]:=
```
Degree10LineLeft = -1 Degree10LineLeft
```

Out[537]=
```
{{{-4.75099}}, {{-11.7028}}, {{-15.2855}},
 {{-18.0175}}, {{-20.2921}}, {{-22.2662}}}
```

In[538]:=
```
Degree10Line = Union[Degree10LineLeft, Degree10LineRight]
```

Out[538]=
```
{{{-22.2662}}, {{-20.2921}}, {{-18.0175}}, {{-15.2855}},
 {{-11.7028}}, {{-4.75099}}, {{0.045343}}, {{7.07936}}, {{10.7465}},
 {{13.5653}}, {{15.9292}}, {{17.9949}}, {{19.8442}}, {{21.5257}}}
```

In[539]:=
```
TableOfValues10a =
 Prepend[Replace[Degree10Line, {x_List} :> x, {0, -1}], {"ϕ(degrees)"}]
```

Out[539]=
```
{{ϕ(degrees)}, {-22.2662}, {-20.2921}, {-18.0175},
 {-15.2855}, {-11.7028}, {-4.75099}, {0.045343}, {7.07936},
 {10.7465}, {13.5653}, {15.9292}, {17.9949}, {19.8442}, {21.5257}}
```

In[541]:=
```
TableOfValues10b = MapThread[Prepend,
  {TableOfValues10a, {"Crossing", -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8}}]
```

Out[541]=
```
{{Crossing, ϕ(degrees)}, {-6, -22.2662}, {-5, -20.2921}, {-4, -18.0175},
 {-3, -15.2855}, {-2, -11.7028}, {-1, -4.75099}, {1, 0.045343}, {2, 7.07936},
 {3, 10.7465}, {4, 13.5653}, {5, 15.9292}, {6, 17.9949}, {7, 19.8442}, {8, 21.5257}}
```

In[621]:=
```
TableOfValues10c = MapThread[Append, {TableOfValues10a,
   {"Wire Number", 13, 14, 15, 16, 17, 18, 18, 17, 16, 15, 14, 13, 12, 11}}]
```

Out[621]=
```
{{ϕ(degrees), Wire Number}, {-22.2662, 13}, {-20.2921, 14},
 {-18.0175, 15}, {-15.2855, 16}, {-11.7028, 17}, {-4.75099, 18},
 {0.045343, 18}, {7.07936, 17}, {10.7465, 16}, {13.5653, 15},
 {15.9292, 14}, {17.9949, 13}, {19.8442, 12}, {21.5257, 11}}
```

In[681]:=
```
Table10degree = Labeled[Grid[TableOfValues10b, Frame → All],
  "Crossing number as a function of ϕ", Top]
```

Out[681]=

Crossing number as a function of ϕ

Crossing	ϕ(degrees)
-6	-22.2662
-5	-20.2921
-4	-18.0175
-3	-15.2855
-2	-11.7028
-1	-4.75099
1	0.045343
2	7.07936
3	10.7465
4	13.5653
5	15.9292
6	17.9949
7	19.8442
8	21.5257

In[682]:=
```
Table10degreeWires =
 Labeled[Grid[TableOfValues10c, Frame → All], "Wire Number as a function of ϕ", Top]
```

Out[682]=

Wire Number as a function of ϕ

ϕ (degrees)	Wire Number
-22.2662	13
-20.2921	14
-18.0175	15
-15.2855	16
-11.7028	17
-4.75099	18
0.045343	18
7.07936	17
10.7465	16
13.5653	15
15.9292	14
17.9949	13
19.8442	12
21.5257	11

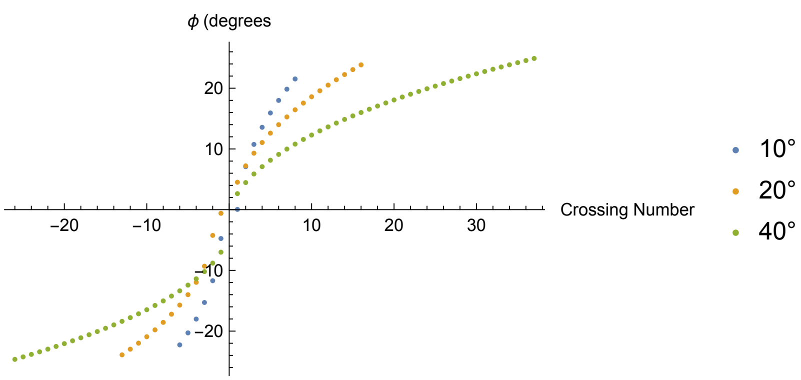
Using the points where the elliptical path cross wires measured in ϕ degrees

In[625]:=
```
CrossingData10 = Drop[TableOfValues10b, 1];
CrossingData20 = Drop[TableOfValues20b, 1];
CrossingData40 = Drop[TableOfValues40b, 1];
```

In[628]:=
```
part1 = ListLinePlot[CrossingData10];
part2 = ListLinePlot[CrossingData20];
part3 = ListLinePlot[CrossingData40];
```

In[648]:=
```
ListPlot[{data10, data20, data40},
 AxesLabel → {"Crossing Number", "ϕ (degrees"}, PlotLegends → {"10°", "20°", "40°"}]
```

Out[648]=



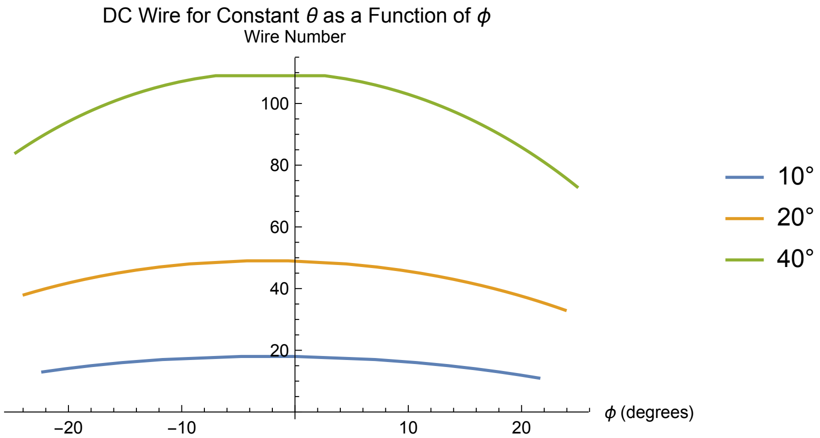
In[637]:=
```
TableOfValues10c
```

Out[637]=
```
{{ϕ(degrees), Wire Number}, {-22.2662, 13}, {-20.2921, 14},
 {-18.0175, 15}, {-15.2855, 16}, {-11.7028, 17}, {-4.75099, 18},
 {0.045343, 18}, {7.07936, 17}, {10.7465, 16}, {13.5653, 15},
 {15.9292, 14}, {17.9949, 13}, {19.8442, 12}, {21.5257, 11}}
```

In[632]:=
```
WireData10 = Drop[TableOfValues10c, 1];
WireData20 = Drop[TableOfValues20c, 1];
WireData40 = Drop[TableOfValues40c, 1];
```

In[649]:=
```
ListLinePlot[{WireData10, WireData20, WireData40},
 AxesLabel → {"ϕ (degrees)", "Wire Number"},
 PlotLabel → "DC Wire for Constant θ as a Function of ϕ",
 PlotLegends → {"10°", "20°", "40°"}]
```

Out[649]=



We can find an equation that gives the wire midpoint crossing as a function of ϕ

In[675]:=
```
parabola10degree = Fit[WireData10, {1, ϕ, ϕ^2}, ϕ];
```

In[674]:=
```
parabola20degree = Fit[WireData20, {1, ϕ, ϕ^2}, ϕ];
```

In[673]:=
```
parabola40degree = Fit[WireData40, {1, ϕ, ϕ^2}, ϕ];
```

In[672]:=
```
Show[ListPlot[{WireData10, WireData20, WireData40},
  PlotStyle → Black, AxesLabel → {"ϕ (degrees)", "Wire Number"},
  PlotLabel → "DC Wire for Constant θ as a Function of ϕ"],
 Plot[{parabola10degree, parabola20degree, parabola40degree}, {ϕ, -25, 25},
  PlotLegends → {parabola10degree, parabola20degree, parabola40degree}]]
```

Out[672]=